Mathematics

(For 8th Class)

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ਸਮੱਗਰ ਸਿੱਖਿਆ ਅਭਿਆਨ

ਪੜ੍ਹੋ ਸਾਰੇ ਵਧੋ ਸਾਰੇ

ਸਿੱਖਿਆ ਅਤੇ ਭਲਾਈ ਵਿਭਾਗ, ਪੰਜਾਬ ਦਾ ਸਾਂਝਾ ਉਪਰਾਲਾ

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Co-ordinator : **Pritpal Singh Kathuria**
Subject Expert, P.S.Ed.B, Mohali

Cover design : **Manjit Singh Dhillon**
Artist, P.S.Ed.B, Mohali

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FOREWORD

The Punjab School Education Board has been continuously engaged in developing syllabi, producing and renewing text books according to the changing educational needs at the state and national level.

This book has been developed in accordance to the guidelines of National Curriculum Framework (NCF) 2005 and PCF 2013, after careful deliberations in workshops involving experienced teachers and experts from the board and field as well. All efforts have been made to make this book interesting with the help of activities and coloured figures. This book has been prepared with the joint efforts of subject experts of Board, SCERT and experienced teachers/experts of mathematics. Board is thankful to all of them.

The authors have tried their best to ensure that the treatment, presentation and style of the book in hand are in accordance with the mental level of the students of classs VIII. The topics, contents and examples in the book have been framed in accordance with the situations existing in the young learner's environment. A number of activities have been suggested in every lesson. These may be modified keeping in view the availability of local resources and real life situations of the learners.

I hope the students will find this book very useful and interesting. The Board will be graceful for suggestions from the field for further improvement of the book.

Chairman

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Text-book Development Committee

WRITERS

- Arun Kumar Garg, G.M.S.S.S. Datewas, Mansa
- Parvinder Singh G.S.S.S. Churiwala Dhanna, Fazilka
- Vikram Sethi G.S.S. Smart S Karni Khera Fazilka

VETTERS

- Jatinder Kumar G.S.S.S Pakkan Kalan, Bathinda.
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- Vikas Julka, G.H.S. Dharamgarh (Lalru) Mohali
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CHAPTER

1

Rational Numbers

Learning Objectives

In this chapter you will learn:

- *About number system.*
- *To apply different operations : Addition, Subtraction, Multiplication and Division on rational numbers.*
- *About the properties of rational numbers under different operations.*

1.1 Introduction

In earlier classes, we have studied about **counting numbers** or **natural numbers** i.e. 1, 2, 3, 4 .... By including 0 to natural numbers, we get **whole numbers** i.e. 0, 1, 2, 3, 4 .... The negative of natural numbers and whole numbers, when put together, we get **integers** i.e. .... –4, –3, –2,–1, 0, 1, 2, 3, ...

Fundamental operations i.e. **addition, subtraction, multiplication and division** were defined on integers and various properties of these operations were also discussed in previous class. The concept of rational numbers was introduced and fundamental operations on rational numbers were also discussed. In this chapter, we shall learn about different properties of these operations on rational numbers.

Let us first recall about rational numbers.

1.2 Rational Numbers:

A number of the form $\frac{p}{q}$ or a number which can be expressed in the form $\left(\frac{p}{q}\right)$, where p and q are integers, $q \neq 0$ and p, q are co-prime is called a **rational number** e.g. $\frac{-2}{5}, \frac{10}{7}, \frac{-3}{8}$, –3, 0 etc.

- **All natural numbers are rational numbers.**
- **All whole numbers are rational numbers.**
- **All integers are rational numbers.**

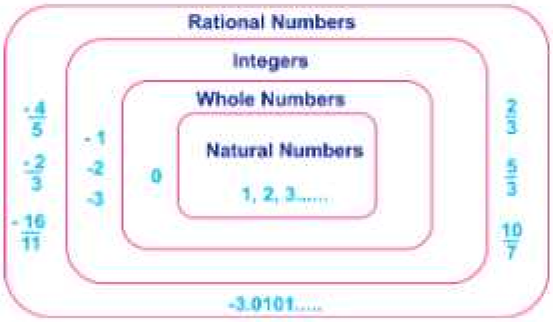


Fig 1.1

1.2.1 Addition of Rational Numbers

In previous class, we have defined the addition of rational numbers. We know if two rational numbers are to be added, then first we have to express each of them as rational number with same and positive denominator (by taking LCM), then we solve.

Let us discuss some examples.

Example 1.1 **Solve the following:**

(i) $\mathbf{\frac{2}{7}+\frac{3}{7}}$ (ii) $\mathbf{\frac{5}{9}+\left(\frac{-1}{9}\right)}$ (iii) $\mathbf{\frac{-3}{11}+\frac{6}{11}}$

(iv) $\mathbf{\frac{5}{-11}+\frac{7}{11}}$ (v) $\mathbf{\frac{-4}{11}+\frac{-3}{11}}$

Solution : **(i)**

$$\frac{2}{7}+\frac{3}{7} = \frac{2+3}{7}$$

$$= \frac{5}{7}$$

(ii)

$$\frac{5}{9}+\left(\frac{-1}{9}\right) = \frac{5}{9}-\frac{1}{9}$$

$$= \frac{5-1}{9}$$

$$= \frac{4}{9}$$

(iii)

$$\frac{-3}{11}+\frac{6}{11} = \frac{-3+6}{11}$$

$$= \frac{3}{11}$$

(iv)

$$\frac{5}{-11}+\frac{7}{11} = \frac{-5}{11}+\frac{7}{11}$$

$$= \frac{-5+7}{11}$$

$$= \frac{2}{11}$$

(v)

$$\frac{-4}{11}+\frac{-3}{11} = \frac{-4}{11}-\frac{3}{11}$$

$$= \frac{-4-3}{11}$$

$$= \frac{-7}{11}$$

Example 1.2 **Solve the following:**

(i) $\frac{5}{12}+\frac{3}{4}$ (ii) $\frac{-3}{8}+\frac{5}{6}$ (iii) $\frac{3}{10}+\frac{-2}{5}$

(iv) $\frac{-5}{8}+\frac{-7}{12}$ (v) $\frac{-7}{15}+\frac{-3}{20}$

Solution : **In these questions, first of all we will make denominator same by taking LCM of denominators.**

(i) $\frac{5}{12}+\frac{3}{4}$

LCM of denominators 12, 4

2	12, 4
2	6, 2
	3, 1

LCM of 12 & 4 = 2 × 2 × 3 =12

Now we express $\frac{5}{12}$ and $\frac{3}{4}$ as rational numbers with same denominators by taking LCM.

Now $\frac{3}{4} = \frac{3\times3}{4\times3}$

$= \frac{9}{12}$

So $\frac{5}{12}+\frac{3}{4} = \frac{5}{12}+\frac{9}{12}$

$= \frac{5+9}{12}$

$= \frac{\overset{7}{\cancel{14}}}{\underset{6}{\cancel{12}}}$

$= \frac{7}{6}$

Or $\frac{5}{12}+\frac{3}{4} = \frac{(5\times1)+(3\times3)}{12}$

12 ÷12 =1

12 ÷ 4 = 3

$= \frac{5+9}{12}$

$= \frac{14}{12}$

$= \frac{7}{6}$

(ii) **We have,** $\frac{-3}{8}+\frac{5}{6}$

Now, we express $\frac{-3}{8}$ and $\frac{5}{6}$ as rational numbres with same denominators by taking LCM.

LCM of denominators 8, 6

2	8, 6
2	4, 3
	2, 3

LCM of 8 & 6 = 2 × 2 × 2 × 3 = 24

We have, $\frac{-3}{8}=\frac{-3\times3}{8\times3}=\frac{-9}{24}$

and $\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$

$$\therefore \quad \frac{-3}{8}+\frac{5}{6} = \frac{-9}{24}+\frac{20}{24}$$

$$= \frac{-9+20}{24}$$

$$= \frac{11}{24}$$

Or

$$\frac{-3}{8}+\frac{5}{6} = \frac{(-3\times 3)+(5\times 4)}{24}$$

$$= \frac{-9+20}{24}$$

$$= \frac{11}{24}$$

24 ÷ 8 = 3
24 ÷ 6 = 4

(iii) **We have,** $\frac{3}{10}+\frac{-2}{5}$

LCM of denominators 10, 5

5	10, 5
	2, 1

LCM of 10 & 5 = 5 × 2 = 10

Now we express $\frac{3}{10}$ **and** $\frac{-2}{5}$ **as rational numbers with same denomiantors by taking LCM.**

We have, $\frac{3}{10}$ **and** $\frac{-2}{5} = \frac{-2\times 2}{5\times 2} = \frac{-4}{10}$

$$\frac{3}{10}+\frac{-2}{5}=\frac{3}{10}+\frac{-4}{10}$$

$$= \frac{3}{10}-\frac{4}{10}$$

$$= \frac{3-4}{10}$$

$$= \frac{-1}{10}$$

Or $\frac{3}{10}+\frac{-2}{5} = \frac{(3\times 1)+(-2\times 2)}{10}$

$$= \frac{3+(-4)}{10}$$

$$= \frac{3-4}{10}$$

$$= \frac{-1}{10}$$

(iv) **We have,** $\frac{-5}{8}+\frac{-7}{12}$

LCM of denominators 8, 12

2	8, 12
2	4, 6
	2, 3

LCM 8 & 12 = 2 × 2 × 2 × 3 = 24

Now, we express $\frac{-5}{8}$ **and** $\frac{-7}{12}$ **as rational numbers with same denominantors by taking LCM.**

We have, $\frac{-5}{8} = \frac{-5\times 3}{8\times 3} = \frac{-15}{24}$

and $\frac{-7}{12} = \frac{-7\times 2}{12\times 2} = \frac{-14}{24}$

$$\therefore \quad \frac{-5}{8}+\frac{-7}{12} = \frac{-15}{24}+\frac{-14}{24}$$

$$= \frac{(-15)+(-14)}{24}$$

$$= \frac{-15-14}{24}$$

$$= \frac{-29}{24}$$

Or $\frac{-5}{8}+\frac{-7}{12} = \frac{-5}{8}+\frac{-7}{12}$

$$= \frac{(-5\times3)+(-7\times2)}{24}$$

$$= \frac{-15-14}{24}$$

$$= \frac{-29}{24}$$

(iv) We have, $\frac{-7}{15}+\frac{-3}{20}$

Now, we express $\frac{-7}{15}$ **and** $\frac{-3}{20}$ **as rational numbers with same denominators by taking LCM.**

LCM of denominators 15 and 20

$$\begin{array}{r|l} 5 & 15,\ 20 \\ \hline & 3,\ 4 \end{array}$$

LCM of 15 & 20

$= 5 \times 3 \times 4 = 60$

We have, $\frac{-7}{15} = \frac{-7\times4}{15\times4} = \frac{-28}{60}$

and $\frac{-3}{20} = \frac{-3\times3}{20\times3} = \frac{-9}{60}$

$$\therefore \quad \frac{-7}{15}+\frac{-3}{20} = \frac{-28}{60}+\frac{-9}{60}$$

$$= \frac{-28+(-9)}{60}$$

$$= \frac{-28-9}{60}$$

$$= \frac{-37}{60}$$

Or $\frac{-7}{15}+\frac{-3}{20} = \frac{(-7\times4)+(-3\times3)}{60}$

$$= \frac{-28-9}{60}$$

$$= \frac{-37}{60}$$

1.2.2 Properties of Addition of Rational Numbers

In this section, we shall learn some basic properties of rational numbers under addition. These properties are similar to those of addition of integers which we have learnt in the previous classes.

- **Closure Property :** The sum of any two rational numbers is always a rational number. e.g. if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

For Example:

(i) $\frac{-2}{3}+\frac{4}{5}=\frac{-10+12}{15}=\frac{2}{15}$, which is a rational number.

(ii) $\frac{5}{8}+\frac{-3}{4}=\frac{5+(-6)}{8}=\frac{5-6}{8}=\frac{-1}{8}$, which is a rational number.

- **Commutative Property :** The two rational numbers can be added in any order, the result will be same. We say that addition is commutative for rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers.

then $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$

e.g. Consider the rational numbers.

$\frac{-3}{8}$ and $\frac{5}{6}$ then

$$\frac{-3}{8}+\frac{5}{6}=\frac{-9+20}{24}-\frac{11}{24}$$

and $\frac{5}{6}+\left(\frac{-3}{8}\right)=\frac{20+(-9)}{24}=\frac{20-9}{24}=\frac{11}{24}$

Thus $\frac{-3}{8}+\frac{5}{6}=\frac{5}{6}+\left(\frac{-3}{8}\right)$

- **Associative Property :** When three rational numbers are to be added by adding the first two rational numbers and then adding the third number in result or by adding the second and third rational numbers and then adding the first number in the result. We get the same result. Then we say that the addition of rational numbers is associative.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ (b, d and f$\neq 0$)

then $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

e.g. Consider three rational numbers $\frac{-1}{2}, \frac{3}{4}$ and $\frac{-5}{6}$ then

$$\left(\frac{-1}{2}+\frac{3}{4}\right)+\left(\frac{-5}{6}\right)=\left(\frac{-2+3}{4}\right)+\left(\frac{-5}{6}\right)=\frac{1}{4}+\left(\frac{-5}{6}\right)$$

$$= \frac{3+(-10)}{12} = \frac{3-10}{12} = \frac{-7}{12}$$

and $\frac{-1}{2} + \left[\frac{3}{4} + \left(\frac{-5}{6}\right)\right] = \frac{-1}{2} + \left(\frac{9+(-10)}{12}\right)$

$$= \frac{-1}{2} + \left(\frac{9-10}{12}\right)$$

$$= \frac{-1}{2} + \left(\frac{-1}{12}\right)$$

$$= \frac{(-1\times 6)+(-1\times 1)}{12}$$

$$= \frac{-6+(-1)}{12}$$

$$= \frac{-6-1}{12}$$

$$= \frac{-7}{12}$$

Thus, $\left(\frac{1}{2}+\frac{3}{4}\right)+\left(\frac{-5}{6}\right) = \frac{-1}{2}+\left(\frac{3}{4}+\frac{-5}{6}\right)$

- **Additive Identity :** When we add 0 to any rational number we get the same rational number. i.e. For any rational number $\frac{a}{b}$, $b \neq 0$ there exists a unique rational number 0 such that

  $\frac{a}{b}+0 = \frac{a}{b} = 0+\frac{a}{b}$

  then 0 is called the identity for the addition of rational numbers. We say '0' is the additive identity for rational numbers.

- **Additive Inverse:** When two rational numbers are added and give the result zero (0)(additive identity) then both rational numbers are called additive inverse of each other.

  i.e. For any rational number $\frac{a}{b}$ $(b \neq 0)$, there exists $\left(\frac{-a}{b}\right)$

  such that $\frac{a}{b}+\left(\frac{-a}{b}\right) = 0 = \left(\frac{-a}{b}\right) + \frac{a}{b}$

then $\frac{-a}{b}$ is additive inverse of $\frac{a}{b}$ and vice-versa.

e.g. $\frac{2}{3}+\left(\frac{-2}{3}\right) = \frac{2+(-2)}{3} = \frac{2-2}{3} = \frac{0}{3} = 0$

$\therefore \quad \frac{2}{3}+\left(\frac{-2}{3}\right) = 0$

And $\frac{-2}{3}+\frac{2}{3} = \frac{-2+2}{3}$

$= \frac{0}{3}$

$= 0$

So, $\frac{2}{3}+\left(\frac{-2}{3}\right) = 0 = \frac{-2}{3}+\frac{2}{3}$

Thus $\frac{-2}{3}$ is additive inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is additive inverse of $\frac{-2}{3}$.

Example 1.3 **Verify commutative property of addition of rational numbers for the following:**

(i) $\mathbf{\frac{-5}{12}}$ **and** $\mathbf{\frac{3}{8}}$ (ii) $\mathbf{\frac{2}{-7}}$ **and** $\mathbf{\frac{-11}{21}}$

Solution : (i) We have, $\frac{-5}{12}$ and $\frac{3}{8}$

Firstly $\frac{-5}{12}+\frac{3}{8} = \frac{(-5\times 2)+(3\times 3)}{24}$

$= \frac{-10+9}{24} - \frac{-1}{24}$

and $\frac{3}{8}+\left(\frac{-5}{12}\right) = \frac{(3\times 3)+(-5\times 2)}{24} = \frac{9+(-10)}{24} = \frac{9-10}{24}$

$= \frac{-1}{24}$

$\therefore \quad \frac{-5}{12}+\frac{3}{8} = \frac{3}{8}+\left(\frac{-5}{12}\right)$

Thus, commutative property under addition holds.

LCM of denominators 12, 8

$$\begin{array}{r|l} 2 & 12,\ 8 \\ \hline 2 & 6,\ 4 \\ \hline & 3,\ 2 \end{array}$$

LCM of 12 & 8

= 2 × 2 × 2 × 3 = 24

(ii) We have, $\frac{2}{-7}$ and $\frac{-11}{21}$

$$\text{Firstly}\quad \frac{2}{-7}+\left(\frac{-11}{21}\right) = \frac{-2}{7}+\left(\frac{-11}{21}\right)$$

$$=\frac{(-2\times3)+(-11\times1)}{21} = \frac{-6+(-11)}{21}$$

$$= \frac{-6-11}{21} = \frac{-17}{21}$$

LCM of denominators 7, 21

$$\begin{array}{r|l} 7 & 7,\ 21 \\ \hline & 1,\ 3 \end{array}$$

LCM of 7 & 21

= 7 × 3 = 21

$$\text{and}\quad \frac{-11}{21}+\frac{2}{-7} = \frac{-11}{21}+\frac{(-2)}{7}$$

$$=\frac{(-11\times1)+(-2\times3)}{21} = \frac{-11+(-6)}{21}=\frac{-11-6}{21} = \frac{-17}{21}$$

$$\therefore\quad \frac{2}{-7}+\left(\frac{-11}{21}\right) = \frac{-11}{21}+\frac{2}{-7}$$

Thus, commutative property under addition holds.

Example 1.4 Verify associative property of addition of rational numbers for the following:

(i) $\frac{5}{3},\frac{1}{6}$ **and** $\frac{-3}{5}$ **(ii)** $-4,\ \frac{3}{7}$ **and** $\frac{-4}{5}$

Solution : (i) We have, $\frac{5}{3},\frac{1}{6}$ and $\frac{-3}{5}$

$$\text{Firstly}\quad \left(\frac{5}{3}+\frac{1}{6}\right)+\left(\frac{-3}{5}\right) = \left[\frac{(5\times2)+(1\times1)}{6}\right]+\left(\frac{-3}{5}\right)$$

$$\begin{array}{r|l} 3 & 3,\ 6 \\ \hline & 1,\ 2 \end{array}$$

LCM of 3 & 6 = 3 × 2 = 6

$$= \left(\frac{10+1}{6}\right)+\left(\frac{-3}{5}\right)$$

$$= \frac{11}{6}+\left(\frac{-3}{5}\right)$$

$$= \frac{(11\times5)+(-3\times6)}{30}$$

$$\begin{array}{r|l} 2 & 6,\ 5 \\ \hline & 3,\ 5 \end{array}$$

LCM of 6 & 5 = 2 × 3 × 5

= 30

$$= \frac{55+(-18)}{30} = \frac{55-18}{30}=\frac{37}{30}$$

and $\frac{5}{3}+\left[\frac{1}{6}+\left(\frac{-3}{5}\right)\right]$ $=\frac{5}{3}+\left[\frac{(1\times5)+(-3\times6)}{30}\right]$

$$=\frac{5}{3}+\left(\frac{5+(-18)}{30}\right)$$

$$=\frac{5}{3}+\left(\frac{5-18}{30}\right)$$

$$=\frac{5}{3}+\left(\frac{-13}{30}\right)$$

$$\begin{array}{r|l} 3 & 3,\ 30 \\ \hline & 1,\ 10 \end{array}$$

LCM of 3 & 30 = 3 × 10 = 30

$$=\frac{(5\times10)+(-13\times1)}{30}$$

$$=\frac{50+(-13)}{30}$$

$$=\frac{50-13}{30}$$

$$=\frac{37}{30}$$

$$\therefore \quad \left(\frac{5}{3}+\frac{1}{6}\right)+\left(\frac{-3}{5}\right)=\frac{5}{3}+\left(\frac{1}{6}+\left(\frac{-3}{5}\right)\right)$$

Thus, associative property under addition holds.

(ii) We have, -4, $\frac{3}{7}$ and $\frac{-4}{5}$

Firstly $\left(\frac{-4}{1}+\frac{3}{7}\right)+\left(\frac{-4}{5}\right)=\left(\frac{-28+3}{7}\right)+\left(\frac{-4}{5}\right)$ (LCM of 1 & 7 is 7)

$=\frac{-25}{7}+\left(\frac{-4}{5}\right)=\frac{-125+(-28)}{35}$ (LCM of 7 & 5 is 35)

$$=\frac{-125+(-28)}{35}=\frac{-153}{35}$$

(LCM of 7 & 5 is 35)

and $-4+\left(\frac{3}{7}+\left(\frac{-4}{5}\right)\right)=-4+\left(\frac{15+(-28)}{35}\right)$

$$=-4+\left(\frac{15-28}{35}\right)=-4+\left(\frac{-13}{35}\right)$$

$$=\frac{-140+(-13)}{35}=\frac{-140-13}{35}=\frac{-153}{35}$$

$$\therefore \quad \left(-4+\frac{3}{7}\right)+\left(\frac{-4}{5}\right) = -4+\left[\frac{3}{7}+\left(\frac{-4}{5}\right)\right]$$

Thus, associative property under addition holds.

Example 1.5 **Solve the following by rearranging and grouping rational numbers.**

(i) $\frac{-3}{5}+\frac{7}{6}+\frac{-2}{5}+\frac{5}{6}$ **(ii)** $\frac{-2}{3}+\frac{7}{5}+\frac{4}{3}+\frac{-4}{5}+(-3)$

Solution : (i) Re-arranging and grouping the numbers in pairs in such a way that each group contain rational numbers with equal denominators.

$$\frac{-3}{5}+\frac{7}{6}+\frac{-2}{5}+\frac{5}{6} = \left(\frac{-3}{5}+\left(\frac{-2}{5}\right)\right)+\left(\frac{7}{6}+\frac{5}{6}\right)$$

$$= \frac{-3+(-2)}{5}+\frac{7+5}{6} = \frac{-5}{5}+\frac{12}{6}$$

$$= -1 + 2 = 1$$

(ii) Re-arranging and grouping the numbers in pairs in such a way that each group contain rational numbers with equal denominators.

$$\frac{-2}{3}+\frac{7}{5}+\frac{4}{3}+\frac{-4}{5}+(-3) = \left(\frac{-2}{3}+\frac{4}{3}\right)+\left(\frac{7}{5}+\frac{-4}{5}\right)+(-3)$$

$$= \frac{-2+4}{3}+\frac{7+(-4)}{5}+(-3)$$

$$= \frac{2}{3}+\frac{3}{5}+(-3) = \frac{10+9+(-45)}{15} \qquad \text{[LCM of 3 \& 5 is 15]}$$

$$= \frac{10+9-45}{15} = \frac{-26}{15}$$

Example 1.6 **Write the additive inverse of each of the following:**

(i) $\frac{-5}{13}$ **(ii)** $\frac{3}{-10}$ **(iii)** $\frac{-7}{-9}$

Solution : (i) Additive inverse of $\frac{-5}{13} = -\left(\frac{-5}{13}\right) = \frac{5}{13}$

(ii) Additive inverse of $\frac{3}{-10} = -\left[\frac{3}{-10}\right] = -\left(\frac{-3}{10}\right) = \frac{3}{10}$

(iii) Additive inverse of $\frac{-7}{-9} = -\left[\frac{-7}{-9}\right] = -\left[\frac{7}{9}\right] = \frac{-7}{9}$

Exercise 1.1

1. **Solve the following:-**

(i) $\frac{-5}{6}+\frac{3}{4}$ (ii) $\frac{6}{11}+\left(\frac{-2}{3}\right)$ (iii) $\frac{-5}{24}+\frac{7}{12}$ (iv) $\frac{-11}{12}+\frac{7}{8}$

(v) $\frac{-3}{10}+\left(\frac{-7}{15}\right)$ (vi) $\frac{-5}{7}+\frac{3}{14}$ (vii) $\frac{7}{6}+\left(\frac{-5}{9}\right)$ (viii) $\frac{-11}{15}+\frac{21}{25}$

2. **Verify commutative property of addition of rational numbers for each of the following.**

(i) $\frac{-5}{8}$ and $\frac{3}{4}$ (ii) $\frac{-2}{5}$ and $\frac{-3}{15}$ (iii) $\frac{-7}{10}$ and $\frac{8}{15}$

(iv) $\frac{-11}{14}$ and $\frac{17}{21}$ (v) -5 and $\frac{2}{3}$

3. **Verify associative property of addition of rational numbers i.e. $(x+y)+z = x+(y+z)$**

(i) $x=\frac{-2}{3}, y=\frac{1}{2}, z=\frac{5}{6}$ (ii) $x=\frac{-3}{4}\ y=\frac{1}{6}, z=\frac{5}{8}$

(iii) $x=2, y=\frac{-5}{12}, z=\frac{-3}{8}$

4. **Write the additive inverse of the following:**

(i) $\frac{-5}{11}$ (ii) $\frac{8}{9}$ (iii) $\frac{-15}{13}$ (iv) $\frac{-2}{-9}$

(v) $\frac{3}{-8}$ (vi) $\frac{2}{-7}$ (vii) $\frac{-18}{-11}$ (viii) 0

5. **Rearrange and Regroup the rational numbers and solve :**

(i) $\frac{2}{5}+\left(\frac{-7}{3}\right)+\frac{4}{5}+\frac{1}{3}$ (ii) $\left(\frac{-3}{8}\right)+\frac{4}{7}+\frac{2}{8}+\left(\frac{-3}{7}\right)$

(iii) $\left(\frac{-6}{7}\right)+\left(\frac{-4}{9}\right)+\left(\frac{-15}{7}\right)+\left(\frac{-5}{6}\right)$ (iv) $\frac{2}{3}+\left(\frac{-4}{5}\right)+\frac{3}{10}+\frac{1}{3}$

(v) $\left(\frac{-1}{8}\right)+\frac{5}{12}+\frac{2}{7}+\frac{5}{7}+\left(\frac{-5}{16}\right)$

6. **Multiple Choice Questions :**

(i) Which of the following is commutative property for addition?

(a) $x \times y = y \times x$ (b) $(x+y) = (y+x)$

(c) $(x+y)+z = x+(y+z)$ (d) $(x-y) = (y-x)$

(ii) Which of the following is associative property for addition?

(a) $x \times y = y \times x$ (b) $x + y = y + x$

(c) $(x+y) + z = x + (y + z)$ (d) $x - y = y - x$

(iii) The additive inverse of $\frac{-5}{-9}$ is

(a) $\frac{5}{9}$ (b) $\frac{5}{-9}$ (c) 0 (d) $\frac{2}{-3}$

(iv) The additive identity of $\frac{2}{3}$ is

(a) 0 (b) $\frac{-2}{3}$ (c) $\frac{-2}{-3}$ (d) $\frac{3}{2}$

1.3 Subtraction of Rational Numbers:

In previous class, we have defined the subtraction of rational numbers. We know if two rational numbers are to be subtracted, firstly we express each one of them as rational number with same denominators if required (by taking LCM) and then we solve.

Example 1.7 **Subtract:**

(i) $\frac{2}{7}$ **from** $\frac{5}{7}$ **(ii)** $\frac{5}{8}$ **from** $\frac{-3}{8}$ **(iii)** $\frac{-3}{10}$ **from** $\frac{2}{5}$

(iv) $\frac{-5}{6}$ **from** $\frac{-3}{4}$ **(v)** $\frac{7}{15}$ **from** $\frac{-7}{10}$

Solution : (i) $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$

(ii) $\frac{-3}{8} - \frac{5}{8} = \frac{-3-5}{8} = \frac{-8}{8} = -1$

(iii) $\frac{2}{5} - \left(\frac{-3}{10}\right) = \frac{4-(-3)}{10} = \frac{4+3}{10} = \frac{7}{10}$

(iv) $\frac{-3}{4} - \left(\frac{-5}{6}\right) = \frac{-9-(-10)}{12} = \frac{-9+10}{12} = \frac{1}{12}$

(v) $\frac{-7}{10} - \frac{7}{15} = \frac{-21-14}{30} = \frac{-35}{30} = \frac{-7}{6}$

1.3.1 Properties of Subtraction of Rational Numbers:

In this section, we shall learn some basic properties of subtraction of rational numbers. These properties are similar to those subtraction of integers which we have learnt in previous classes.

- **Closure Property :** The subtraction or difference of any two rational numbers is also a rational number.

  i.e. For any two rational number $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0)

  $$\frac{a}{b}-\frac{c}{d} \text{ is also a rational number.}$$

  (i) $\frac{5}{6}-\frac{2}{3}=\frac{5-4}{6}=\frac{1}{6}$, rational number.

  (ii) $\frac{-3}{8}-\left(\frac{-5}{6}\right)=\frac{-9-(-20)}{24}=\frac{-9+20}{24}=\frac{11}{24}$, rational number.

  (iii) $\frac{7}{10}-\left(\frac{-2}{5}\right)=\frac{7-(-4)}{10}=\frac{7+4}{10}=\frac{11}{10}$, rational number.

- **Commutative Property :** The subtraction of rational numbers is not commutative i.e. For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

  We have $\frac{a}{b}-\frac{c}{d}\neq\frac{c}{d}-\frac{a}{b}$

  e.g. $\frac{5}{4}-\frac{3}{5}=\frac{25-12}{20}=\frac{13}{20}$

  and $\frac{3}{5}-\frac{5}{4}=\frac{12-25}{20}=\frac{-13}{20}$

  $\therefore$ $\frac{5}{4}-\frac{3}{5}\neq\frac{3}{5}-\frac{5}{4}$

- **Associative Property :** The subtraction of rational numbers is not associative i.e. For any rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$; b, d, f $\neq$ 0, We have

  $$\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}\neq\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)$$

  e.g. $\left(\frac{3}{4}-\frac{2}{3}\right)-\frac{1}{2}=\left(\frac{9-8}{12}\right)-\frac{1}{2}=\frac{1}{12}-\frac{1}{2}=\frac{1-6}{12}=\frac{-5}{12}$

and $\frac{3}{4}-\left(\frac{2}{3}-\frac{1}{2}\right) \quad =\frac{3}{4}-\left(\frac{4-3}{6}\right)=\frac{3}{4}-\frac{1}{6}=\frac{9-2}{12}=\frac{7}{12}$

$\therefore \quad \left(\frac{3}{4}-\frac{2}{3}\right)-\frac{1}{2} \neq \frac{3}{4}-\left(\frac{2}{3}-\frac{1}{2}\right)$

- **Existence of Identity :** Identity does not exist in subtraction as commutative property does not hold in subtraction. Because for any rational number a, a – 0 = a but 0 – a ≠ a, so in case of subtraction identity does not exist.

Example 1.8 **Verify that x – y ≠ y – x when**

(i) $x=\frac{-2}{5}, y=\frac{-3}{4}$ **(ii)** $x=\frac{5}{12}, y=\frac{-7}{8}$

Solution : (i) LHS : $x-y = \frac{-2}{5}-\left(\frac{-3}{4}\right)=\frac{-2}{5}+\frac{3}{4}$

$=\frac{-8+15}{20}=\frac{7}{20}$

RHS : $y-x = \frac{-3}{4}-\left(\frac{-2}{5}\right)=\frac{-15-(-8)}{20}=\frac{-15+8}{20}$

$=\frac{-7}{20}$

∴ LHS ≠ RHS thus, x–y ≠ y–x

(ii) LHS : $x-y = \frac{5}{12}-\left(\frac{-7}{8}\right)=\frac{10-(-21)}{24}$

$=\frac{10+21}{24}=\frac{31}{24}$

RHS : $y-x = \frac{-7}{8}-\frac{5}{12}=\frac{-21-10}{24}=\frac{-31}{24}$

∴ LHS ≠ RHS

Thus, x – y ≠ y – x

Example 1.9 **Verify (x – y) – z ≠ x – (y – z) when**

(i) $x=\frac{-2}{3}, y=\frac{5}{8}, z=\frac{-7}{12}$ **(ii)** $x=\frac{1}{2}, y=\frac{-2}{5}, z=\frac{3}{10}$

Solution : (i) LHS : $(x-y)-z \quad = \left(\frac{-2}{3}-\frac{5}{8}\right)-\left(\frac{-7}{12}\right)$

$$= \frac{-16-15}{24} - \left(\frac{-7}{12}\right) = \frac{-31}{24} - \left(\frac{-7}{12}\right)$$

$$= \frac{-31-(-14)}{24} = \frac{-31+14}{24} = \frac{-17}{24}$$

RHS : $x - (y-z) = \frac{-2}{3} - \left(\frac{5}{8} - \left(\frac{-7}{12}\right)\right)$

$$= \frac{-2}{3} - \left(\frac{15-(-14)}{24}\right) = \frac{-2}{3} - \left(\frac{15+14}{24}\right)$$

$$= \frac{-2}{3} - \frac{29}{24} = \frac{-16-29}{24} = \frac{-45}{24}$$

$\therefore$ LHS $\neq$ RHS

Thus, $(x - y) - z \neq x - (y - z)$

(ii) LHS : $(x-y) - z = \left(\frac{1}{2} - \left(\frac{-2}{5}\right)\right) - \frac{3}{10}$

$$= \frac{5-(-4)}{10} - \frac{3}{10} = \frac{5+4}{10} - \frac{3}{10} = \frac{9}{10} - \frac{3}{10}$$

$$= \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$$

RHS : $x - (y-z) = \frac{1}{2} - \left(\frac{-2}{5} - \frac{3}{10}\right)$

$$= \frac{1}{2} - \left(\frac{-4-3}{10}\right) = \frac{1}{2} - \left(\frac{-7}{10}\right) = \frac{5-(-7)}{10} = \frac{5+7}{10}$$

$$= \frac{12}{10} = \frac{6}{5}$$

$\therefore$ LHS $\neq$ RHS

Thus, $(x-y) - z \neq x - (y-z)$

Exercise 1.2

1. Subtract:-

(i) $\frac{2}{5}$ from $\frac{4}{5}$ (ii) $\frac{-3}{7}$ from $\frac{4}{7}$ (iii) $\frac{-5}{8}$ from $\frac{3}{4}$ (iv) $\frac{-8}{21}$ from $\frac{5}{14}$

(v) $\frac{-7}{10}$ from $\frac{-8}{15}$ (vi) $\frac{6}{11}$ from $\frac{5}{6}$ (vii) $\frac{-3}{4}$ from $\frac{-5}{12}$ (viii) $\frac{13}{10}$ from $\frac{-8}{25}$

2. **Verify that x – y ≠ y – x when**

(i) $x = \frac{-5}{12}, y = \frac{-3}{8}$ (ii) $x = \frac{7}{15}, y = \frac{-3}{10}$

(iii) $x = \frac{-15}{16}, y = \frac{7}{12}$ (iv) $x = \frac{-3}{4}, y = \frac{-5}{6}$

3. **Verify that (x–y) – z ≠ x – (y – z) when**

(i) $x = \frac{-7}{12}, y = \frac{-3}{4}, z = \frac{2}{3}$ (ii) $x = \frac{3}{8}, y = \frac{-2}{5}, z = \frac{-7}{10}$

(iii) $x = \frac{-1}{2}, y = \frac{-5}{4}, z = \frac{3}{8}$

4. **Solve the following:-**

(i) $\frac{3}{4} + \frac{5}{6} - \frac{7}{8}$ (ii) $\frac{-11}{2} + \frac{7}{6} - \frac{5}{8}$ (iii) $\frac{-4}{5} - \left(\frac{-7}{10}\right) + \left(\frac{-8}{15}\right)$

(iv) $\frac{-2}{5} - \left[\frac{-3}{10} - \left(\frac{-4}{15}\right)\right]$ (v) $\frac{3}{8} - \left(\frac{-2}{9}\right) + \left(\frac{5}{-36}\right)$

1.4 Multiplication of Rational Numbers:

In earlier classes, we have learnt the multiplication of two fractions. The product of two given fractions is a fraction whose numerator is the product of the numerators of the given fractions and whose denominator is the product of the denominators of the given fractions.

i.e.

$$\text{Product of Fractions} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

This same rule is applicable to the product of rational numbers.

$$\therefore \quad \text{Product of Rational Numbers} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

i.e. For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$

then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Let's discuss some examples.

Example 1.10 Multiply :

(i) $\frac{3}{4}$ by $\frac{5}{11}$ (ii) $\frac{-2}{3}$ by $\frac{5}{9}$ (iii) $\left(\frac{-7}{8}\right)$ by 5

(iv) $\left(\frac{-5}{8}\right)$ **by** $\frac{4}{3}$ **(v)** $\left(\frac{-10}{7}\right)$ **by** $\left(\frac{-14}{15}\right)$

Solution : (i) $\frac{3}{4}\times\frac{5}{11} = \frac{3\times5}{4\times11} = \frac{15}{44}$

(ii) $\left(\frac{-2}{3}\right)\times\frac{5}{9} = \frac{(-2)\times5}{3\times9} = \frac{-10}{27}$

(iii) $\left(\frac{-7}{8}\right)\times5 = \frac{-7}{8}\times\frac{5}{1} = \frac{-7\times5}{8\times1} = \frac{-35}{8}$

(iv) $\left(\frac{-5}{8}\right)\times\frac{4}{3} = \frac{(-5)\times\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}\times3} = \frac{-5}{6}$

(v) $\left(\frac{-10}{7}\right)\times\left(\frac{-14}{15}\right) = \frac{\left(-\overset{2}{\cancel{10}}\right)\times\left(-\overset{2}{\cancel{14}}\right)}{\underset{1}{\cancel{7}}\times\underset{3}{\cancel{15}}} = \frac{4}{3}$

Example 1.11 Simplify :

(i) $\left(\frac{-8}{9}\right)\times\frac{3}{64}$ **(ii)** $\left(\frac{-9}{16}\right)\times\left(\frac{-64}{27}\right)$ **(iii)** $\left(\frac{-10}{9}\right)\times\left(\frac{36}{-25}\right)$

(iv) $\frac{15}{32}\times\left(\frac{-18}{25}\right)$ **(v)** $\frac{13}{20}\times\left(\frac{25}{-26}\right)$

Solution : (i) $\left(\frac{-8}{9}\right)\times\frac{3}{64} = \frac{\left(\left(-\overset{1}{\cancel{8}}\right)\times\overset{1}{\cancel{3}}\right)}{\underset{3}{\cancel{9}}\times\underset{8}{\cancel{64}}} = \frac{-1}{24}$

(ii) $\left(\frac{-9}{16}\right)\times\left(\frac{-64}{27}\right) = \frac{\left(-\overset{1}{\cancel{9}}\right)\times\left(-\overset{4}{\cancel{64}}\right)}{\underset{1}{\cancel{16}}\times\underset{3}{\cancel{27}}} = \frac{4}{3}$

(iii) $\left(\frac{-10}{9}\right)\times\left(\frac{36}{-25}\right) = \frac{\left(-\overset{2}{\cancel{10}}\right)\times\overset{4}{\cancel{36}}}{\underset{1}{\cancel{9}}\times\left(-\underset{5}{\cancel{25}}\right)} = \frac{8}{5}$

(iv) $$\frac{15}{32}\times\left(\frac{-18}{25}\right)=\frac{\overset{3}{\cancel{15}}\times\left(-\overset{9}{\cancel{18}}\right)}{\underset{16}{\cancel{32}}\times\underset{5}{\cancel{25}}}=\frac{-27}{80}$$

(v) $$\frac{13}{20}\times\left(\frac{25}{-26}\right)=\frac{\overset{1}{\cancel{13}}\times\overset{5}{\cancel{25}}}{\underset{4}{\cancel{20}}\times\left(-\underset{2}{\cancel{26}}\right)}=\frac{-5}{8}$$

1.4.1 Properties of Multiplication of Rational Numbers:

In this section, we shall learn some basic properties of multiplication of rational numbers. These properties are similar to those of multiplication of integers which we have learnt in previous classes.

- **Closure Property :** The product of any two rational numbers is always a rational number.

  i.e.

  if $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0) are any two rational numbers.

  then $\frac{a}{b}\times\frac{c}{d}$ is also a rational number.

e.g. (i) $\frac{-5}{8}\times\frac{3}{4}=\frac{-5\times3}{8\times4}=\frac{-15}{32}$ is a rational number.

(ii) $\frac{-10}{9}\times\frac{3}{5}=\frac{-\overset{2}{\cancel{10}}\times\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}\times\underset{1}{\cancel{5}}}=\frac{-2}{3}$ is a rational number.

- **Commutative Property :** When two rational numbers can be multiplied in any order then we say that multiplication is commutative for rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0) are any two rational numbers then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$

e.g. Consider two rational numbers $\frac{-5}{6}$ and $\frac{2}{3}$

then $$\left(\frac{-5}{6}\right)\times\frac{2}{3}=\frac{(-5)\times\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}\times3}=\frac{-5}{9}$$

and $$\frac{2}{3}\times\left(\frac{-5}{6}\right)=\frac{\overset{1}{\cancel{2}}\times(-5)}{3\times\underset{3}{\cancel{6}}}=\frac{-5}{9}$$

$$\therefore\quad\left(\frac{-5}{6}\right)\times\frac{2}{3}=\frac{2}{3}\times\left(\frac{-5}{6}\right)$$

- **Associatve Property :** When three rational numbers are to be multiplied, by multiplying the first two rational numbers and then multiply the third number or by multiplying the second and third rational numbers and then multiplying the first number If we get the same result, then we say that the multiplication of rational numbers is associative.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ (b, d, f $\neq$ 0)

We have $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{c}{f} = \frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$

e.g. Consider three rational numbers $\frac{-5}{8}, \frac{4}{9}$ and $\frac{-3}{10}$ then

$$\left(\frac{-5}{8}\times\frac{4}{9}\right)\times\left(\frac{-3}{10}\right) = \frac{(-5)\times\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}\times 9}\times\left(\frac{-3}{10}\right) = \frac{-5}{18}\times\left(\frac{-3}{10}\right)$$

$$= \frac{\left(\overset{1}{\cancel{5}}\right)\times\left(-\overset{1}{\cancel{3}}\right)}{\underset{6}{\cancel{18}}\times\underset{2}{\cancel{10}}} = \frac{1}{12}$$

and
$$\left(\frac{-5}{8}\right)\times\left(\frac{4}{9}\times\left(\frac{-3}{10}\right)\right) = \left(\frac{-5}{8}\right)\times\frac{\overset{2}{\cancel{4}}\times\left(-\overset{1}{\cancel{3}}\right)}{\underset{3}{\cancel{9}}\times\underset{5}{\cancel{10}}}$$

$$= \left(\frac{-5}{8}\right)\times\left(\frac{-2}{15}\right) = \frac{\left(-\overset{1}{\cancel{5}}\right)\times\left(-\overset{1}{\cancel{2}}\right)}{\underset{4}{\cancel{8}}\times\underset{3}{\cancel{15}}} = \frac{1}{12}$$

$$\therefore \quad \left(\frac{-5}{8}\times\frac{4}{9}\right)\times\left(\frac{-3}{10}\right) = \left(\frac{-5}{8}\right)\times\left(\frac{4}{9}\times\left(\frac{-3}{10}\right)\right)$$

- **Multiplicative Identity :** When we multiply 1 by any rational number then the product is the same rational number.

i.e. For any rational number $\frac{a}{b}$, (b$\neq$0) there exists a unique natural number 1 such that $1\times\frac{a}{b}=\frac{a}{b}=\frac{a}{b}\times 1$

So 1 is the multiplicative identity for rational numbers.

- **Multiplicative Inverse:** When two rational numbers are multiplied and give result 1 (multiplicative identity) then one rational number is called the multiplicative inverse of other i.e. For any rational number $\frac{a}{b}$ (b $\neq$ 0) there exist $\frac{b}{a}$ (a $\neq$ 0) such that

$$\frac{a}{b}\times\frac{b}{a}=1=\frac{b}{a}\times\frac{a}{b}$$

Note :- 0 has no multiplicative inverse. As reciprocal of 0 is $\frac{1}{0}$, which does not exists.

e.g. $$\frac{5}{6}\times\frac{6}{5} = \frac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}}\times\underset{1}{\cancel{5}}} = 1$$

and $$\frac{6}{5}\times\frac{5}{6} = \frac{\overset{1}{\cancel{6}}\times\overset{1}{\cancel{5}}}{\underset{1}{5}\times\underset{1}{\cancel{6}}} = 1$$

∴ $$\frac{5}{6}\times\frac{6}{5} = \mathbf{1} = \frac{6}{5}\times\frac{5}{6}$$

Thus $\frac{5}{6}$ is the multiplicative inverse (reciprocal) of $\frac{6}{5}$ and vice versa.

1.4.2 Distributive Property of Multiplication over Addition and Subtraction

The multiplication of rational numbers is distributive over their addition and subtraction.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, (b, d, f ≠ 0)

$$\frac{a}{b}\times\left[\frac{c}{d}+\frac{e}{f}\right] = \frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$$

e.g. Consider three rational numbers $\frac{-2}{3}, \frac{5}{6}$ and $\frac{-3}{4}$

$$\frac{-2}{3}\times\left(\frac{5}{6}+\left(\frac{-3}{4}\right)\right) = \frac{-2}{3}\times\left(\frac{10+(-9)}{12}\right)$$

$$= \frac{-2}{3}\times\frac{1}{12} = \frac{-\overset{1}{\cancel{2}}\times 1}{3\times\underset{6}{\cancel{12}}} = \frac{-1}{18}$$

and $$\left(\frac{-2}{3}\right)\times\frac{5}{6}+\left(\frac{-2}{3}\right)\times\left(\frac{-3}{4}\right) = \frac{\left(-\overset{1}{\cancel{2}}\right)\times 5}{3\times\underset{3}{\cancel{6}}} + \frac{\left(-\overset{1}{\cancel{2}}\right)\times\left(-\overset{1}{\cancel{3}}\right)}{\underset{1}{\cancel{3}}\times\underset{2}{\cancel{4}}}$$

$$= \frac{-5}{9}+\frac{1}{2} = \frac{-10+9}{18} = \frac{-1}{18}$$

$$\therefore \qquad \frac{-2}{3}\times\left(\frac{5}{6}+\left(\frac{-3}{4}\right)\right) = \left(\frac{-2}{3}\right)\times\frac{5}{6}+\left(\frac{-2}{3}\right)\times\left(\frac{-3}{4}\right)$$

Example 1.12 **Verify x × y = y × x when**

(i) $x = \frac{-5}{12}$, $y = \frac{4}{15}$ **(ii)** $x = \frac{-6}{7}$, $y = \frac{-14}{9}$

Solution : (i) LHS : $x \times y = \frac{-5}{12}\times\frac{4}{15} = \frac{\left(-\overset{1}{\cancel{5}}\right)\times\overset{1}{\cancel{4}}}{\underset{3}{\cancel{12}}\times\underset{3}{\cancel{15}}} = \frac{-1}{9}$

RHS : $y \times x = \frac{4}{15}\times\left(\frac{-5}{12}\right) = \frac{\overset{1}{\cancel{4}}\times\left(-\overset{1}{\cancel{5}}\right)}{\underset{3}{\cancel{15}}\times\underset{3}{\cancel{12}}} = \frac{-1}{9}$

$\therefore$ LHS = RHS

Thus, x × y = y × x

(ii) LHS : $x \times y = \left(\frac{-6}{7}\right)\times\left(\frac{-14}{9}\right) = \frac{\left(-\overset{2}{\cancel{6}}\right)\times\left(-\overset{2}{\cancel{14}}\right)}{\underset{1}{\cancel{7}}\times\underset{3}{\cancel{9}}} = \frac{4}{3}$

RHS : $y \times x = \left(\frac{-14}{9}\right)\times\left(\frac{-6}{7}\right) = \frac{\left(-\overset{2}{\cancel{14}}\right)\times\left(-\overset{2}{\cancel{6}}\right)}{\underset{3}{\cancel{9}}\times\underset{1}{7}} = \frac{4}{3}$

$\therefore$ LHS = RHS

Thus, x × y = y × x

Example 1.13 **Verify x × (y × z) = (x×y)×z when**

(i) $x = \frac{-7}{3}$, $y = \frac{12}{5}$, $z = \frac{4}{9}$ **(ii)** $x = \frac{-1}{2}$, $y = \frac{5}{4}$, $z = \frac{-7}{5}$

Solution : (i) LHS : $x \times (y\times z) = \frac{-7}{3}\times\left(\frac{12}{5}\times\frac{4}{9}\right)$

$$= \frac{-7}{3}\times\frac{\overset{4}{\cancel{12}}\times 4}{5\times\underset{3}{\cancel{9}}} = \frac{-7}{3}\times\frac{16}{15} = \frac{-7\times16}{3\times15} = \frac{-112}{45}$$

RHS : $(x \times y) \times z = \left(\frac{-7}{3}\times\frac{12}{5}\right)\times\frac{4}{9}$

$$= \frac{-7\times\overset{4}{\cancel{12}}}{\underset{1}{\cancel{3}}\times5}\times\frac{4}{9} = \frac{-28}{5}\times\frac{4}{9} = \frac{-28\times4}{5\times9} = \frac{-112}{45}$$

$\therefore$ LHS = RHS

Thus, $x\times(y\times z) = (x\times y)\times z$

(ii) LHS : $x\times(y\times z) = \frac{-1}{2}\times\left(\frac{5}{4}\times\left(\frac{-7}{5}\right)\right)$

$$= \frac{-1}{2}\times\frac{\overset{1}{\cancel{5}}\times(-7)}{4\times\underset{1}{\cancel{5}}} = \frac{-1}{2}\times\left(\frac{-7}{4}\right) = \frac{(-1)\times(-7)}{2\times4} = \frac{7}{8}$$

RHS : $(x\times y)\times z = \left(\left(\frac{-1}{2}\right)\times\frac{5}{4}\right)\times\left(\frac{-7}{5}\right)$

$$= \frac{(-1)\times5}{2\times4}\times\left(\frac{-7}{5}\right) = \frac{-5}{8}\times\left(\frac{-7}{5}\right) = \frac{\left(-\overset{1}{\cancel{5}}\right)\times(-7)}{8\times\underset{1}{\cancel{5}}} = \frac{7}{8}$$

$\therefore$ LHS = RHS

Thus, $x\times(y\times z) = (x\times y)\times z$

Example 1.14 **Write the reciprocal (multiplicative inverse) of each of the following rational numbers :**

(i) -5 (ii) $\frac{-2}{3}$ (iii) $\frac{7}{15}$ (iv) $\frac{-2}{5}\times\frac{3}{7}$ (v) $\frac{-5}{8}\times\frac{4}{3}$

Solution : (i) Reciprocal of -5 i.e. $\frac{-5}{1} = \frac{1}{-5} = \frac{-1}{5}$

(ii) Reciprocal of $\frac{-2}{3} = \frac{3}{-2} = \frac{-3}{2}$

(iii) Reciprocal of $\frac{7}{15} = \frac{15}{7}$

(iv) We have, $\frac{-2}{5}\times\frac{3}{7} = \frac{(-2)\times3}{5\times7} = \frac{-6}{35}$

$\therefore$ Reciprocal of $\frac{-6}{35} = \frac{35}{-6} = \frac{-35}{6}$

(v) We have, $\frac{-5}{8}\times\frac{4}{3} = \frac{-5\times\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}\times3} = \frac{-5}{6}$

$\therefore$ Reciprocal of $\frac{-5}{6} = \frac{6}{-5} = \frac{-6}{5}$

Example 1.15 Verify the property : $x \times (y + z) = x \times y + x \times z$ when

(i) $x = \frac{-3}{7}, y = \frac{12}{13}, z = \frac{-5}{6}$ (ii) $x = \frac{-3}{4}, y = \frac{5}{2}, z = \frac{-7}{6}$

Solution : (i) LHS : $x\times(y+z) = \left(\frac{-3}{7}\right)\times\left(\frac{12}{13}+\left(\frac{-5}{6}\right)\right)$

$$= \left(\frac{-3}{7}\right)\times\left(\frac{72+(-65)}{78}\right) = \left(\frac{-3}{7}\right)\times\frac{7}{78} = \frac{\left(-\overset{1}{\cancel{3}}\right)\times\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}}\times\underset{26}{\cancel{78}}} = \frac{-1}{26}$$

(ii) RHS : $x\times y + x\times z = \left(\frac{-3}{7}\right)\times\left(\frac{12}{13}\right)+\left(\frac{-3}{7}\right)\times\left(\frac{-5}{6}\right)$

$$= \frac{(-3)\times12}{7\times13}+\frac{(-3)\times(-5)}{7\times6} = \frac{-36}{91}+\frac{15}{14}$$

$$= \frac{-72+65}{182} = \frac{\overset{1}{\cancel{-7}}}{\underset{26}{\cancel{182}}} = \frac{-1}{26}$$

∴ LHS = RHS

Thus, $x\times(y+z) = x \times y + x \times z$

(ii) LHS : $x\times(y+z) = \left(\frac{-3}{4}\right)\times\left(\frac{5}{2}+\left(\frac{-7}{6}\right)\right)$

$$= \left(\frac{-3}{4}\right)\times\left(\frac{15+(-7)}{6}\right) = \left(\frac{-3}{4}\right)\times\frac{8}{6} = \frac{\left(-\overset{1}{\cancel{3}}\right)\times\overset{\overset{1}{2}}{\cancel{8}}}{\underset{1}{\cancel{4}}\times\underset{\underset{1}{2}}{\cancel{6}}} = -1$$

RHS : $x\times y + x\times z = \left(\frac{-3}{4}\right)\times\frac{5}{2}+\left(\frac{-3}{4}\right)\times\left(\frac{-7}{6}\right)$

$$= \frac{(-3)\times5}{4\times2}+\frac{\left(-\overset{1}{\cancel{3}}\right)\times(-7)}{4\times\underset{2}{\cancel{6}}} = \frac{-15}{8}+\frac{7}{8} = \frac{-15+7}{8}$$

$$= \frac{-\overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}}} = -1$$

∴ LHS = RHS

Thus, $x \times (y + z) = x \times y + x \times z$

Example 1.16 **Using distributive property, simplify :**

(i) $\left(\frac{-5}{4}\right)\times\frac{8}{5}+\left(\frac{-5}{4}\right)\times\frac{16}{5}$ (ii) $\frac{2}{7}\times\frac{7}{16}-\frac{-2}{7}\times\frac{21}{4}$

Solution : (i) We have, $\left(\frac{-5}{4}\right)\times\frac{8}{5}+\left(\frac{-5}{4}\right)\times\frac{16}{5}$

$\left(\text{By using } x \times y - x \times z = x \times (y - z) \text{ where } x = \frac{2}{7}, y = \frac{7}{16}, z = \frac{21}{4}\right)$

$$= \left(\frac{-5}{4}\right)\times\left(\frac{8}{5}+\frac{16}{5}\right) \qquad \left(\text{Taking } \frac{-5}{4} \text{ Common}\right)$$

$$= \left(\frac{-5}{4}\right)\times\left(\frac{8+16}{5}\right) = \frac{-5}{4}\times\frac{24}{5} = \frac{-\overset{1}{\cancel{5}}\times\overset{6}{\cancel{24}}}{\underset{1}{\cancel{4}}\times\underset{1}{\cancel{5}}} = -6$$

(ii) We have, $\frac{2}{7}\times\frac{7}{16}-\frac{2}{7}\times\frac{21}{4}$

$\left(\text{By using } x \times y - x \times z = x \times (y - z) \text{ where } x = \frac{2}{7}, y = \frac{7}{16}, z = \frac{21}{4}\right)$

$$= \frac{2}{7}\times\left(\frac{7}{16}-\frac{21}{4}\right) \qquad \left(\text{Taking } \frac{2}{7} \text{ common}\right)$$

$$= \frac{2}{7}\times\left(\frac{7-84}{16}\right) = \frac{2}{7}\times\left(\frac{-77}{16}\right) = \frac{\overset{1}{\cancel{2}}\times\left(-\overset{11}{\cancel{77}}\right)}{\underset{1}{\cancel{7}}\times\underset{8}{\cancel{16}}} = \frac{-11}{8}$$

Exercise 1.3

1. **Solve the following:-**

(i) $\frac{7}{11}\times\frac{5}{4}$ (ii) $\frac{5}{7}\times\left(\frac{-3}{4}\right)$ (iii) $\frac{2}{9}\times\frac{-5}{11}$

(iv) $\frac{-3}{5}\times\frac{4}{7}$ (v) $\left(\frac{-8}{7}\right)\times\left(\frac{-14}{5}\right)$ (vi) $\left(\frac{-5}{9}\right)\times\left(\frac{36}{-25}\right)$

(vii) $\left(\frac{-8}{25}\right)\times\left(\frac{-15}{16}\right)$ (viii) $\left(\frac{-6}{11}\right)\times\left(\frac{-44}{30}\right)$ (ix) $\frac{5}{17}\times\left(\frac{-51}{30}\right)$

(x) $\left(\frac{-7}{18}\right)\times\left(\frac{15}{-7}\right)$ (xi) $\left(\frac{-16}{5}\right)\times\frac{20}{9}\times\left(\frac{-3}{4}\right)$ (xii) $\frac{9}{10}\times\left(\frac{-15}{27}\right)\times\frac{18}{5}$

2. **Verify that x× y = y×x for the following when**

(i) $x = \frac{-5}{7}$ $y = \frac{9}{13}$ (ii) $x = \frac{3}{10}, y = \frac{-15}{8}$ (iii) $x = \frac{-7}{8}, y = \frac{-4}{9}$

(iv) $x = 5, y = \frac{-9}{10}$

3. **Verify that x × (y × z) = (x×y) × z for the following when**

(i) $x = \frac{-7}{6}, y = \frac{12}{5}, z = \frac{-2}{9}$ (ii) $x = \frac{1}{2}, y = \frac{-5}{8}, z = \frac{-3}{5}$

(iii) $x = \frac{5}{7}, y = \frac{-12}{10}, z = \frac{-4}{9}$ (iv) $x = \frac{-3}{5}, y = \frac{2}{9}, z = \frac{10}{7}$

4. **Write the reciprocal of each of the following:**

(i) –2 (ii) $\frac{-5}{8}$ (iii) $\frac{7}{-9}$ (iv) $\frac{-3}{4}$

(v) $\frac{2}{7}\times\left(\frac{-3}{15}\right)$ (vi) $\left(\frac{-3}{8}\right)\times\left(\frac{-12}{9}\right)$ (vii) $(-8)\times\frac{5}{6}$ (viii) $3\times\left(\frac{-7}{9}\right)$

5. **Verify that x × (y + z) = x × y + x × z when**

(i) $x = \frac{3}{5}, y = \frac{25}{24}, z = 10$ (ii) $x = \frac{-5}{4}, y = \frac{8}{5}, z = \frac{16}{15}$

(iii) $x = \frac{-2}{7}, y = \frac{14}{10}, z = \frac{3}{5}$

6. **Verify that x × (y – z) = x × y – x × z when**

(i) $x = \frac{-2}{3}, y = \frac{3}{4}, z = \frac{6}{7}$ (ii) $x = \frac{-1}{2}, y = \frac{5}{6}, z = \frac{-3}{10}$

(iii) $x = \frac{3}{4}, y = \frac{8}{9}, z = -10$

7. **Name the property of multiplication of rational numbers represented by the following statement:**

(i) $\frac{-2}{5}\times\frac{3}{4} = \frac{3}{4}\times\left(\frac{-2}{5}\right)$ ....

(ii) $\frac{-3}{8}\times 1 = \frac{-3}{8} = 1\times\frac{-3}{8}$ ....

(iii) $\frac{5}{8}\times\left(\frac{3}{4}+\frac{2}{3}\right) = \frac{5}{8}\times\frac{3}{4}+\frac{5}{8}\times\frac{2}{3}$ ....

(iv) $\left(\frac{-2}{7}\times\frac{5}{4}\right)\times\frac{7}{10} = \frac{-2}{7}\times\left(\frac{5}{4}\times\frac{7}{10}\right)$ ....

(v) $\left(\frac{-7}{9}\right)\times\frac{3}{4}-\left(\frac{-7}{9}\right)\times\frac{5}{10} = \left(\frac{-7}{9}\right)\times\left(\frac{3}{4}-\frac{5}{10}\right)$ ....

8. Multiple Choice Questions :

(i) For rational numbers x, y and z, which of the following is not true :

(a) $x \times y = y \times x$ (b) $x \times (y - z) = x \times y - x \times z$

(c) $x - y = y - x$ (d) $x \times (y \times z) = (x \times y) \times z$

(ii) If $\frac{-5}{8}\times\frac{4}{7}=\frac{4}{7}\times\left(\frac{-5}{8}\right)$ then which of the following property it holds

(a) Closure (b) Commutative (c) Associative (d) Identity

(iii) The multiplicative identity of rational number 'a' is

(a) 1 (b) 0 (c) $\frac{1}{a}$ (d) $-a$

(iv) The statement $\frac{-5}{8}\times\left(\frac{3}{4}-\frac{2}{3}\right)=\left(\frac{-5}{8}\right)\times\frac{3}{4}-\left(\frac{-5}{8}\right)\times\frac{2}{3}$ holds under the property.

(a) Associative of multiplication

(b) Associative of subtraction

(c) Distribution of multiplication over addition

(d) Distribution of multiplication over subtraction

(v) Which of the following number does not have multiplicative inverse?

(a) 0 (b) –1 (c) 1 (d) $\frac{-2}{-3}$

(vi) Which of the following number is multiplicative inverse of itself ?

(a) 0 (b) –1 (c) 1 (d) Both b and c

1.5 Division of Rational Numbers:

In earlier classes, we have learnt the division of two fractions. We know the division of fractions is the inverse of multiplication. The same rule is applicable for the rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq 0$) are two rational numbers such that $\frac{c}{d} \neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times(\text{Reciprocal of }\frac{c}{d})=\frac{a}{b}\times\frac{d}{c}$$

Here, $\frac{a}{b}$ is called the dividend, $\frac{c}{d}$ is called the divisor and $\frac{a}{b}\times\frac{d}{c}$ is called the quotient.

Note : Division by 0 is not defined.

Let's discuss some examples.

Example 1.17 **Divide :**

(i) $\frac{3}{10}$ by $\frac{4}{25}$ (ii) $\left(\frac{-8}{9}\right)$ by $\left(\frac{-4}{3}\right)$ (iii) $\left(\frac{-8}{13}\right)$ by $\left(\frac{-5}{26}\right)$

(iv) $\left(\frac{-5}{8}\right)$ by $\left(\frac{15}{16}\right)$ (v) $\frac{7}{15}$ by $\frac{21}{20}$

Solution : (i)

$$\frac{3}{10} \div \frac{4}{25} = \frac{3}{10} \times \text{(Reciprocal of } \frac{4}{25}\text{)}$$

$$= \frac{3}{10} \times \frac{25}{4} = \frac{3 \times \overset{5}{\cancel{25}}}{\underset{2}{\cancel{10}} \times 4} = \frac{15}{8}$$

(ii)

$$\left(\frac{-8}{9}\right) \div \left(\frac{-4}{3}\right) = \left(\frac{-8}{9}\right) \times \left(\text{Reciprocal of } \frac{-4}{3}\right)$$

$$= \left(\frac{-8}{9}\right) \times \left(\frac{3}{-4}\right) = \frac{\left(-\overset{2}{\cancel{8}}\right) \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \left(-\underset{1}{\cancel{4}}\right)} = \frac{2}{3}$$

(iii)

$$\left(\frac{-8}{13}\right) \div \left(\frac{-5}{26}\right) = \left(\frac{-8}{13}\right) \times \left(\text{Reciprocal of } \frac{-5}{26}\right)$$

$$= \left(\frac{-8}{13}\right) \times \left(\frac{26}{-5}\right) = \frac{(-8) \times \overset{2}{\cancel{26}}}{\underset{1}{\cancel{13}} \times (-5)} = \frac{-16}{-5} = \frac{16}{5}$$

(iv)

$$\left(\frac{-5}{8}\right) \div \frac{15}{16} = \left(\frac{-5}{8}\right) \times \left(\text{Reciprocal of } \frac{15}{16}\right)$$

$$= \left(\frac{-5}{8}\right) \times \frac{16}{15} = \frac{\left(-\overset{1}{\cancel{5}}\right) \times \overset{2}{\cancel{16}}}{\cancel{8} \times \underset{3}{\cancel{15}}} = \frac{-2}{3}$$

(v)

$$\frac{7}{15} \div \frac{21}{20} = \frac{7}{15} \times \left(\text{Reciprocal of } \frac{21}{20}\right)$$

$$= \frac{7}{15} \times \frac{20}{21} = \frac{\overset{1}{\cancel{7}} \times \overset{4}{\cancel{20}}}{\underset{3}{\cancel{15}} \times \underset{3}{\cancel{21}}} = \frac{4}{9}$$

Example 1.18 **The product of two numbers is $\frac{-14}{27}$. If one of the number is $\frac{-7}{9}$ then find the other.**

Solution : Let the other number be x then

$$\left(\frac{-7}{9}\right) \times x = \frac{-14}{27}$$

$$x = \left(\frac{-14}{27}\right) \div \left(\frac{-7}{9}\right)$$

$$= \frac{-\overset{2}{\cancel{14}}}{\underset{3}{\cancel{27}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{-1}{\cancel{-7}}}$$

$$= \frac{2}{3}$$

Example 1.19 **By what number should we multiply $\frac{3}{-7}$, so that the product may be $\frac{-18}{49}$.**

Solution : Let the required number be x then

$$x \times \left(\frac{3}{-7}\right) = \left(\frac{-18}{49}\right)$$

$$x = \left(\frac{-18}{49}\right) \div \left(\frac{3}{-7}\right) = \left(\frac{-18}{49}\right) \times \left(\frac{-7}{3}\right)$$

$$= \frac{\left(-\overset{6}{\cancel{18}}\right) \times \left(-\overset{1}{\cancel{7}}\right)}{\underset{7}{\cancel{49}} \times \underset{1}{\cancel{3}}} = \frac{6}{7}$$

1.5.1 Properties of Division of Rational Numbers

- **Closure Property :** The division of two rational numbers (divisor ≠ 0) is always a rational number.

i.e. for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$

Such that $\frac{c}{d} \neq 0$ then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number.

e.g. (i) $\left(\frac{-2}{5}\right) \div \frac{4}{9} = \frac{-2}{5} \times \frac{9}{4} = \frac{(-2)\times 9}{5\times 4} = \frac{-9}{10}$ is a rational number.

(ii) $\left(\frac{-3}{10}\right) \div \left(\frac{-5}{9}\right) = \left(\frac{-3}{10}\right) \times \left(\frac{9}{-5}\right) = \frac{(-3)\times(-9)}{10\times 5} = \frac{27}{50}$ is a rational number.

- **Commutative property :** The division of rational numbers is not commutative.

  i.e. For any two non-zero rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

- **Associative property :** The division of rational numbers ia not associative.

  i.e. For any three non-zero rational numbers.

  $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ then

  $$\left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f} \neq \frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right)$$

1.5.2 Properties of Rational Numbers

Properties/Operations	**Addition**	**Subtraction**	**Multiplication**	**Division**
Closure	✓	✓	✓	✓
Commutative	✓	×	✓	×
Associative	✓	×	✓	×
Identity	✓	×	✓	×
Inverse	✓	×	✓	×

Example 1.20 **Verify $x \div y \neq y \div x$ when**

(i) $x = \frac{-2}{5}, y = \frac{3}{4}$ **(ii)** $x = \frac{-3}{13}, y = \frac{-7}{9}$

Solution : (i) LHS : $x \div y = \left(\frac{-2}{5}\right) \div \frac{3}{4} = \left(\frac{-2}{5}\right) \times \frac{4}{3} = \frac{(-2)\times 4}{5\times 3} = \frac{-8}{15}$

RHS : $y \div x = \frac{3}{4} \div \left(\frac{-2}{5}\right) = \frac{3}{4} \times \left(\frac{5}{-2}\right) = \frac{3\times 5}{4\times(-2)} = \frac{-15}{8}$

$\therefore$ LHS ≠ RHS

Thus, $x \div y \neq y \div x$

(ii) LHS : $x \div y = \left(\frac{-3}{13}\right) \div \left(\frac{-7}{9}\right) = \left(\frac{-3}{13}\right) \times \left(\frac{9}{-7}\right)$

$$= \frac{(-3)\times 9}{13\times(-7)} - \frac{27}{91}$$

RHS : $y \div x = \left(\frac{-7}{9}\right) \div \left(\frac{-3}{13}\right) = \left(\frac{-7}{9}\right) \times \left(\frac{13}{-3}\right)$

$$= \frac{(-7)\times(-13)}{9\times 3} = \frac{91}{27}$$

∴ LHS ≠ RHS

Thus, $x \div y \neq y \div x$

Example 1.21 **Verify $x + (y + z) \neq (x + y) + z$, when $x = \frac{-2}{3}$, $y = \frac{-5}{6}$, $z = 3$**

Solution : LHS : $x \div (y \div z) = \left(\frac{-2}{3}\right) \div \left(\left(\frac{-5}{6}\right) \div 3\right) = \left(\frac{-2}{3}\right) \div \left(\left(\frac{-5}{6}\right) \div \frac{3}{1}\right)$

$$= \left(\frac{-2}{3}\right) \div \left(\frac{-5}{6} \times \frac{1}{3}\right) = \left(\frac{-2}{3}\right) \div \left(\frac{-5}{18}\right)$$

$$= \left(\frac{-2}{3}\right) \times \left(\frac{18}{-5}\right) = \frac{(-2)\times \overset{6}{\cancel{18}}}{\underset{1}{\cancel{3}}\times(-5)} = \frac{12}{5}$$

RHS : $(x \div y) \div z = \left[\left[\frac{-2}{3}\right] \div \left[\frac{-5}{6}\right]\right] \div 3$

$$= \left(\left(\frac{-2}{3}\right) \times \left(\frac{-6}{5}\right)\right) \div 3 = \left(\frac{(-2)\times\left(-\overset{2}{\cancel{6}}\right)}{\underset{1}{\cancel{3}}\times 5}\right) \div 3$$

$$= \frac{4}{5} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

So, $x \div (y \div z) \neq (x \div y) \div z$

Exercise 1.4

1. **Divide:-**

(i) $\frac{2}{5}$ by $\frac{3}{4}$ (ii) $\left(\frac{-3}{8}\right)$ by $\left(\frac{-2}{3}\right)$ (iii) $\left(\frac{-5}{6}\right)$ by $\frac{3}{4}$

(iv) $\left(\frac{-5}{8}\right)$ by (–3) (v) $\left(\frac{-3}{4}\right)$ by (–6) (vi) $\left(\frac{-2}{3}\right)$ by $\left(\frac{-7}{12}\right)$

(vii) $\left(\frac{-16}{21}\right)$ by $\left(\frac{-4}{9}\right)$ (viii) $\frac{10}{9}$ by $\left(\frac{-25}{12}\right)$

2. **Verify $x \div y \neq y \div x$, when**

(i) $x = \frac{5}{7}$ $y = \frac{-3}{4}$ (ii) $x = \frac{-7}{10}, y = \frac{-5}{12}$

(iii) $x = \frac{-3}{4}, y = \frac{-9}{16}$

3. **Verify $x \div (y \div z) \neq (x \div y) \div z$, when**

(i) $x = \frac{-3}{15}, y = \frac{-2}{3}, z = 2$ (ii) $x = \frac{-1}{4}, y = \frac{-3}{2}, z = \frac{-5}{6}$

4. **The product of two rational numbers is $\frac{-8}{9}$. If one of the number is $\frac{-2}{5}$, find the other.**

5. **The product of two rational numbers is –10. If one of the number is 15, find the other.**

6. **By what number should $\frac{-3}{4}$ be multiplied so that the product is $\frac{15}{16}$?**

Looking Outcome

After completion of this chapter, the students are now able to:

- *Know about number system.*
- *Apply different operations addition,subtraction, multiplication and division on rational numbers.*
- *Know about the properties of rational numbers under different operation.*

Answer

Exercise 1.1

1. (i) $\frac{-1}{12}$ (ii) $\frac{-4}{33}$ (iii) $\frac{3}{8}$ (iv) $\frac{-1}{24}$ (v) $\frac{-23}{30}$
(vi) $\frac{-1}{2}$ (vii) $\frac{11}{18}$ (viii) $\frac{8}{75}$

4. (i) $\frac{5}{11}$ (ii) $\frac{-8}{9}$ (iii) $\frac{15}{13}$ (iv) $\frac{-2}{9}$ (v) $\frac{3}{8}$
(vi) $\frac{2}{7}$ (vii) $\frac{-18}{11}$ (viii) 0

5. (i) $\frac{-12}{15}$ (ii) $\frac{1}{56}$ (iii) $\frac{-77}{18}$ (iv) $\frac{1}{2}$ (v) $\frac{47}{48}$

6. (i) b (ii) c (iii) b (iv) a

Exercise 1.2

1. (i) $\frac{2}{5}$ (ii) 1 (iii) $\frac{11}{8}$ (iv) $\frac{31}{42}$ (v) $\frac{1}{6}$

(vi) $\frac{19}{66}$ (vii) $\frac{1}{3}$ (viii) $\frac{-81}{50}$

4. (i) $\frac{17}{24}$ (ii) $\frac{-119}{24}$ (iii) $\frac{-19}{30}$ (iv) $\frac{-11}{30}$ (v) $\frac{33}{72}$

Exercise 1.3

1. (i) $\frac{35}{44}$ (ii) $\frac{-15}{28}$ (iii) $\frac{-10}{99}$ (iv) $\frac{-112}{35}$ (v) $\frac{16}{5}$ (vi) $\frac{4}{5}$

(vii) $\frac{3}{10}$ (viii) $\frac{4}{5}$ (ix) $\frac{-1}{2}$ (x) $\frac{5}{6}$ (xi) $\frac{16}{3}$ (xii) $\frac{-9}{5}$

4. (i) $\frac{-1}{2}$ (ii) $\frac{-8}{5}$ (iii) $\frac{-9}{7}$ (iv) $\frac{-4}{3}$ (v) $\frac{-35}{2}$

(vi) 2 (vii) $\frac{-3}{20}$ (viii) $\frac{-3}{7}$

7. (i) Commutative Property (ii) Existence of Identity (iii) Distribution over addition
(iv) Associative property (v) Distribution over subtraction

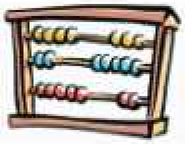
8. (i) c (ii) b (iii) a (iv) d (v) a (vi) d

Exercise 1.4

1. (i) $\frac{8}{15}$ (ii) $\frac{9}{16}$ (iii) $\frac{-10}{9}$ (iv) $\frac{5}{24}$ (v) $\frac{1}{8}$ (vi) $\frac{8}{7}$ (vii) $\frac{12}{7}$

(viii) $\frac{-8}{15}$

4. $\frac{20}{9}$ 5. $\frac{-2}{3}$ 6. $\frac{-5}{4}$



CHAPTER

2 Linear Equations in One Variable

Learning Objectives

In this chapter you will learn:

- *To solve the different types of linear equations.*
- *To use the linear equations in daily life.*
- *To tackle the practical life situations using variables.*

2.1 Introduction :-

In class VII, we have already discussed the concept of an equation in one variable and its solution. e.g. $4x = 12$, $3y = 15$, $2y + 1 = 9$ etc.

In this section, we shall discuss this concept in detail.

2.2 Equation in one Variable:-

The equation in which algebraic expression have one variable is called equation in one variable. e.g. $2y + 5 = 9$, $3z^2 - 1 = 7$, $4x^2 + 5x = 8$ etc.

In above examples, all the equations have one variable.

But $2x + 3y = 7$, $2a + b + c = 7$, $4abc = 5$ etc. are not equations in one variable as

- 1st equation has two variables i.e. x and y
- 2nd equation has three variables i.e. a, b and c.
- 3rd equation also has three variables i.e. a, b and c.

2.2.1 Linear Equation in one variable:-

The equation in one variable having degree (highest power of variable) 1 is called linear equation in one variable e.g.(i) $2x + 3 = 12$ is linear equation in one variable (x).

(ii) $5y - 3 = 8$ is linear equation in one variable (y).

(iii) $2x + 3 = 5y$ is not a linear equation in one variable. Here, we have two variables (x and y)

(iv) $3z^2 - 1 = 7$ is not a linear equation in one variable because its degree is 2.

2.2.2 Solution of Linear Equation in one variable:-

In equation, Equal (=) sign divides the equation in two parts i.e. Left part is called Left Hand Side (LHS) and Right part is called Right Hand Side (RHS) i.e. value of left part = value of right part. This is true for particular values of the variable and these particular values are called the solutions of the equation. Linear equation in one variable has one solution. That means only one value of the variable satisfies the equation.

e.g: $4x = 8$

$\text{LHS} = 4x$

(i) If $x = 1$

$4x = 4\times1=4$

$\text{RHS} = 8$

$\text{LHS} \neq \text{RHS}$

For $x = 1$,

Value of left part ≠ value of right part

$\therefore x = 1$ is not a solution of the given equation.

(ii) If $x = 2$ then LHS $= 4x = 4(2) = 8$

RHS = 8

LHS = RHS

Here , value of left part = value of right part

$\therefore x = 2$ is a solution of the given equation.

2.3 Solving equations having variables on both sides

In last section, we have discussed the equation having variable on one side and In this section, we shall discuss the equations having variables on both sides.

First, we adjust variable on one side and constants on other side then we shall solve as discussed in previous sections.

Example 2.1 : Solve:-

(i) $4x - 3 = 3x + 2$ **(ii)** $3x + 4 = x - 6$

(iii) $4x - 5 = 7x + 8$

Sol. (i) We have $4x - 3 = 3x + 2$

$4x = 3x + 2 + 3$ (transposing–3 to RHS)

$4x = 3x + 5$

transposing 3x to LHS we get

$4x - 3x = 5$

$\Rightarrow x = 5$

which is the required answer.

Or We can solve this equation by transposing variable and constant together.

we have $4x - 3 = 3x + 2$

Transposing – 3 to RHS and 3x to LHS, we get

$4x - 3x = + 2 + 3$

$\Rightarrow x = 5$

which is the required solution.

(ii) we have $3x + 4 = x - 6$

Transposing +4 to RHS and x to LHS, we get

$3x - x = -6 - 4$

$\Rightarrow 2x = -10 \quad \Rightarrow x = \frac{-10}{2} = -5$

which is the required solution.

(iii) we have, $4x - 5 = 7x + 8$

Transposing –5 to RHS and 7x to LHS, we get

$4x - 7x = + 8 + 5$

$-3x = 13$

$x = \frac{13}{-3} \Rightarrow x = \frac{-13}{3}$

which is the required solution.

Example 2.2 : Solve the following equations and verify the answer:-

(i) $8x + 4 = 3(x - 1) + 7$ **(ii)** $2x - 5 = 14 - (x - 2)$

(iii) $3(\ell - 3) = 5(2\ell + 1)$

Sol. (i) We have, $8x + 4 = 3(x-1)+7$

$\Rightarrow 8x + 4 = 3x - 3 + 7$

$\Rightarrow 8x + 4 = 3x + 4$

Transposing +4 to RHS and 3x to LHS, we get

$8x - 3x = +4 - 4$

$\Rightarrow \quad 5x = 0$

Transposing 5 to RHS, we get

$$x = \frac{0}{5} = 0$$

Verification

put x = 0

in the given equation, we get

L.H.S.	R.H.S.
$8(0) + 4$	$3(0 - 1) + 7$
$= 0 + 4$	$= -3 + 7$
$= 4$	$= 4$

L.H.S. = R.H.S.

Both sides are equal. Hence, the solution is verified.

(ii) We have, $2x - 5 = 14 - (x - 2)$

$\Rightarrow 2x - 5 = 14 - x + 2$

$\Rightarrow 2x - 5 = 16 - x$

Transposing –5 to RHS and –x to LHS, we get.

$\Rightarrow 2x + x = 16 + 5$

$\Rightarrow 3x = 21$

$$\Rightarrow x = \frac{21}{3} = 7$$

Verification

Put x = 7 in the given equation, we get

L.H.S.	R.H.S.
$2(7) - 5$	$14 - (7 - 2)$
$= 14 - 5$	$= 14 - 5$
$= 9$	$= 9$

L.H.S. = R.H.S.

Hence the solution is verified.

(iii) We have $3(\ell - 3) = 5(2\ell + 1)$

$3\ell - 9 = 10\ell + 5$

Transpose –9 to RHS and 10ℓ to LHS, we get

$3\ell - 10\ell = 5 + 9$

$-7\ell = 14$

Transpose–7 to RHS, we get

$$\ell = \frac{14}{-7} = -2$$

Verification

Put $\ell = -2$ in the given equation,

LHS	RHS
$3(-2 - 3)$	$5[2(-2) + 1]$
$= 3(-5)$	$= 5(-4 + 1)$
$= -15$	$= 5(-3)$
	$= -15$

LHS = RHS

Hence, the solution is verified

Exercise 2.1

Solve the following equations and verify the result :-

(1) $2x - 3 = x + 2$

(2) $5x - 6 = 2x + 9$

(3) $5a - 3 = 3a - 5$

(4) $5x + 9 = 5 + 3x$

(5) $4y + 3 = 6 + 2y$

(6) $3x - 1 = 15 - x$

(7) $4x + 3 = 2(x - 1) + 5$

(8) $3\ell - 5 = 4(\ell + 2) - 6$

(9) $6x = 5(x + 10) - 2$

2.4 Some Practical Applications

Example 2.3 : A positive number is 5 times another number. If 21 is added to both the numbers then larger number becomes twice the shorter number. What are the numbers?

Sol. Suppose the shorter number be x

As given : Larger number = 5 × (shorter number) = 5x

According to another condition

If 21 is added to both numbers then numbers are (x + 21) and (5x + 21)

Now, larger number = 2 × shorter number

$\Rightarrow 5x + 21 = 2(x + 21)$

$\Rightarrow 5x + 21 = 2x + 42 \Rightarrow 5x - 2x = 42 - 21$

$\Rightarrow 3x = 21$

$$\Rightarrow x = \frac{21}{3} = 7$$

So, numbers are x and 5x i.e. 7 and 5 × 7 = 35

Example 2.4 : A number consists of two digits and sum of digits is 8. If 18 is added to the number, its digits are reversed. Find the number.

Sol. We are given a two-digit number which consists ones place digit and tens place digit.

Given condition:- Sum of digits = 8

i.e. sum of ones digit & tens digit = 8

Let ones digit be x

then tens digit be 8 – x

∴ Two digit number = 10 × (tens digit) + (ones digit)

$= 10(8 - x) + x = 80 - 10x + x$

$= 80 - 9x$

Now, number obtained by reversing the digits

= 10 × (ones digit) + (tens digit)

$= 10 \times x + (8 - x) = 9x + 8$

2nd condition :- 18 added to original number = number obtained by reversing the digits

i.e. 18 + (Two-Digit number) = number obtained by reversing digits

$\Rightarrow 18 + (80 - 9x) = 9x + 8$

$\Rightarrow 98 - 9x = 9x + 8$

$\Rightarrow 98 - 8 = 9x + 9x \Rightarrow 90 = 18x$

$$\Rightarrow x = \frac{90}{18} = 5$$

∴ Two-digit number = $80 - 9x = 80 - 9(5)$

$= 80 - 45 = 35$

Example 2.5 : Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

Sol. Given Condition:-

Shobo's mother's present age = 6 × Shobo's present age

Suppose shobo's present age be x years

Shobo's mother's present age be 6x years

Now,

Shobo's age five years from now = $\frac{1}{3}$ of Mother's present age

$\Rightarrow x + 5 = \frac{1}{3} \times 6x \Rightarrow x + 5 = 2x$

$\Rightarrow 5 = 2x - x = x \quad \Rightarrow \quad x = 5$

Hence, Shobo's present age = 5 years

and Shobo's mother's present age = 6 × 5 = 30 years

Exercise 2.2

1. A number is such that it is as much greater than 84 as it is less than 108. Find it.
2. Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.
3. Find a number such that when 5 is subtracted from 5 times the number, the result is 4 more than twice the number.
4. The digits of a two digit number are differ by 3. If the digits are inter changed and the resulting number is added to the original number, we get 143. Find the original number.
5. Sum of digits of a two digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. Find the two-digit number.
6. Preet is 6 years older than Abdul. Six years ago, Preet's age was four times Abdul's age. Find their present ages.
7. After 12 years, I Shall be 3 times as old as was 4 years ago. Find my present age.
8. Jiya is twice as old as Kavya. If six years is subtracted from Kavya's age and four years added to Jiya's age, then Jiya will be four times Kavya's age. Find their present ages.

2.5 Reducing Equation to simplest form :

Example 2.6 : Solve the following equations:-

(i) $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ (ii) $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + 2$

(iii) $\frac{3x}{4} - \frac{3}{2} - \frac{-2}{3} - \frac{5x}{6}$

Sol. (i) We have, $\frac{2x}{3}+1=\frac{7x}{15}+3$

Multiply both sides of the equation by LCM of denominators (3 & 15) i.e. 15, we get.

$$\left(\frac{2x}{3}+1\right)\times 15=\left(\frac{7x}{15}+3\right)\times 15$$

$$\Rightarrow \frac{2x}{3}\times 15+1\times 15=\frac{7x}{15}\times 15+3\times 15$$

$\Rightarrow 10x + 15 = 7x + 45$

Transposing + 15 to RHS and 7x to LHS, we get

$\Rightarrow 10x - 7x = + 45 - 15$

$\Rightarrow 3x = 30$

$$\Rightarrow x=\frac{30}{3}=10$$

(ii) We have $\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+2$

Multiply both sides of the equation by LCM of denominators (2, 5, 3) i.e. 30, we get

$$\left(\frac{x}{2}-\frac{1}{5}\right)\times 30=\left(\frac{x}{3}+2\right)\times 30$$

$$\Rightarrow \frac{x}{2}\times 30-\frac{1}{5}\times 30=\frac{x}{3}\times 30+2\times 30$$

$\Rightarrow 15x - 6 = 10x + 60$

Transposing –6 to RHS and 10x to LHS, we get

$15x - 10x = 60 + 6$

$\Rightarrow \quad 5x = 66$

$$\Rightarrow \quad x=\frac{66}{5}$$

which is the required solution.

(iii) We have $\frac{3x}{4}-\frac{3}{2}=\frac{-2}{3}-\frac{5x}{6}$

Multiply both sides of the equation by LCM of denominators (4, 2, 3, 6) i.e. 12, we get

$$\left(\frac{3x}{4}-\frac{3}{2}\right)\times 12=\left(\frac{-2}{3}-\frac{5x}{6}\right)\times 12$$

$$\Rightarrow \frac{3x}{4}\times 12-\frac{3}{2}\times 12=\frac{-2}{3}\times 12-\frac{5x}{6}\times 12$$

$\Rightarrow 9x - 18 = -8 - 10x$

Transposing –18 to RHS and–10x to LHS, we get

$9x + 10x = -8 + 18$

$\Rightarrow 19x = 10 \Rightarrow x = \frac{10}{19}$

which is the required solution.

Example 2.7 : Solve the following equations:-

(i) $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$ (ii) $\frac{5x}{3} - \frac{x-1}{4} = \frac{x-3}{5}$

(iii) $\left(\frac{3a-2}{4}\right) - \frac{2a+3}{3} = \frac{2}{3} - a$

Sol. (i) We have $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$

Multiply both sides of the equation by LCM of denominators (2, 3) i.e. 6, we get

$$\left(\frac{6x+1}{2} + 1\right) \times 6 = \left(\frac{7x-3}{3}\right) \times 6$$

$$\Rightarrow \left[\frac{6x+1}{2}\right] \times 6 + 1 \times 6 = \left[\frac{7x-3}{3}\right] \times 6$$

$\Rightarrow 3(6x + 1) + 6 = 2\ (7\ x - 3)$

$\Rightarrow 18\ x + 3 + 6 = 14x - 6$

$\Rightarrow 18x + 9 = 14x - 6$

$\Rightarrow 18x - 14x = -6 - 9$

$\Rightarrow 4x = -15$

$\Rightarrow x = \frac{-15}{4}$

(ii) We have, $\frac{5x}{3} - \frac{x-1}{4} = \frac{x-3}{5}$

Multiply both sides of the equation by the LCM of denominators (3, 4, 5) i.e. 60, we get

$$\left(\frac{5x}{3} - \frac{x-1}{4}\right) \times 60 = \left(\frac{x-3}{5}\right) \times 60$$

$$\Rightarrow \frac{5x}{3} \times 60 - \left[\frac{x-1}{4}\right] \times 60 = \left[\frac{x-3}{5}\right] \times 60$$

$\Rightarrow 100x - 15\ (x - 1) = 12\ (x - 3)$

$\Rightarrow 100x - 15x + 15 = 12x - 36$

$\Rightarrow 85x + 15 = 12x — 36$

$\Rightarrow 85x - 12x = -36 - 15$

$\Rightarrow 73x = -51$

$$\Rightarrow x = \frac{-51}{73}$$

(iii) We have, $\frac{3a-2}{4} - \frac{2a+3}{3} = \frac{2}{3} - a$

Multiply both sides of the equation by LCM of denominators (4, 3, 3) i.e. 12, we get

$$\left[\frac{3a-2}{4} - \frac{(2a+3)}{3}\right] \times 12 = \left(\frac{2}{3} - a\right) \times 12$$

$$\Rightarrow \left(\frac{3a-2}{4}\right) \times 12 - \left[\frac{2a+3}{3}\right] \times 12 = \frac{2}{3} \times 12 - a \times 12$$

$\Rightarrow 3\,(3a - 2) - 4\,(2a + 3) = 8 - 12a$

$\Rightarrow 9a - 6 - 8a - 12 = 8 - 12a$

$\Rightarrow a - 18 = 8 - 12a$

$\Rightarrow a + 12a = 8 + 18 \Rightarrow 13a = 26$

$$\Rightarrow \quad a = \frac{26}{13} = 2$$

Example 2.8 : Solve the equation $15(y - 4) -2(y - 9) + 5\,(y + 6) = 0$

Sol: $15\,(y - 4) - 2\,(y - 9) + 5\,(y + 6) = 0$

$15y - 60 - 2y + 18 + 5y + 30 = 0$

$15y - 2y + 5y - 60 + 18 + 30 = 0$

$18y - 12 = 0$

$18y = 12$

$$y = \frac{12}{18}$$

$$y = \frac{2}{3}$$

Exercise 2.3

Solve the following Equations :

(1) $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$

(2) $\frac{l}{2} - \frac{1}{5} = \frac{l}{3} + \frac{1}{4}$

(3) $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

(4) $5x - 2\,(2x - 7) = 2\,(3x - 1) + \frac{7}{2}$

(5) $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

(6) $\frac{3a}{4} - \frac{a-1}{2} = \frac{a-2}{3}$

(7) $4\,(x + 2) \text{ — } 5 = 2\,(x - 1) + 7$

(8) $7(2a - 3) = 4 - 3\,(1 - a)$

(9) $3(5x - 7) - 2(9x - 11) = 4(8x - 13) - 17$

(10) $15(a - 4) - 2(a - 9) + 5(a + 6) = 0$

Example 2.23 : The Present ages of Hari and Harry are in the ratio 5:7. Four years from

Learning Outcomes

After completion of the chapter, students are now able to:

- *Solve different types of linear equations.*
- *Use linear equations in daily life.*
- *Tackle the pratical life situations using variables*

Answers

Exercise 2.1

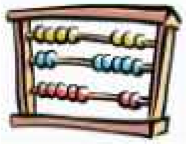
1. $x = 5$
2. $x = 5$
3. $a = -1$
4. $x = -2$
5. $y = \frac{3}{2}$
6. $x = 4$
7. $x = 0$
8. $\ell = -7$
9. $x = 48$

Exercise 2.2

1. 96
2. 14, 20
3. 3
4. 85 or 58
5. 36
6. Preet 's Present Age = 14 Years
   Abdul's Present Age = 8 years
7. 12 Years
8. Kavya's Present Age = 14 Years
   Jiya's Present Age = 28 Years

Exercise 2.3

1. $n = 36$
2. $\ell - \frac{27}{10}$
3. $m - \frac{7}{5}$
4. $x = \frac{5}{2}$
5. $x = -1$
6. $a = 14$
7. $x = 1$
8. $a = 2$
9. $x = 2$
10. $a = \frac{2}{3}$



CHAPTER

3

Understanding Quadrilaterals

Learning Objectives

In this chapter you will learn:

- *To differentiate between different types of quadrilaterals on basis of their properties and establish relationship between them.*

3.1 Introduction :-

In our daily life, we come across various plane surfaces such as blackboard in the class, a page of a book, top of a table etc.

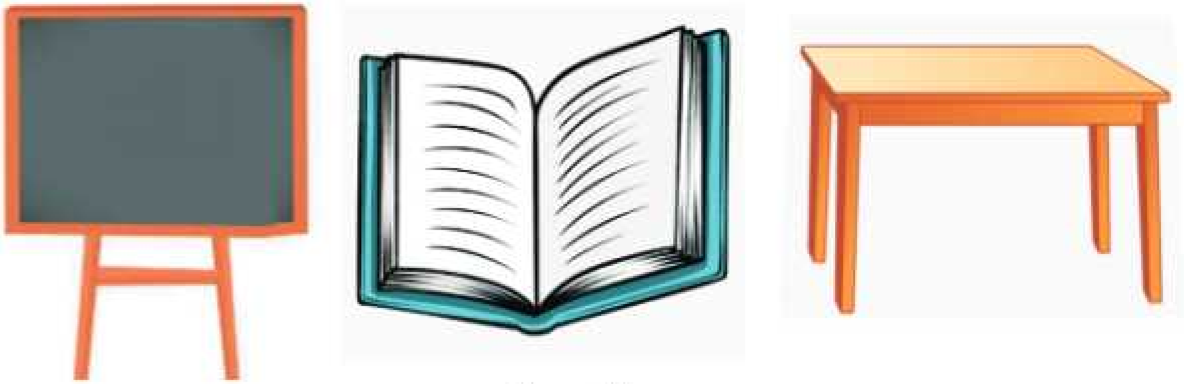


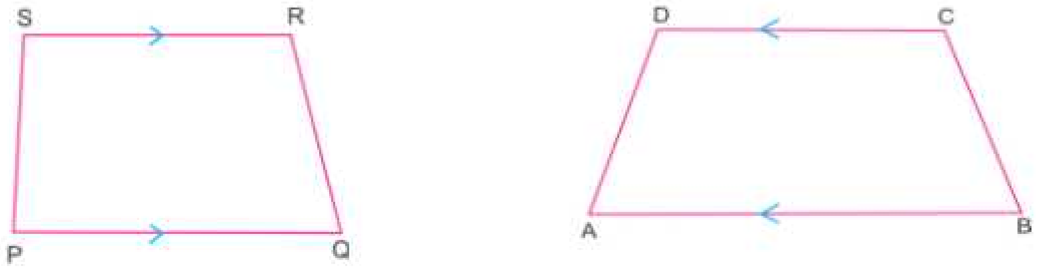
Figure 3.1

These are perfect samples or models for a plane surface. We know that a paper is the sample or model of the surface.

3.2 Types of Quadrilaterals :

In this section, we shall learn about some special types of quadrilaterals and their properties.

Trapezium : A quadrilateral having exactly one pair of parallel sides, is called a trapezium.



(i) (ii)

Figure 3.2

Note : The arrow marks indicate parallel lines.

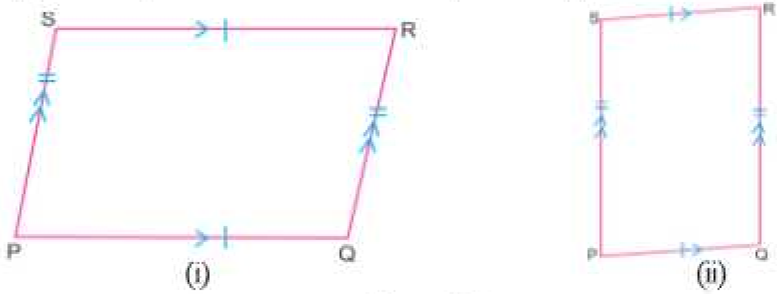
Isosceles Trapezium : A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal.

Figure 3.3

Here, quadriateral ABCD is an isosceles trapezium in which AB||DC and AD = BC.

Parallelogram : As the name of this quadrilateral suggests that it has some concern with parallel lines.

"Parallelogram is a quadrilateral if both of its pair of opposite sides are parallel or equal."



(i) (ii)

Figure 3.4

Here, quadrilateral PQRS is a parallelogram in which
PQ||RS and PS||QR or PQ=RS and PS=QR.

3.3 Properties of a Parallelogram :

Property 1 : In a parallelogram, opposite sides are equal.

Proof : Let ABCD be a parallelogram. Draw its diagonal AC.

In ΔABC and ΔCDA, We get

$\angle 3 = \angle 1$

$\angle 2 = \angle 4$ [alternate interior angles

AC = AC (Common)

∴ ΔABC ≅ ΔCDA (ASA congruency)

⇒ AB = CD and BC = DA

Thus, in a parallelogram, the opposite sides are equal.

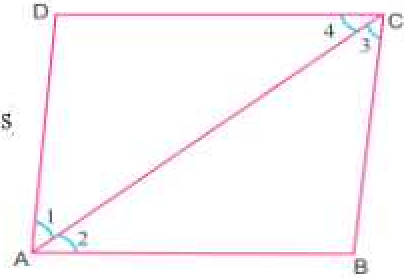


Figure 3.5

Example 3.1 : Find x and y in the following parallelograms

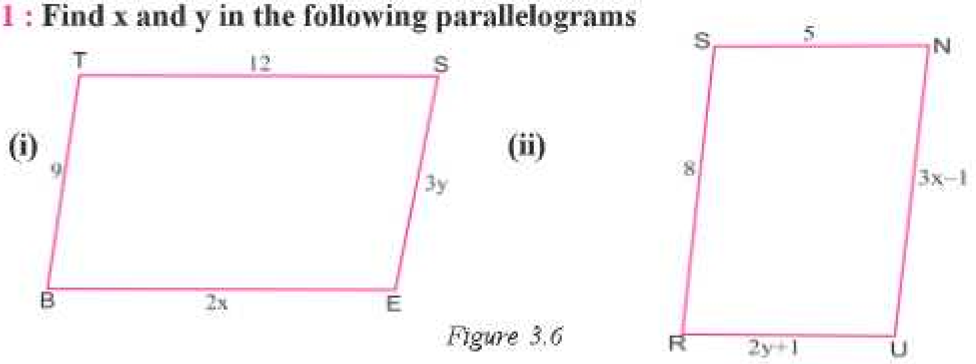


Figure 3.6

Sol. (i) Given, BEST is a parallelogram.

We know, the opposite sides of a parallelogram are equal.

$\therefore$ BE = ST and ES = BT

$2x = 12$ | $3y = 9$

$x = \frac{12}{2} = 6$ | $y = \frac{9}{3} = 3$

(ii) Given, RUNS is a parallelogram.

We know, the opposite sides of a parallelogram are equal.

$\therefore$ RU = NS and UN = RS

$\Rightarrow$ $2y + 1 = 5$ | $3x - 1 = 8$

$2y = 5 - 1 = 4$ | $3x = 8 + 1 = 9$

$y = \frac{4}{2} = 2$ | $x = \frac{9}{3} = 3$

Property 2 : In a parallelogram, opposite angles are equal.

Proof : Let ABCD be a parallelogram. Draw its diagonal AC.

In ΔABC and ΔCDA, We get

$\angle 2 = \angle 4$

$\angle 3 = \angle 1$ [alternate interior angles]

AC = AC (common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (ASA congruency)

$\Rightarrow$ $\angle B = \angle D$ (c.p.c.t.)

Similarly, we can prove $\angle A = \angle C$ by joining diagonal BD.

Thus, In a parallelogram the opposite angles are equal.

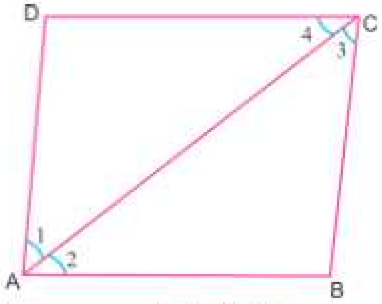


Figure 3.7

Property 3 : In a parallelogram, the sum of any two adjacent angles is 180°.

Proof : Let ABCD be a parallelogram. So AB||CD and AD is a transversal line which intersects them at A and D respectively.

As, we know that sum of interior angles on the same side of transversal between parallel lines is 180°.

$\therefore$ $\angle A + \angle D = 180°$

Similarly, we can prove that

$\angle A + \angle B = 180°$

and $\angle B + \angle C = 180°$

and $\angle C + \angle D = 180°$

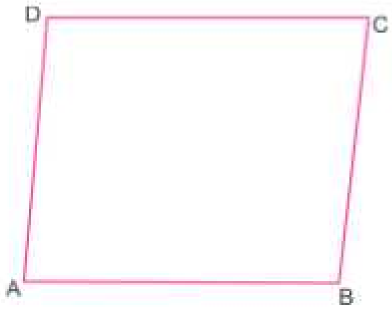


Figure 3.8

Example 3.2 : Two adjacent angles of a parallelogram are equal. What is the measure of each angle?

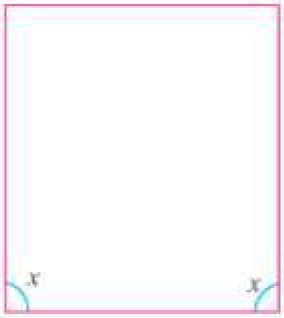


Figure 3.9

Sol. Let the measure of each of adjacent angles be x.

We know that sum of adjacent angles of a parallelogram is 180°.

$\therefore \quad x + x = 180° \quad \Rightarrow \quad 2x = 180°$

$$x = \frac{180°}{2} = 90°$$

Hence, the measure of each angle is 90°.

Example 3.3 : Two adjacent angles of a parallelogram are in 2:3. Find the measure of all the angles.

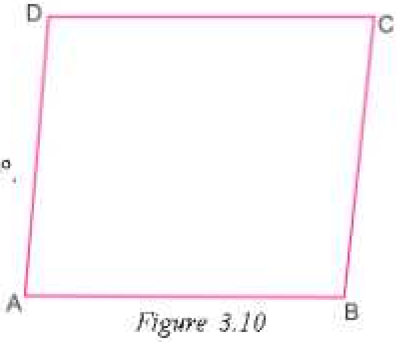


Figure 3.10

Sol. Let ABCD be a parallelogram.

Such that $\angle A : \angle B = 2 : 3$

Let $\angle A = 2x$ and $\angle B = 3x$

We know that sum of adjacent angles of a parallelogram is 180°.

$\therefore \quad \angle A + \angle B = 180°$

$\Rightarrow \quad 2x + 3x = 180° \quad \Rightarrow \quad 5x = 180°$

$$\Rightarrow \quad x = \frac{180°}{5} = 36°$$

$\therefore \quad \angle A = 2x = 2 \times 36° = 72°\ ,\ \angle B = 3x = 3 \times 36° = 108°$

Since, opposite angles of a parallelogram are equal.

$\Rightarrow \quad \angle C = \angle A = 72° \qquad \text{and} \qquad \angle D = \angle B = 108°$

Example 3.4 : In fig, RING is a parallelogram,

If ∠R = 70° then find all other angles.

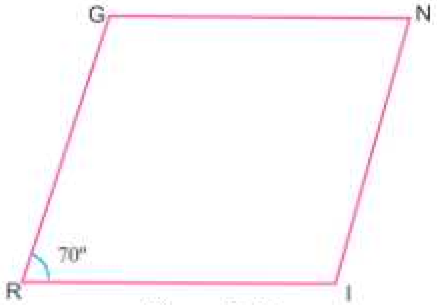


Figure 3.11

Sol. Since, sum of adjacent angles of a parallelogram is 180°

$\therefore \quad \angle R + \angle I = 180°$

$\Rightarrow \quad 70° + \angle I = 180° \quad \Rightarrow \quad \angle I = 180° - 70° = 110°$

Since, opposite angles of a parallelogram are equal.

$\Rightarrow \quad \angle N = \angle R = 70° \qquad \text{and} \qquad \angle G = \angle I = 110°$

Example 3.5 : In Fig, PQRS is a parallelogram. Find the values of x, y and z.

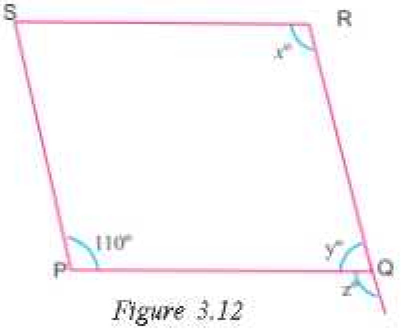


Figure 3.12

Sol. Since, opposite angles of a parallelogram are equal.

$\therefore \qquad x = 110°$

Since, sum of adjacent angles of a parallelogram is 180°

$\therefore \quad y + 110° = 180°$

$\Rightarrow \quad y = 180° - 110° = 70°$

Clearly $y + z = 180°$ (Linear Pair)

$\Rightarrow \quad 70° + z = 180° \Rightarrow 180° - 70° = 110°$

Property 4 : In a parallelogram, diagonals bisect each other.

Proof : Let ABCD be a parallelogram and draw its diagonals AC and BD intersecting each other at O.

In ΔAOB and ΔCOD, We have

$AB = CD$ (Opposite sides of a parallelogram are equal

$\angle OAB = \angle OCD$ (alternate interior angles)

$\angle OBA = \angle ODC$ (alternate interior angles)

$\therefore$ $\Delta AOB \cong \Delta COD$ (ASA congruency criterion)

$\Rightarrow$ $OA = OC$ and $OB = OD$ (cpct.)

Thus, In a parallelogram the diagonals bisect each other.

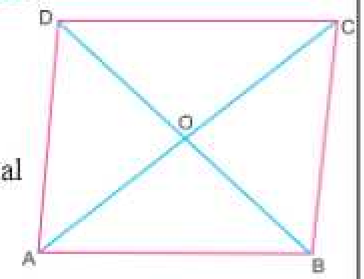


Figure 3.13

Example 3.6 : In parallelogram ABCD, the diagonals AC and BD intersect at O. If OA=3cm and OB=2.5cm then find the length of AC and BD.

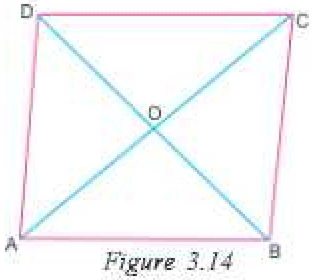


Figure 3.14

Sol. We know, the diagonals of a parallelogram bisect each other.

$\therefore$ $AC = 2\,(OA) = 2\times 3 = 6cm$

and $BD = 2(OB) = 2\times 2.5 = 5cm$

Example 3.7 : In parallelogram HELP, HL = 10cm and PE = 9cm, then find the length of HO and EO.

Sol. We know, the diagonals of a parallelogram bisect each other.

$$\therefore \quad OH = \frac{1}{2}HL = \frac{1}{2}\times 10 = 5cm$$

$$\text{and} \quad EO = \frac{1}{2}PE = \frac{1}{2}\times 9 = 4.5cm$$

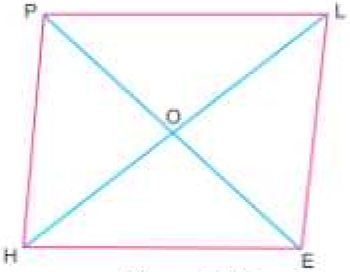


Figure 3.15

Example 3.8 : Find x & y in fig, if PQRS is a paralellogram.

Sol. We know, the diagonals of a parallelogram bisect each other.

$\therefore$ $OP = OR$ and $OQ = OS$

$\Rightarrow$ $3x + 2 = 17$ | $11 = 4y - 1$

$\Rightarrow$ $3x = 17-2 = 15$ | $4y = 11 + 1 = 12$

$x = \frac{15}{3} = 5$ | $y = \frac{12}{4} = 3$

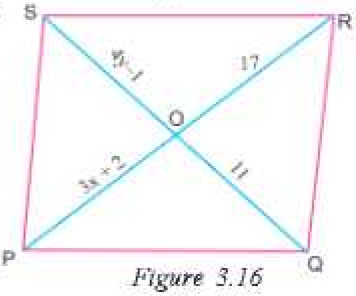


Figure 3.16

Exercise 3.1

1. The two adjacent sides of a parallelogram are 6cm and 8cm. Find the perimeter of the parallelogram.
2. Given below a parallelogram PQRS, then complete the following (Using properties of paralellogram)

(i) PQ = ...

(ii) QR = ...

(iii) ∠P = ...

(iv) ∠S = ...

(v) ∠P + ∠Q = ...

(vi) ∠R + ∠S = ...

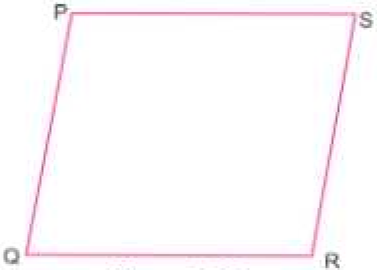
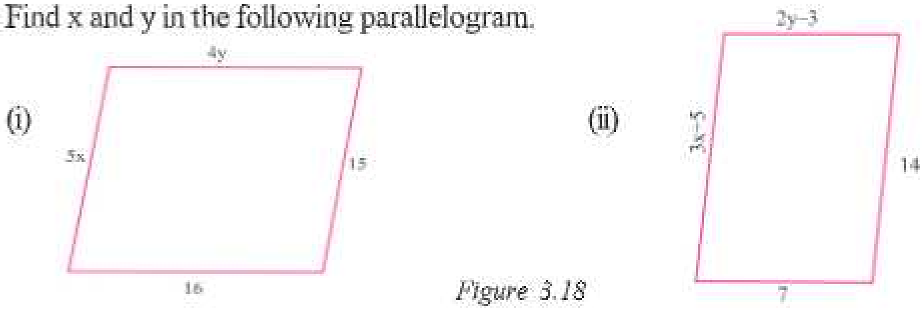


Figure 3.17

3. Find x and y in the following parallelogram.



(i) (ii)

Figure 3.18

4. Two adjacent angles of a parallelogram are in ratio 4:5. Find the measure of all the angles.

5. Two adjacent angles of a parallelogram are in ratio 3:7. Find the measure of all the angles.

6. In parallelogram WXYZ, ∠Y = 80°. Find the measure of all the angles.

Figure 3.19

7. In parallelogram BEST, ∠B = 105°, Find the measure of all the angles.

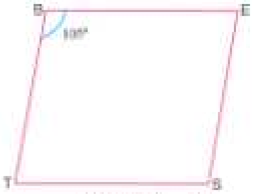


Figure 3.20

8. Find x & y in the following parallelogram.

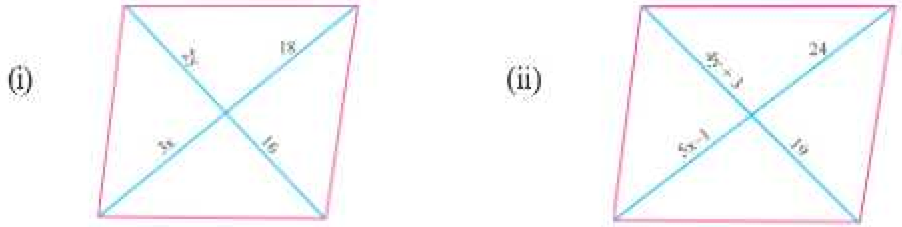
(i) (ii)

Figure 3.21

9. In parallelogram ABCD, diagonals AC and BD intersect at O. If AC = 12cm and BD = 16cm then find OA and OD.

10. In parallelogram PQRS, diagonals PR and QS intersect at O. If OP = 6cm and OS = 7cm then find PR and QS.

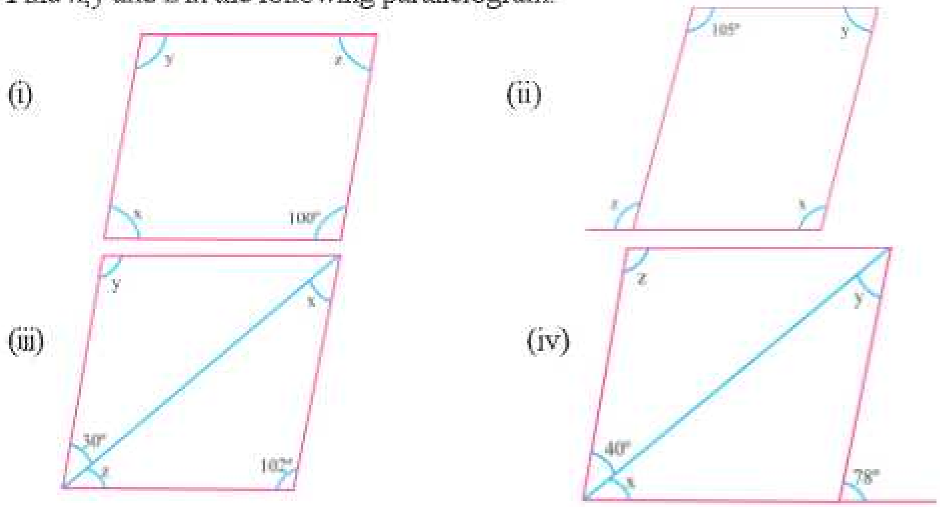
11. Find x and y in the following parallelogram.



(i) (ii)

Figure 3.22

12. Find x,y and z in the following parallelogram.



(i) (ii) (iii) (iv)

Figure 3.23

13. Multiple Choice Questions :

(i) If length of one diagonal of a rectangle is 6cm, what is the length of other diagonal?

(a) 3cm (b) 6cm (c) 12cm (d) 4cm

(ii) If 6x and 24 are two opposite sides of parallelogram, what is the value of x?

(a) 4 (b) 6 (c) 24 (d) 12

(iii) If 3x–2 and 7 are the two equal parts of a diagonal of parallelogram, what is the value of x ?

(a) 5 (b) 4 (c) 3 (d) 6

(iv) If 4y° and 100° are two opposite angles of a parallelogram then find the value of y?

(a) 25 (b) 20 (c) 100 (d) 10

3.4 Rhombus :

A rhombus is quadrilateral with sides of equal length.

Or A parallelogram having its adjacent sides equal is called a rhomb

ABCD is a rhombus in which AB = BC = CD = DA

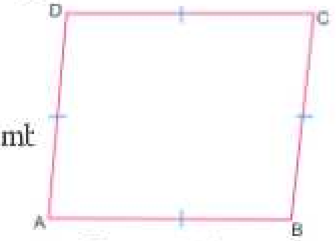


Figure 3.24

3.4.1 Properties of Rhombus:

Since rhombus is a parallelogram, so all properties of parallelogram will be contained in rhombus i.e.

(i) Opposite pair of sides are parallel.

(ii) Opposite angles are equal.

(iii) Sum of adjacent angles is 180°.

(iv) Diagonals bisect each other.

Property 5 : The diagonals of a rhombus bisect each other at right angles.

Proof : Let ABCD be a rhombus and its diagonals AC and BD intersect at O.

Since every rhombus is a parallelogram.

We know, the diagonals of a parallelogram bisect each other.

Now, To prove that the diagonals of the rhombus ABCD are perpendicular to each other.

So in ΔAOB and ΔBOC, We have

AO = OC (O is mid point of AC)

OB = OB (common)

AB = BC (Sides of a rhombus)

$\Delta AOB \cong \Delta BOC$ (SSS congruency criterion)

$\Rightarrow \quad \angle AOB = \angle BOC$ [c.p.c.t.]

Since $\quad \angle AOB + \angle BOC = 180^\circ$ (Linear Pair)

$\therefore \quad \angle AOB + \angle AOB = 180^\circ$

$$\Rightarrow \quad 2\angle AOB = 180^\circ \Rightarrow \angle AOB = \frac{180}{2} = 90^\circ$$

Thus, the diagonals of a rhombus bisect each other at 90°.

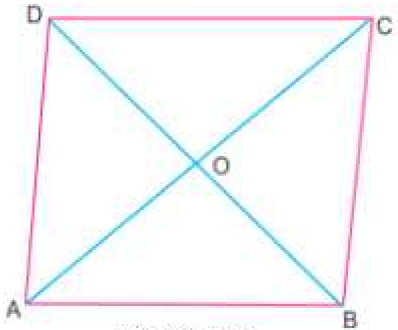


Figure 3.25

Example 3.9 : In the given figure, RICE is a rhombus. Find x, y and z.

Sol. We know, diagonals of a rhombus bisect each other.

$\therefore \quad OR = OC \Rightarrow y = 12$

and $OE = OI \Rightarrow x = 5$

Since all sides of the rhombus are equal.

IR = ER

$\therefore \quad z = 13$

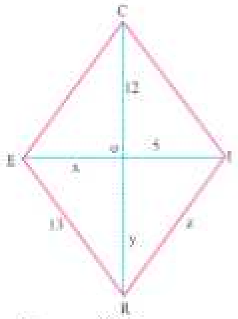


Figure 3.26

Example 3.10 : The diagonals of a rhombus are 6cm and 8cm. Find the side of the rhombus.

Sol. Let ABCD be a rhombus in which diagonals AC = 8cm and BD = 6cm.

Since diagonals of a rhombus bisect each other at 90°.

$$OA = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4cm$$

$$\text{and } OB = \frac{1}{2} BD = \frac{1}{2} \times 6 = 3cm$$

In right ΔOAB.

By Pythagoras theorem,

$AB^2 = OA^2 + OB^2$

$\Rightarrow AB^2 = 4^2 + 3^2 = 16 + 9 = 25 = 5^2$

$\Rightarrow$ AB = 5cm. Thus, side of rhombus is 5cm.

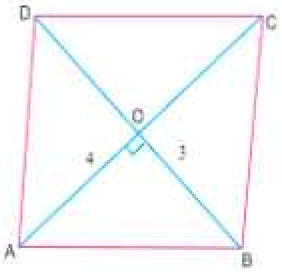


Figure 3.27

3.5 Rectangle :-

A parallelogram having all angles equal, is called rectangle.

Property 1 : Each angle of a rectangle is a right angle.

Proof: Let ABCD be a rectangle.

Since rectangle is a parallelogram having all angles equal.

$\Rightarrow \quad \angle A = \angle B = \angle C = \angle D$

We know, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\angle A + \angle A + \angle A + \angle A = 360^\circ$

$\Rightarrow \quad 4\angle A = 360^\circ \Rightarrow \angle A = \frac{360^\circ}{4} = 90^\circ$

$\therefore \quad \angle A = \angle B = \angle C = \angle D = 90^\circ$

Thus, Each angle of the rectangle is right angle.

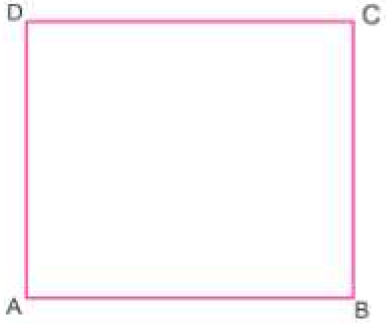


Figure 3.28

Property 2 : The diagonals of a rectangle are equal.

Proof: Let ABCD be a rectangle with diagonals AC and BD.

In ΔDAB and ΔCBA, we have

AD = BC (opposite sides of a rectangle)

AB = AB (common)

$\angle DAB = \angle CBA$ (Each 90°)

$\therefore \quad \Delta DAB \cong \Delta CBA$ (SAS congruency criterion)

$\Rightarrow \quad BD = AC$ (c.p.c.t.)

Hence, the diagonals of a rectangle are equal.

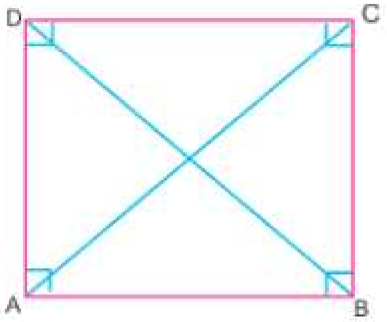


Figure 3.29

Example 3.11 : In the given figure PQRS is a rectangle in which $\angle QPR = 32^\circ$. Find $\angle PRQ$.

Sol. We know, each angle of a rectangle is 90°.

In ΔPQR, we have

$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$

$\Rightarrow \quad 32^\circ + 90^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \quad 122^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \quad \angle PRQ = 180^\circ - 122^\circ = 58^\circ$

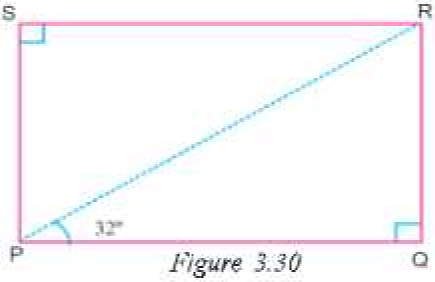


Figure 3.30

Example 3.12 : In the given figure, ABCD is a rectangle. Find x and y.

Sol. We know, opposite sides of a rectangle are equal.

$\therefore \quad BC = AD$	$AB = CD$
$\Rightarrow \quad 5x - 1 = 24$	$\Rightarrow \quad 2y - 3 = 5$
$\Rightarrow \quad 5x = 24 + 1 = 25$	$\Rightarrow \quad 2y = 5 + 3 = 8$
$\Rightarrow \quad x = \frac{25}{5} = 5$	$\Rightarrow \quad y = \frac{8}{2} = 4$

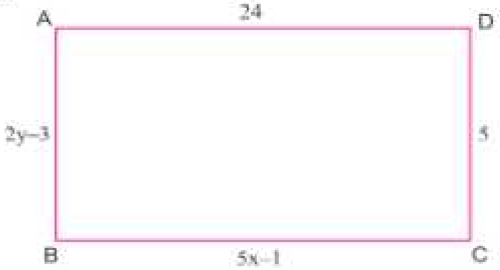


Figure 3.31

Example 3.13 : In the given figure, RENT is a rectangle and its diagonals meet at O. Find x.

Sol. Since, diagonals of a rectangle are equal and bisect each other.

$$\therefore \quad RN = TE \Rightarrow \frac{1}{2}RN = \frac{1}{2}TE$$

$$\Rightarrow \quad OR = OT$$

$$\Rightarrow \quad 3x+4 = 2x+7 \quad \Rightarrow \quad 3x - 2x = 7 - 4$$

$$x = 3$$

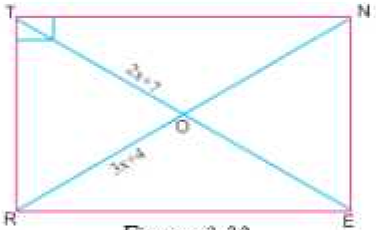


Figure 3.32

3.6 Square :-

A rectangle having all sides equal is called a square.

OR

A rhombus having all angles equal is called a square.

Since square is rhombus as well as rectangle. So all properties of rhombus and rectangle contained in square i.e.

Figure 3.33

(i) All the sides are equal.

(ii) Each angle is 90°.

(iii) The diagonals are of equal length.

(iv) The diagonals bisect each other at 90°.

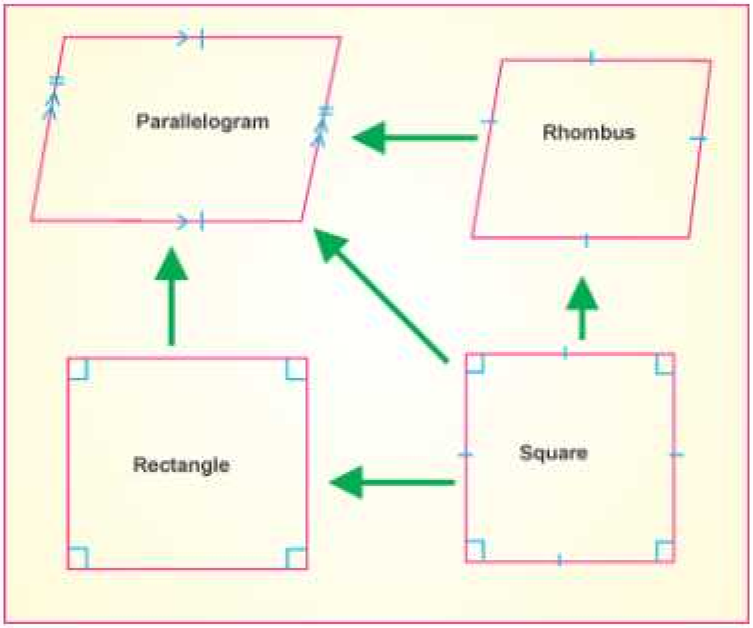


Figure 3.34

Exercise 3.2

1. Identify the quadrilateral in which
   (i) all angles are equal.
   (ii) opposite sides are equal.
2. Identify the quadrilateral in which
   (i) all the sides are equal
   (ii) each of the angle is 90°.
3. Identify the quadrilateral in which
   (i) diagonals bisect each other at 90°
   (ii) diagonals are equal in length.

4. In the given figure, RACE is a rectangle find x, y and z.

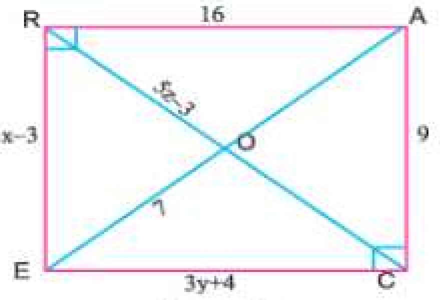


Figure 3.35

5. In the given figure, PQRS is a rhombus find x and y.

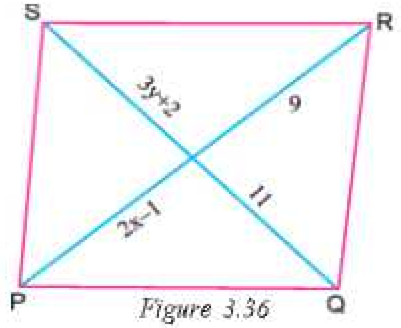


Figure 3.36

6. In the given figure, ABCD is a rectangle ∠BAC = 36°, find ∠ACB.

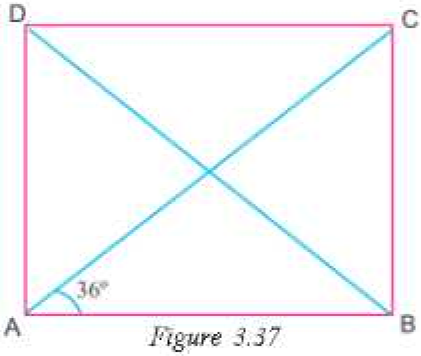


Figure 3.37

7. **Multiple Choice Questions :**

(i) Sum of adjacent angles in a parallelogram is :

(a) 90° (b) 180° (c) 360° (d) None of them

(ii) If adjacent angles of a parallelogram are equal then which polygon it will become :

(a) Rectangle (b) Rhombus (c) Square (d) Trapezium

(iii) If adjacent angles of a rhombus are equal then which polygon it will become :

(a) Rectangle (b) Square (c) Trapezium (d) Parallelogram

(iv) If 3y° and 120° are the adjacent angles of rhombus then find the value of y.

(a) 15° (b) 90° (c) 20° (d) 60°

Activities

Activity 1: Prove that the sum of interior angles of a quadrilateral is 360°.

Required Material : chart paper, geometry box, coloured pen or pencil.

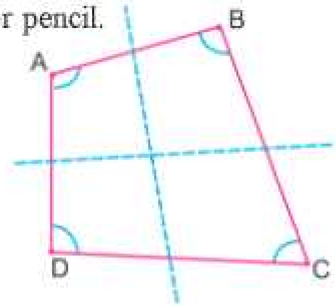


Figure 3.38

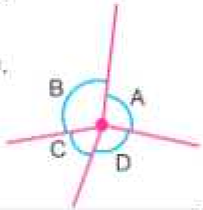
Procedure :

1. Take a chart paper and draw a quadrilateral ABCD.
2. Cut the quadrilateral from the chart as shown.
3. Cut all the four angles from the quadrilateral.
4. Draw a dot on another chart paper.
5. Paste all the angles ∠A, ∠B, ∠C and ∠D along their vertices on the dot.

Observation: All four angles after pasting along a dot form a complete circle.

∴ $\angle A + \angle B + \angle C + \angle D = 360^\circ$

i.e. Sum of all interior angles of a quadrilateral is 360°



VIVA VOCE

Q. 1. How many diagonals does a quadrilateral have?

Ans: 2

Q. 2. What is the sum of interior angles of a quadrilateral?

Ans: 360°.

Q. 3. What is the sum of exterior angles of a quadrilateral?

Ans: 360°

Activity 2 : Prove by cutting and pasting the paper that sum of exterior angles, taken in order, of any polygon is 360°.

Material Required : chart paper, geometry box, colour pen Or pencil.

Procedure:

Triangle

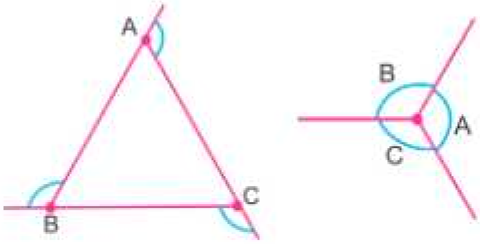


Figure 3.39

1. Take a chart paper and draw a triangle ABC.
2. Cut ΔABC from the chart paper along its exterior angles as shown.
3. Cut the exterior angles A, B, C from the triangle.
4. Draw a dot on another chart paper.
5. Paste all exterior angles ∠A, ∠B, ∠C along their vertices on dot as shown

Quadrilateral

1. Take a chart paper and draw a quadrilateral PQRS.
2. Cut PQRS from the chart paper along its exterior angles as shown.
3. Cut the exterior angles P, Q, R, S from the quadrilateral.
4. Draw a dot on another chart paper.
5. Paste all exterior angles ∠P, ∠Q, ∠R, ∠S along their vertices on a dot.

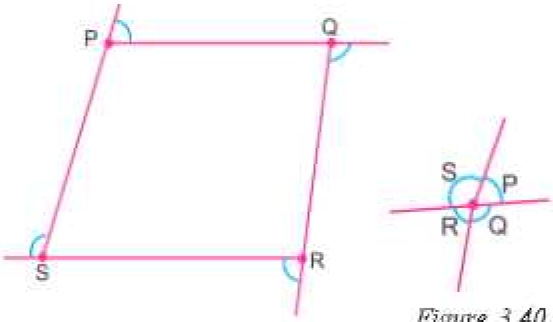


Figure 3.40

Observation: In both cases, exterior angles form a complete circle:

∴ $\angle A + \angle B + \angle C = 360^\circ$

and $\angle P + \angle Q + \angle R + \angle S = 360^\circ$

Similary, students can verify the result for other polygons.

i.e. Sum of exterior angles of a polygon is 360°.

VIVA VOCE

Q. 1. What is the sum of interior angles of a triangle?

Ans: 180°

Q. 2. What is the sum of interior angles of a pentagon?

Ans: 540°

Q. 3. What is the sum of exterior angles of hexagon?

Ans: 360°

Activity 3: Verify (i) The diagonals of a rectangle are equal in length.

(ii) The diagonals of a square are equal in length.

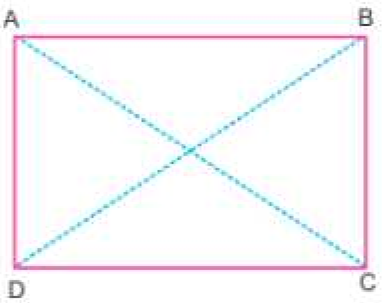
(iii) The diagonals of a rhombus and parallelogram are not equal in length.

Required Material: Chart paper, Geometry Box, Coloured Pen or Pencil.

Procedure :

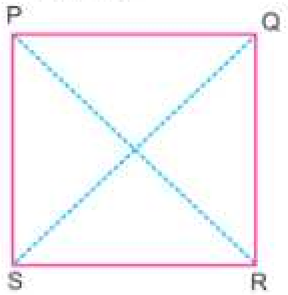
1. Take a chart paper and draw a rectangle, square, parallelogram and rhombus as shown.
2. Join the diagonals of all quadrilaterals.
3. Measure the lengths of diagonals.

(i) Rectangle

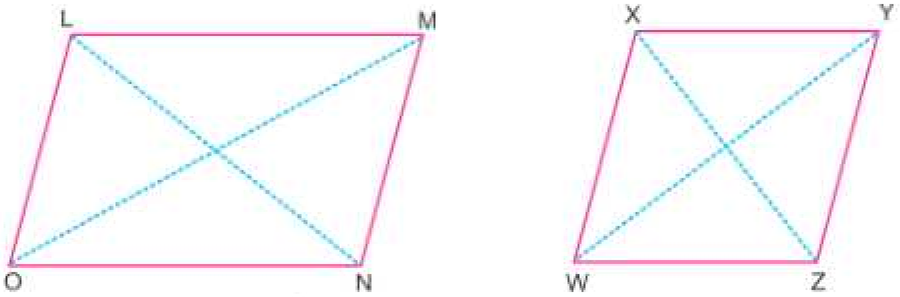


AC = ______, BD = ______

(ii) Square



PR = ______, QS = ______



(iii) Parallelogram (iv) Rhombus

NL = ______, OM = ______ XZ = ______, YW = ______

Figure 3.41

Observation:

(i) The diagonals of rectangle are equal in length.

(ii) The diagonals of the square are equal in length.

(iii) The diagonals of the parallelogram are not equal in length.

(iv) The diagonals of the rhombus are not equal in length.

VIVA VOCE

1. If one diagonal of rectangle is 6cm then what is the length of other diagonal?

Ans: 6cm.

2. In rhombus PQRS, diagonals PR=6cm and QS=8cm intersect at O then OP = ... & OS = ...

Ans: 3cm and 4cm

3. The diagonals of a square bisect each other at............ .

Ans: 90°

Learning Outcomes

After completion of the chapter, students are now able to

- *Differentiate between different types of quadrilaterals on the basis of their properties and establish relationship between them.*

Answers

Exercise 3.1

1. 28cm
2. (i) SR (ii) PS (iii) ∠R (iv) ∠Q (v) 180° (vi) 180°
3. (i) x = 3, y=4 (ii) x=3, y = 5

4. 80°, 100°, 80°, 100° **5.** 54°, 126°, 54°, 126°

6. $\angle X = 100^\circ$, $\angle W = 80^\circ$, $\angle Z = 100^\circ$ **7.** $\angle E = 75^\circ$, $\angle S = 105^\circ$, $\angle T = 75^\circ$

8. (i) $x = 20^\circ$, $y = 20^\circ$ (ii) $x = 20^\circ$, $y = 24^\circ$

9. OA = 6cm, OD = 8cm 10. PR = 12cm, QS = 14cm

11. (i) $x = 6$, $y = 8$ (ii) $x = 5$, $y = 4$

12. (i) $x = 80^\circ$, $y = 100^\circ$, $z = 80^\circ$ (ii) $x = 105^\circ$, $y = 75^\circ$, $z = 105^\circ$

(iii) $x = 30^\circ$, $y = 102^\circ$, $z = 48^\circ$ (iv) $x = 38^\circ$, $y = 40^\circ$, $z = 102^\circ$

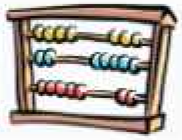
13. (i) b (ii) a (iii) c (iv) a

Exercise 3.2

1. Rectangle **2.** Square **3.** Square 4. $x = 3$, $y = 4$, $z = 2$

5. $x = 5$, $y = 3$ **6.** 54°

7. (i) b (ii) a (iii) b (iv) c



CHAPTER

4

Data Handling

Learning Objective

In this chapter, you will learn:-

- *About collection and presentation of Data in various forms.*
- *To represent the data as histograms & pie charts*
- *To interpret the histrograms and pie charts.*
- *To Use the histograms and pie chart in daily life situation.*
- *Introduction of chances and probability.*
- *To relate chances and probability to real life problems..*

4.1 Raw Data / Primary Data:

Statistics has gained very important place in the modern world and **data** is the base on which the structure of statistical investigation is made. The success and failure of investigation mainly depends upon the quality and accuracy of data. The word 'data' means 'information'. **Collection of data** is the first step in any statistical investigation. The numerical observations collected by an observer cannot be put to any use immediately and directly.So it is called **raw data** or **primary data**.

For example, the marks (out of 20) of 10 students of class 8th are 12, 15, 18, 10, 13, 19, 20, 14, 12, 10

Here, each entry in the above list is a numerical fact which is called an **observation.** Such a collection of observations gathered or collected initially is called **raw data.** So *"Primary data or raw data is the data which is originally collected by an investigator or agency for the first time for some specific purpose."*

4.2 Presentation of Data:-

After collecting data, the investigator has to find ways to present them in tabular form. Such an arrangement is called **presentation of data**.

The raw data can be arranged in following ways:

(i) Alphabetical order or Serial order.

(ii) Ascending or Descending order.

The raw data when put in ascending or descending order is called **an array**.

For Example,

The marks obtained by 10 students in a class test, out of 25 marks, according to their roll numbers be.

18, 21, 17, 13, 5, 14, 20, 24, 19, 16

This data in this form is called raw data or primary data or ungrouped data. The above data can be arranged in serial order as follows:-

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks obtained	18	21	17	13	5	14	20	24	19	16

Table 4.1

But data in this group does not give us a clear picture of the standard achievement of the group. If we arrange them in ascending or descending order then it gives us a slighty better picture.

In ascending order:-

5, 13, 14, 16, 17, 18, 19, 20, 21, 24

In descending order:-

24, 21, 20, 19, 18, 17, 16, 14, 13, 5

If the number of observations is large then arranging data in ascending or descending or serial order is very tough and tedious job. To make it easily understandable and clear, we can tabulate data in the form of **Frequency Distribution**

4.3 Frequency Distribution or Frequency Table:-

It is a method to present raw data in the form which we can easily understand. Frequency distributions or tables are of two types:-

(i) Discrete frequency distribution

(ii) Continuous frequency distribution

4.3.1. Discrete frequency Distribution:- The construction of a discrete frequency distribution from the given raw data is done by the use of tally marks.

Tally marks. In tally marks, we use the following symbols.

Numbers	1	2	3	4	5	6	7	8	9	10
Tally Marks	I	II	III	IIII	𝍸	𝍸 I	𝍸 II	𝍸 III	𝍸 IIII	𝍸 𝍸

Table 4.2

Let's discuss some examples:

Example 4.1 : A die was thrown 25 times and following scores were obtained:

1, 5, 2, 4, 3, 6, 1, 4, 2, 5, 1, 6, 2, 6, 3, 5, 4, 1, 3, 2, 3, 6, 1, 5, 2.

Prepare a frequency table for the scores.

Sol.

Number	Tally Marks	Frequency
1	𝍸	5
2	𝍸	5
3	IIII	4
4	III	3
5	IIII	4
6	IIII	4
	Total	**25**

Table 4.3

Example 4.2 : Prepare a frequency distribution of the favourite subject of a group of 22 students which is as follows:

Punjabi, Mathematics, Science, English, Punjabi, English, Mathematics, Mathematics, Science, Punjabi, Punjabi, Science, Science, Mathematics, Punjabi, English, Punjabi, English, Science, Punjabi, Science, Mathematics.

Also answer that which is the most liked subject and the least liked?

Sol. Frequency Distribution of the favourite subjects

Subjects	Tally Marks	(Number of students) Frequency
Punjabi	𝍸 II	7
Mathematics	𝍸	5
Science	𝍸 I	6
English	IIII	4
	Total	**22**

Table 4.4

It is clear from the table that maximum students like Punjabi subject and least number of students like English Subject.

Frequency:- The numbers appearing on a dice in example 4.1 and the number of students liking a subject in example 4.2 corresponding to tally marks gives frequency. "Frequency gives the number of times that a particular occurs".

4.3.2 Continuous Frequency Distribution:-

The above method of presenting the raw data is convenient and easy where the values in the raw data are largely repeating. But If the number of observations are not repeating and difference between the greatest and the smallest observations is large then we arrange the data into classes or groups. Let's discuss with some examples.

Example 4.3 : The weekly wages (in ₹) of 30 workers in a factory are

830, 835, 890, 810, 835, 869, 836, 890, 898, 845, 832, 820, 860, 833, 845, 855, 812, 808, 804, 835, 840, 835, 885, 836, 878, 840, 868, 890, 806, 840.

Using tally marks, make a frequency table with intervals as 800-810, 810-820 and so on.

Sol.

Wages	Tally marks	Frequency
800-810	III	3
810-820	II	2
820-830	I	1
830-840	𝍸 IIII	9
840-850	𝍸	5
850-860	I	1
860-870	III	3
870-880	I	1
880-890	I	1
890-900	IIII	4
	Total	**30**

Table 4.5

Note: Data presented in this manner is said to be **grouped** and the distribution obtained is called **grouped frequency distribution.** It helps us to draw meaningful results.

In Above Example:

(i) The weekly wages of most workers are between ₹ 830 and ₹ 840.

(ii) 4 workers are taking highest wages more than ₹ 890.

Now, here we shall defi2ne some terms related to **grouped frequency distribution.**

* **Class Interval:-** Each of the groups 800-810, 810-820 and so on are called **class-intervals** Here observe that 810 occurs in both classes ie 800-810 and 810-820. But it is not possible that an observation can belong simultaneously to two classes.

"To avoid this, we adopt the convention that the common observation will belong to the higher class i.e. 810 belongs to 810-820 (not to 800-810)." Similarly 820 belongs to 820-830 (not to 810-820)

* **Lower class limit:-** In class interval, smaller value is called **lower class limit.**
For example, in class interval 800-810, the **lower class limit is 800.**
Similarly, In 810-820, the lower class limit is 810.

* **Upper Class limit:-** In class interval, higher value is called **upper class limit.**
For example:- In class interval 800-810, the upper class limit is 810.
Similiarly, in 810-820, the upper class limit is 820.

* **Class width or class size:-** The difference between the upper class limit and the lower class limit is called the **class width or size** of the class interval.
Here in all class interval 800-810, 810-820 and so on, class width is 10.

* **Class Mark:-** Mid value of the each class interval is called class mark.

$$\text{i.e. class mark} = \frac{\text{Lower limit + Upper limit}}{2}$$

For example the class mark of the class interval 800-810 is as follow

$$\text{Classmark} = \frac{800+810}{2} = \frac{1610}{2} = 805$$

Now we are showing all the above terms in a single table

Class Interval	Lower Class limit	Upper Class limit	Class width of size	Class marks
800-810	800	810	10	805
810-820	810	820	10	815
820-830	820	830	10	825
830-840	830	840	10	835
840-850	840	850	10	845
850-860	850	860	10	855
860-870	860	870	10	865
870-880	870	880	10	875
880-890	880	890	10	885
890-900	890	900	10	895

Table 4.6

Example 4.4 : The marks obtained by 40 students of class VIII in Mathematics test are given below:-

18, 12, 8, 6, 8, 5, 16, 23, 12, 2, 16, 2, 23, 10, 9, 20 , 12, 5, 3, 5, 6, 7, 15, 21, 13, 13, 20, 7, 1, 21, 24, 16, 23, 18, 13, 18, 3, 7, 16, 17.

Represent the data in the form of a frequency distribution using classes 15-20, 20-25 and so on.

Sol. The minimum and maximum marks in the given raw data are 1 and 24 respectively. So classes of the above data can be 0-5, 5-10 and so on. Thus, the frequency distribution is as given below:

Frequency Distribution

Marks	Tally Marks	Frequency
0-5	𝍸	5
5-10	𝍸 𝍸 |	11
10-15	𝍸 ||	7
15-20	𝍸 ||||	9
20-25	𝍸 |||	8
Total		**40**

Table 4.7

Example 4.5 : The heights (in cm) of 30 students of class VIII are given below:-

155, 158, 154, 158, 149, 148, 160, 150, 148, 159, 161, 153, 157, 153, 162, 157, 154, 159, 151, 160, 156, 156, 152, 163, 147, 155, 152, 157, 153, 155.

Prepare frequency table with class size of 3cm.

Sol. The minimum and maximum height in the given raw data are 147 and 163. It is mentioned that class size is 3cm. So classes of the above data be 147-150, 150-153, 153-156 and so on.

Frequency Distribution

Height (in cm)	Tally marks	Frequency
147-150	||||	4
150-153	||||	4
153-156	𝍸 |||	8
156-159	𝍸 ||	7
159-162	𝍸	5
162-165	||	2
	Total	**30**

Table 4.8

Exercise 4.1

1. Following data gives the number of children in 40 families: 1, 2, 1, 5, 6, 2, 1, 3, 5, 4, 2, 6, 3, 0, 2, 4, 0, 0, 2, 3, 2, 0, 4, 1, 4, 2, 2, 3, 2, 1, 0, 5, 4, 2, 4, 3, 6, 2, 1, 2. Represent it in the form of frequency distribution.

2. In a study of number of accidents per day, the observations for 30 days were obtained as follows:-

   6, 5, 6, 3, 5, 2, 4, 3, 4, 2, 4, 2, 1, 2, 0, 2, 5, 1, 6, 4, 3, 0, 6, 5, 5, 1, 5, 6, 2, 6.

   Prepare a frequency distribution table.

3. Prepare a frequency table of the following ages (in years) of 25 students of class VIII in your school:

   13, 14, 12, 13, 14, 13, 15, 14, 13, 13, 14, 14, 12, 16, 14, 13, 14, 16, 15, 14, 13, 13, 17, 12, 13.

4. The shoppers who come to a departmental store are marked as: man (M), woman (W), girl (G) or boy (B). The following list gives the shoppers who came during the first hour in the morning.

   W W G W B M G G W W M M W W W G B M W W B G M G M W M W W W
   W M W B W M G W W W G W W M M W W M W G G M W M M W B W G G

   Prepare a frequency distribution table

5. Construct a frequency distribution table for the data on weights (in kg) of 20 students of a class using interval 30-35, 35-40 and so on

   40, 48, 33, 38, 31, 60, 53, 49, 36, 46, 34, 65, 55, 49, 41, 47, 44, 39, 38, 42.

6. Prepare a frequency distribution table of the marks (out of 50) obtained in test by 60 students of class VIII.

   21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

   Use class interval 0-10, 10-20 and so on.

7. The water tax bills (in ₹) of 30 houses in a locality are given below. Construct a grouped frequency distribution with class size of 10.

   30, 32, 54, 45, 78, 74, 112, 66, 108, 76, 14, 20, 88, 40, 44, 35, 15, 66, 95, 84, 75, 96, 110, 74, 88, 102, 34, 14, 110, 44.

8. Construct a frequency table with class intervals 0-5, 5-10 and so on of the following marks obtained by a group of 30 students in a test.

   10, 7, 5, 12, 0, 15, 25, 20, 22, 27, 17, 11, 8, 9, 6, 17, 23, 19, 31, 21, 29, 37, 31, 35, 45, 40, 49, 42, 50, 16.

9. Multiple Choice Questions :

(i) The upper limit of 20-30 is:

(a) 25 (b) 20 (c) 30 (d) 50

(ii) The lower limit of 25-35 is:

(a) 25 (b) 30 (c) 35 (d) 60

(iii) The class size of 40-60 is:

(a) 10 (b) 20 (c) 40 (d) 60

(iv) The class marks of 100-150 is:

(a) 100 (b) 120 (c) 150 (d) 125

(v) The classes 0-10, 10-20, 20-30 are

(a) Continuous (b) Discontinuous

(c) Insufficient data (d) Ungrouped data

(vi) .... gives the number of times a particular entry occurs in the given data.

(a) Frequency (b) Lower Limit (c) Upper limit (d) Class mark

(vii) If 50-60 is the class interval of grouped data, then the lower class limt is:

(a) 50 (b) 10 (c) 60 (d) 110

(viii) Tally marks are used to find which of the following?

(a) Frequency (b) Lower limits (c) Upper limits (d) None of these

(ix) Study the following frequency distribution table and answer the following:

Class Interval	**Frequency**
5-10	8
10-15	10
15-20	25
20-25	10
25-30	12
30-35	6

(A) What is the size of the class intervals?

(a) 5 (b) 10 (c) 15 (d) 20

(B) Which class has the highest frequency?

(a) 5-10 (b) 15-20 (c) 30-35 (d) 20-25

(C) Which class has the lowest frequency?

(a) 5-10 (b) 15-20 (c) 30-35 (d) 10-15

(D) Which two classes have the same frequency?

(a) 5-10 and 10-15 (b) 5-10 and 30-35

(c) 25-30 and 30-35 (d) 10-15 and 20-25

4.4 Graphical Method of Representing the data

In previous section, we have learnt how to prepare a frequency distribution table. We have discussed two types of frequency distributions. Here we shall discuss the graphical representation of the frequency distribution. In previous classes, we have learnt some graphical representations like pictograph and Bar graph. We know "A bar graph is the quickest way to represent a frequency distribution pictorially." In this section, we shall learn how to represent a grouped frequency distribution graphically. The most common graphical representation is the **Histogram.**

4.5 Histogram:- "A histogram is a graphical representation of a frequency distribution in the form of rectangles with class intervals as bases and heights proportional to corresponding frequencies such that there is no gap between any two successive intervals".

In this chapter, we shall learn construction of histogram for a continuous grouped frequency distribution with equal class intervals.

Let's discuss it with some examples!

Example 4.6 : The following table gives the marks scored by 60 students in a test.

Marks	0-10	10-20	20-30	30-40	40-50	Total
No. of students (frequency)	3	10	21	19	7	60

Table 4.9

Represent this data in the form of a histogram

Sol. We represent the marks (class intervals) along x-axis with a suitable scale and the number of students (frequency) along y-axis on a suitable scale.

- Draw the rectangular bars according to their values.

Taking class intervals as bases and the corresponding frequencies as heights we construct rectangles to obtain the histogram (Fig 5.1)

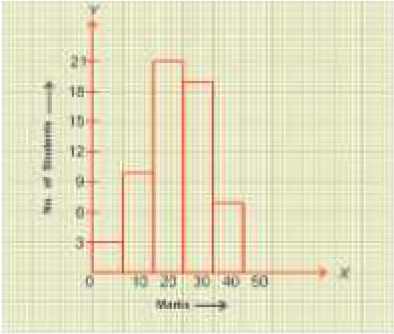


Figure 4.1

Example 4.7 : The following is the distribution of weights (in kg) of 50 persons:-

Weight (in kg)	45-50	50-55	55-60	60-65	65-70	70-75
No of persons	12	8	8	4	10	8

Table 4.10

Draw a histogram of the above data

Sol. Here, We shall represent the weight along X-axis and number of persons along y-axis with a suitable scale.

- Since the scale on X-axis starts at 45,so a **kink (break)** is indicated near the origin to signify that the graph is drawn to scale beginning at 45 and not at origin.
- Draw the rectangular bars according to their values.

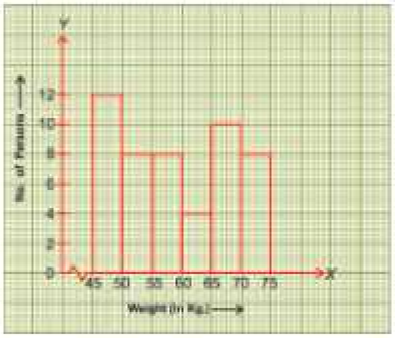


Figure 4.2

Example 4.8 : The weekly wages (in ₹) of 30 workers in a factory are:

830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840

Prepare a frequency distribution table using intervals 800-810, 810-820 and so on.

(i) Draw a histogram of the above distribution.

(ii) Which group has the maximum number of workers?

(iii) How many workers earn ₹ 850 and more?

(iv) How many workers earn less than ₹ 850?

Sol.

Frequency Distribution

Wages	Tally Marks	No of Workers (Frequency)
800-810	III	3
810-820	II	2
820-830	I	1
830-840	𝍸 IIII	9
840-850	𝍸	5
850-860	I	1
860-870	III	3
870-880	I	1
880-890	I	1
890-900	IIII	4
Total		**30**

Table 4.11

(i) Now we shall draw histogram for the above table. For this, we represent wages along x-axis and number of workers along y-axis on a suitable scale. since the scale on x-axis starts at 800. So a kink (—√—) is indicated near the origin. Now, draw the rectangular bars according to the values.

(ii) Clearly, the bar of 830-840 is highest. So group 830-840 has the maximum number of workers.

(iii) Number of workers earn ₹850 and more = 1 + 3 + 1 + 1 + 4 = 10

Figure 4.3

(iv) Number of workers earn less than ₹850 = 3 + 2 + 1 + 9 + 5 = 20

Example 4.9 : The following histogram shows the number of literate females in the age group of 10 to 40 years in a town.

(i) Write the age group in which the number of literate female is the highest?

(ii) What is the class width?

(iii) What is the lowest frequency?

(iv) In which group literate females are least?

(v) Which information does this histrogram represents?

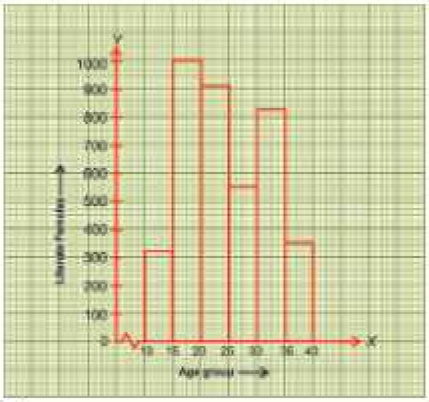


Figure 4.4

Sol. (i) The number of literate females is highest in 15-20.

(ii) Class width of all class intervals is 5.

(iii) The lowest frequency is 320.

(iv) Literate females are least in group 10-15.

(v) The Histogram represents the number of Literate females in different age group.

Example 4.10 : The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

Answer the following questions:

(i) For how many hours did the maximum number of students watch TV?

(ii) How many students watch TV for less than 4 hours?

(iii) How many students spent more than 5 hours in watching TV?

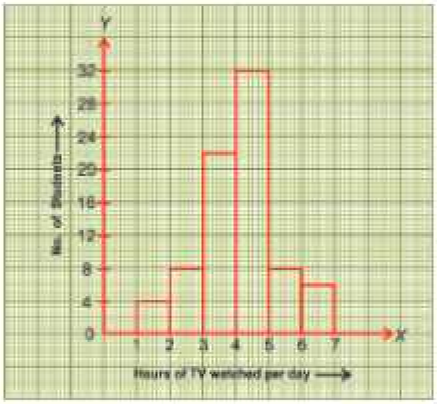


Figure 4.5

Sol. (i) The maximum number of students watch TV = 4-5 hours.

(ii) The number of students watch TV for less than 4 hours = 4 + 8 + 22 = 34

(iii) The number of students watch TV for more than 5 hours = 8 + 6 = 14

Exercise 4.2

Draw Histogram of the Following:-

1.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	6	9	12	8	10	5

2.

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	15	12	18	24	21

3.

Weight (in kg)	25-30	30-35	35-40	40-45	45-50	50-55
No. of persons	4	8	13	15	10	12

4.

Class Intervals	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	26	18	30	15	24	35

5.

Daily earnings (in ₹)	450-500	500-550	550-600	600-650	650-700	700-750
Number of persons	16	10	12	20	25	12

6. In a survey of 20 people, the amount of money with them is found to be as follows:-
104, 98, 98, 88, 91, 99, 107, 109, 116, 121, 121, 133, 146, 159, 172, 185, 197, 209, 225, 108.

Draw a histogram of the frequency distribution of above data (taking one of class intervals 50-100)

7. The marks obtained by 40 students of class VIII in an examination are given below:-
18, 8, 12, 6, 8, 16, 12, 5, 23, 16, 2, 23, 7, 12, 20, 16, 9, 7, 5, 6, 5, 3, 13, 21, 13, 20, 15, 7, 1, 21, 20, 18, 13, 23, 15, 18, 7, 17, 16, 3.
Prepare a frequency distribution using one of the class as 15-20. Draw histogram also.

8. The following histogram depicts the marks obtained by 46 students of a class.
Observe the histogram and answer the following :
   (i) What is the class size?
   (ii) How many students obtained less than 20 marks?
   (iii) How many students obtained 30 or more marks but less than 60?
   (iv) If passing marks are 30. What is the number of failures?

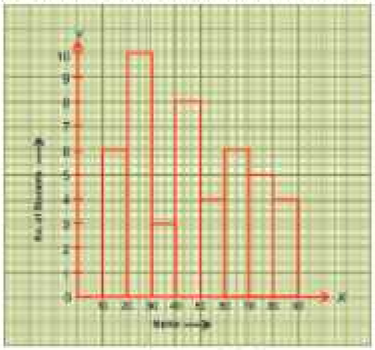


Figure 4.6

9. Observe the following histogram and answer the questions given below:-
   (i) What information is given by the graph?
   (ii) Which group has maximum girls?
   (iii) How many girls have a height of 145 cm and more?

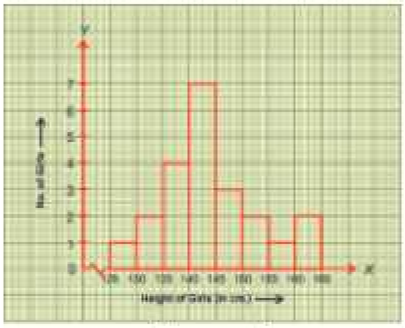


Figure 4.7

10. The following histogram shows the frequency distribution of the ages of teachers in a school :
   (i) What is the number of eldest and youngest teachers in the school?
   (ii) Which age group teachers are more in the school and which least?
   (iii) What is the class size?

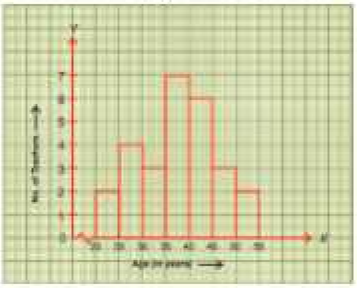


Figure 4.8

11. Multiple Choice Questions :

(A) Below is the histrogram depicting marks obtained by 44 students of a class?

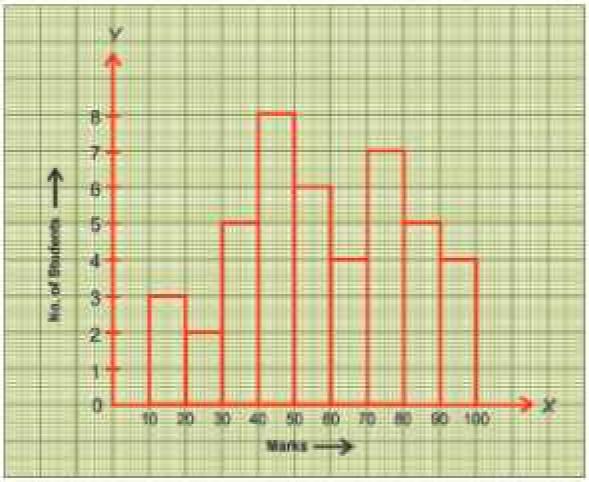


Figure 4.9

Choose the correct answer of the following:-

(i) What is the class size?

(a) 5 (b) 10 (c) 20 (d) 43

(ii) Write the number of students getting highest marks?

(a) 1 (b) 2 (c) 3 (d) 4

(iii) How many students get more than 60 marks?

(a) 20 (b) 21 (c) 22 (d) 24

(iv) How many students get less than 40 marks?

(a) 13 (b) 18 (c) 8 (d) 10

(v) In which class interval number of students are maximum?

(a) 20-30 (b) 30-40 (c) 40-50 (d) 90-100

(B) In a histogram, if all the intervals are of same size, then area of each bar is equal to

(a) Frequency (b) Height of bar

(c) Class Size (d) Class Size × frequency

12. State whether the following statements are True or False:

(i) There is no difference between bar graphs and histograms. (T/F)

(ii) Histogram is a bar graph with gap between two adjacent rectangles. (T/F)

(iii) In histogram the height of rectangle is meaningless. (T/F)

4.6 Circle Graph or Pie Chart:-

In previous section, we have learnt about classification and tabulation of data and their graphical representation by using histograms. In this section, we shall learn diagramatic representation of data.

The diagramatic representation has more preference than the graphic representatian as we can draw diagrams on any type of paper but for graphical representation, we generally need a graph paper.

In this section, we shall discuss a particular type of diagram, known as **circle-graphs** or **pie chart** to represent the given data.

"A pie chart is a pictorial representation of the numerical data by non-intersecting adjacent sectors of the circle such that area of each sector is proportional to the magnitude of the data represented by the sector.

4.6.1 Drawing a Pie Chart:- Pie Chart shows the relationship between a whole and its parts. The whole is divided into sectors. The size of each sector is proportional to the information it represents.

We know that the area of a sector is proportional to the angle subtended by it at the centre by its arc. So, sector angles or central angles are proportional to the component values of the components represented by the sectors. The total angle of a circle is 360°.

$$\text{Thus, central angle of the components} = \frac{\text{Value of the component}}{\text{Sum of the Component values}} \times 360^\circ$$

Let's discuss some examples for construction of a pie chart.

Example 4.11 : The favourite flavours of ice-creams for students of a school are given in percentage as follows:-

Flavours	Vanilla	Chocolate	Other Flavours
Percentage of students	50%	25%	25%

Represent this data in a pie chart.

Sol. The total angle at the centre of the circle is 360°. The central angles of the sectors will be a fraction of 360°.

$$\therefore \text{ Central angle of the components} = \frac{\text{Value of the component}}{\text{Sum of the Component values}} \times 360^\circ$$

Flavours	**Percentage of Students**	**Fraction** = $\frac{\textbf{components}}{\textbf{sum of components}}$	**Central Angle**
Vanilla	50%	$\frac{50}{100}=\frac{1}{2}$	$\frac{1}{2}\times 360^\circ = 180^\circ$
Chocolate	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}\times 360^\circ = 90^\circ$
Others	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}\times 360^\circ = 90^\circ$
Total	100		360°

Table 4.12

Steps for construction of a Pie Chart

1. Draw a circle with any convenient radius. Mark its centre O and radius OA.

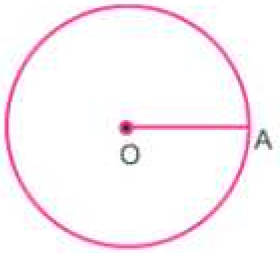


Figure 4.10

2. The angle of the sector of Ist component (Vanilla) is 180°. Use the protractor to draw $\angle AOB = 180°$ with base OA.

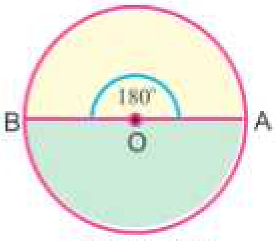


Figure 4.11

3. The angle of the sector of 2^{nd} component (chocolate) is 90°. Use the protractor to draw $\angle BOC = 90°$ with base OB.

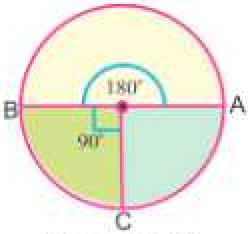


Figure 4.12

4. Now, we observe that the remaining angle of the sector $\angle COA = 90°$ with base OC, which represents the last component. It is our required Pie chart

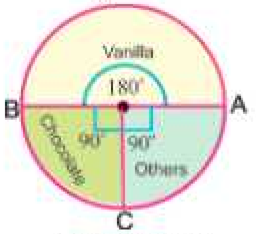


Figure 4.13

Example 4.12 : Draw a pie chart showing the following information. The table shows the colour preferred by a group of people.

Colours	Blue	Green	Red	Yellow	Total
No. of People	9	18	6	3	36

Sol. We know that

$$\text{Central angle of a component} = \frac{\text{component's value}}{\text{sum of all the values}} \times 360°$$

Colours	No. of People	Fractions	Central angle
Blue	9	$\frac{9}{36} = \frac{1}{4}$	$\frac{1}{4} \times 360° = 90°$
Green	18	$\frac{18}{36} = \frac{1}{2}$	$\frac{1}{2} \times 360° = 180°$
Red	6	$\frac{6}{36} = \frac{1}{6}$	$\frac{1}{6} \times 360° = 60°$
Yellow	3	$\frac{3}{36} = \frac{1}{12}$	$\frac{1}{12} \times 360° = 30°$
Total	**36**		**360°**

Table 4.13

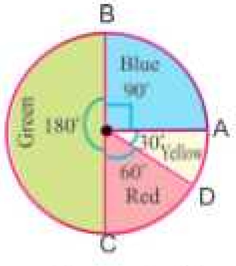


Figure 4.14

Example 4.13 : The number of hours spent by a school boy on different activities in a working day, is given below:-

Activity	Sleep	School time	Home work	Play	Others	Total
No. of hours	8	7	4	2	3	24

Draw a pie chart for this data.

Sol. We know

$$\text{Central angle of a component} = \frac{\text{Component's value}}{\text{Sum of all values}} \times 360^\circ$$

Activity	No. of hours	Fraction Part	Central angle
Sleep	8	$\frac{8}{24} = \frac{1}{3}$	$\frac{1}{3} \times 360^\circ = 120^\circ$
School time	7	$\frac{7}{24}$	$\frac{7}{24} \times 360^\circ = 105^\circ$
Home work	4	$\frac{4}{24} = \frac{1}{6}$	$\frac{1}{6} \times 360^\circ = 60^\circ$
Play	2	$\frac{2}{24} = \frac{1}{12}$	$\frac{1}{12} \times 360^\circ = 30^\circ$
Others	3	$\frac{3}{24} = \frac{1}{8}$	$\frac{1}{8} \times 360^\circ = 45^\circ$
Total	**24**		**360°**

Table 4.14

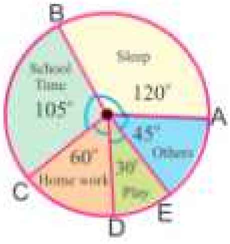


Figure 4.15

Example 4.14 : The following data shows the expenditure of a person on different items during a month. Represent the data by a pie chart

Items	Rent	Education	Food	Clothing	Others
Amount (in ₹)	2700	1800	2400	1500	2400

Sol.

Items	Amount	Fraction	Central angle
Rent	2700	$\frac{2700}{10800} = \frac{1}{4}$	$\frac{1}{4} \times 360^\circ = 90^\circ$
Education	1800	$\frac{1800}{10800} = \frac{1}{6}$	$\frac{1}{6} \times 360^\circ = 60^\circ$
Food	2400	$\frac{2400}{10800} = \frac{2}{9}$	$\frac{2}{9} \times 360^\circ = 80^\circ$
Clothing	1500	$\frac{1500}{10800} = \frac{5}{36}$	$\frac{5}{36} \times 360^\circ = 50^\circ$
Others	2400	$\frac{2400}{10800} = \frac{2}{9}$	$\frac{2}{9} \times 360^\circ = 80^\circ$
Total	**10800**		**360°**

Table 4.15

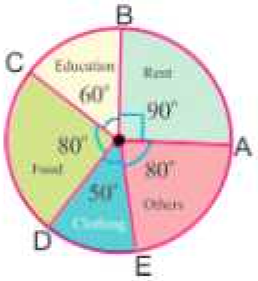


Figure 4.16

4.6.2 Reading of Pie Chart:-

In previous section, we have learnt about construction of a pie chart. In this section, we shall obtain information related to given data from the given pie chart.

We know,

$$\text{Central Angle} = \frac{\text{Value of Component}}{\text{Sum of all values}} \times 360^\circ$$

$$\text{Or} \quad \text{Value of Component} = \frac{\text{Central angles} \times \text{sum of values of all components}}{360^\circ}$$

$$\text{and Percentages value of a component} = \frac{\text{Central angle of component}}{360^\circ} \times 100$$

These formulae will be used to find the values of various components of the data from pie chart.

Example 4.15 : The given pie chart shows the marks obtained by Anita in differnt subjects. If the total marks is 540 then find the marks obtained in each subject

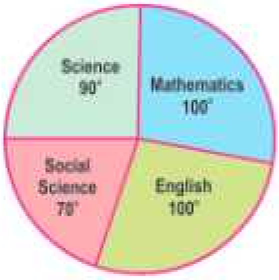


Figure 4.17

Sol. Here total marks = 540

We know that

$$\text{Marks obtained in each subject} = \frac{\text{Central angle of subject}}{360^\circ} \times \text{Total marks}$$

$$= \frac{\text{Central angle of subject}}{360^\circ} \times 540$$

Subject	**Central Angle**	**Fraction**	**Marks**
Mathematics	100°	$\frac{100^\circ}{360^\circ} = \frac{5}{18}$	$\frac{5}{18} \times 540 = 150$
Science	90°	$\frac{90^\circ}{360^\circ} = \frac{1}{4}$	$\frac{1}{4} \times 540 = 135$
Social Science	70°	$\frac{70^\circ}{360^\circ} = \frac{7}{36}$	$\frac{7}{36} \times 540 = 105$
English	100°	$\frac{100^\circ}{360^\circ} - \frac{5}{18}$	$\frac{5}{18} \times 540 - 150$
Total	**360°**		**540**

Table 4.16

Example 4.16 : The pie chart shows the marks obtained by a student in various subjects. If the student scored 180 marks in Mathematics. Find the

(i) Total Marks obtained

(ii) Marks obtained in each subject

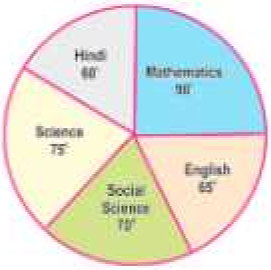


Figure 4.18

Sol. (i) Marks obtained in Mathematics = 180

We know that

$$\text{Value of a Component} = \frac{\text{Central angle}}{360^\circ} \times \text{Sum of all values}$$

$$\Rightarrow 180 = \frac{\text{Central Angle of Mathematics}}{360^\circ} \times \text{Total marks}$$

$$\Rightarrow 180 = \frac{90^\circ}{360^\circ} \times \text{Total marks}$$

$$\Rightarrow \text{Total marks} = 180 \times \frac{360^\circ}{90^\circ} = 720$$

(ii)

Subject	Central Angle	Marks
Hindi	60°	$\frac{60^\circ}{360^\circ} \times 720 = 120$
Science	75°	$\frac{75^\circ}{360^\circ} \times 720 = 150$
Social Science	70°	$\frac{70^\circ}{360^\circ} \times 720 = 140$
English	65°	$\frac{65^\circ}{360^\circ} \times 720 = 130$
Mathematics	90°	$\frac{90^\circ}{360^\circ} \times 720 = 180$
Total	**360°**	**720**

Table 4.17

Exercise 4.3

Draw a pie chart for the following (1-5) :-

1. The number of Students in a hostel speaking different languages is given below:-

Language	Hindi	Punjabi	English	Marathi	Tamil	Bengali	Total
No. of students	10	30	12	9	7	4	72

2. The number of students admitted in different faculties of a college are given below:-

Faculty	Science	Arts	Commerce	Law	Education	Total
No. of Students	1000	1200	650	450	300	3600

3. The following data represents the expenditure of a family in different items:

Items	Food	Clothing	Rent	Education	Others
Expenditure (in ₹)	4000	2000	1500	1500	1000

4. In one day, the sales (in ₹) of different items of a bakery are given below:-

Items	Ordinary Bread	Fruit Bread	Cakes	Biscuits	others
Sales (in ₹)	260	40	100	60	20

5. The following data gives the amount spent on the construction of a house.

Items	Cement	Timber	Bricks	Labour	Steel	others
Expenditure (in thousand rupees)	60	30	45	75	45	45

6. The given pie chart shows the annual agricultural production of an Indian state. If the production of all the commodities is 8100 tonnes, find the production of each product.

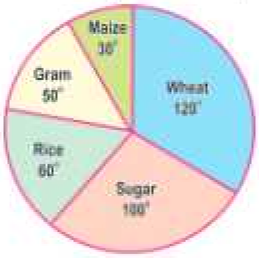


Figure 4.19

7. The given pie chart shows the marks obtained by a student in an examination. If the student secure 880 marks in all, calculate her marks in each subject.

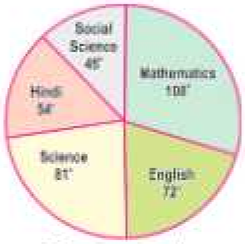


Figure 4.20

8. The following pie chart shows the monthly expenditure of a family. If the family spends ₹1650 on clothing. Answer the following question.

(i)What is the total monthly expenditure of family?

(ii) Find expenditure on each item.

Figure 4.21

9. The given pie chart shows the marks obtained by a student in various subjects. If the student scored 135 marks in mathematics , find the

(i) Total Marks obtained

(ii) Marks obtained in each subject

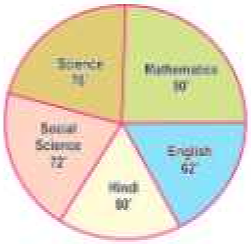


Figure 4.22

10. Multiple Choice Questions :

(i) A diagram which is used to represent data by dividing a circle into sectors is called

(a) Bar Graph (b) Histogram (c) Pie Chart (d) None of these

(ii) Sum of all central angles in a pie chart is

(a) 90° (b) 180° (c) 360° (d) 270°

(iii) In a class of 40 students, if 8 students take gardening as a hobby, the central angle of the sector representing the students who have taken gardening as a hobby.

(a) 72° (b) 90° (c) 50° (d) 30°

(iv) If 60% of students of a school speak Punjabi, then what is the central angle of the sector representing the students who speak punjabi.

(a) 126° (b) 216° (c) 144° (d) 162°

(v) If the central angle of a sector representing the playing cricket in a particular school is 108°, the percentage of the students playing cricket in a school?

(a) 30% (b) 45% (c) 90% (d) 72%

11. Multiple Choice Questions :

The following pie chart shows the monthly expenditure of Shikha on various items. If she spends ₹ 36000 per month, then choose the correct option of the following questions:

Figure 4.23

(i) How much does she spend on food?

(a) ₹ 8100 (b) ₹ 3600

(c) ₹ 13500 (d) ₹ 10800

(ii) How much does she spend on rent?

(a) ₹ 8100 (b) ₹ 3600 (c) ₹ 13500 (d) ₹ 10800

(iii) How much does she spend on others?

(a) ₹ 8100 (b) ₹ 3600 (c) ₹ 13500 (d) ₹ 10800

(iv) How much does she spend on Education?

(a) ₹ 8100 (b) ₹ 3600 (c) ₹ 13500 (d) ₹ 10800

4.7 Chance and Probability:-

In our daily life, we come across many statements like

(i) Probably you are right

(ii) It may rain today

(iii) Indian team has good chance of winning.

In such statements, we generally use the terms: May, probable, likely, chance etc. All these terms represent the same sense that the event is not certain to take place. Such examples where the chances of a certain thing happening or not happening are not equal, This is the rough idea of meaning of 'probability'. However, in the theory of probability, we assign numerical value to the degree of uncertainity. The concept of probability originated in the beginning of 18th century in problems like throwing a die, tossing a coin, drawing a card from a pack of cards etc. Starting with game of chance, Now it is commonly used in our day-to-day conversation.

Getting a Result:- We know that before a cricket match starts, captains of the two teams go out to toss a coin to decide that which team will bat first.

- When a coin is tossed then the possible results are head or tail.

Such an experiment is called a **random experiment** and Head or Tail are the two outcomes of this experiment and getting a head or tail is called an **event**.

For example:- When a die is thrown then outcomes are 1, 2, 3, 4, 5 or 6 and throwing a die is called a random experiment.

Random experiment or Event	Outcomes
1. Tossing a coin	Head or tail
2. Throwing a die	1, 2, 3, 4, 5, or 6

4.7.1 Linking chances to probability:-

- Consider the experiment of tossing a coin once. The outcomes are head or tail. Since both outcomes has same chance of occuring. Such outcomes are called **'equally likely outcomes'**. Here, total number of outcomes = 2.

  Now, getting a head is one out of two outcomes i.e. $\frac{1}{2}$, we say that the probability of getting a head = $\frac{1}{2}$ and getting a tail is one out of two outcomes, i.e. $\frac{1}{2}$, we say that the probability of getting a tail = $\frac{1}{2}$

Total outcomes	**Events**	**Probability**
2	Number of heads = 1	1 out of 2 i.e. $\frac{1}{2}$
	Number of tails = 1	1 out of 2 i.e. $\frac{1}{2}$

Table 4.18

- Consider the experiment of throwing a die once. The outcomes are 1, 2, 3, 4, 5, or 6. All outcomes are equally likely outcomes.

Total outcomes	**Events**		**Probability**
6	Number 1	1	1 out of 6, i.e. $\frac{1}{6}$
	Number 2	1	1 out of 6, i.e. $\frac{1}{6}$
	Number 3	1	1 out of 6, i.e. $\frac{1}{6}$
	Number 4	1	1 out of 6, i.e. $\frac{1}{6}$
	Number 5	1	1 out of 6, i.e. $\frac{1}{6}$
	Number 6	1	1 out of 6, i.e. $\frac{1}{6}$

Table 4.19

From the above table, it is clear that probability of an event is

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes of the experiment}}$$

Let's discuss some examples :

Example 4.17 : A coin is tossed once. What is the probability of getting a (i) Head (ii) Tail.

Sol. In a single toss of a coin, we can get any one of two faces, head or tail

So, total number of outcomes of the experiment = 2

(i) Getting a head is 1 out of 2 outcomes i.e. P (getting a head) = $\frac{1}{2}$

(ii) Getting a tail is 1 out of 2 outcomes i.e. P (getting a tail) = $\frac{1}{2}$

Figure 4.24

Example 4.18 : A bag has 5 red balls and 3 blue balls. A ball is drawn from the bag without looking into the bag. What is the probability of getting

(i) a red ball (ii) a blue ball?

Sol. Total number of balls (outcomes) in the bag = 5 + 3 = 8

(i) Number of Red balls = 5 out of 8

∴ P (getting a red ball) = $\frac{5}{8}$

(ii) Number of Blue balls = 3 out of 8

∴ P (getting a blue ball) = $\frac{3}{8}$

Example 4.19 : In the given figure a square is divided into 9 equal parts. One part is selected at random, find the probability that selected part is

(i) Shaded part (ii) Unshaded part

Sol. Total boxes = 9

(i) Number of shaded boxes = 5 out of 9

∴ P (shaded part) = $\frac{5}{9}$

(ii) Number of unshaded boxes = 4 out of 9

∴ P (unshaded part) = $\frac{4}{9}$

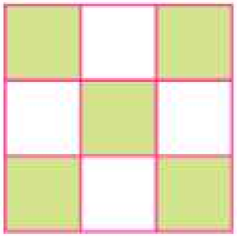


Figure 4.25

Example 4.20 : In the given figure, a circle is divided into 8 equal parts where, R represents red colour and G represents green colour. One part is selected at random then find the probability of getting

(i) A green colour (ii) A red colour

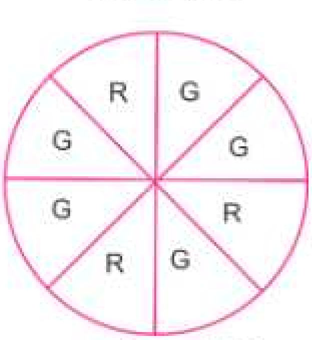


Figure 4.26

Sol. Total parts = 8

(i) Number of parts having G = 5 out of 8

$$\therefore \text{P (green colour)} = \frac{5}{8}$$

(ii) Number of parts having R = 3 out of 8

$$\therefore \text{P (red colour)} = \frac{3}{8}$$

Example 4.21 A die is thrown, Find the probability of getting

(i) a prime number (ii) number 2 or 4

(iii) number greater than 4 (iv) an odd number

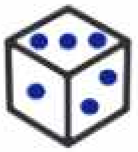


Figure 4.27

Sol. Total outcomes = 6

(i) Prime numbers = 2, 3, 5

Number of prime numbers = 3 out of 6

$$\therefore \text{P (prime number)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Number of outcomes of 2 or 4 = 2 out of 6

$$\therefore \text{P (getting 2 or 4)} = \frac{2}{6} = \frac{1}{3}$$

(iii) Number greater than 4 = 5, 6

so number of outcomes of greater than 4 = 2 out of 6

$$\therefore \text{P (number greater than 4)} = \frac{2}{6} = \frac{1}{3}$$

(iv) Odd numbers = 1, 3, 5

Number of odd numbers = 3 out of 6

$$\therefore \text{P (an odd number)} = \frac{3}{6} = \frac{1}{2}$$

Example 4.22 A bag contains 5 red marbles, 7 white marbles, 3 blue marbles. What is the probability that if one marble is taken out of the bag at random, it will be

(i) red (ii) white (iii) blue (iv) not white

Sol. Total marbles in the bag = 5 + 7 + 3 = 15

(i) Number of red marbles = 5 out of 15

$$\therefore \text{P (a red marble)} = \frac{5}{15} = \frac{1}{3}$$

(ii) Number of white marbles = 7 out of 15

$$\therefore \text{P (a white marble)} = \frac{7}{15}$$

(iii) Number of blue marbles = 3 out of 15

$\therefore$ P (a blue marble) = $\frac{3}{15} = \frac{1}{5}$

(iv) Number of marbles which are not white = 5 + 3 = 8 (out of 15)

$\therefore$ P (not white) = $\frac{8}{15}$

Example 4.23 20 cards numbered 1,2,3, ___, 20 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is

(i) an odd number (ii) a prime number
(iii) divisible by 3 (iv) single digit

Sol. Total number of cards = 20

(i) Odd numbers = 1, 3, 5, 7, 9, 11,13, 15, 17, 19
Number of odd numbers = 10 out of 20

$\therefore$ P (an odd number) = $\frac{10}{20} = \frac{1}{2}$

(ii) Prime numbers =2,3,5,7,11,13,17,19
Number of prime numbers = 8 out of 20

$\therefore$ P (a prime number) = $\frac{8}{20} = \frac{2}{5}$

(iii) Numbers divisible by 3 = 3, 6, 9, 12, 15, 18
Number of numbers divisible by 3 = 6 out of 20

$\therefore$ P (a number divisible by 3) = $\frac{6}{20} = \frac{3}{10}$

(iv) Single Digits = 1, 2, 3, 4, 5, 6, 7, 8, 9
Number of single digits = 9 out of 20.

$\therefore$ P (single digit) = $\frac{9}{20}$

Exercise 4.4

1. A die is thrown, Find the probability of getting a number (i) less than 3 (ii) greater than 5 (iii) number 4.
2. An urn contains 7 green balls and 5 red balls. One ball is selected at random. Find the probability of getting (i) a green ball (ii) a red ball.
3. A Bag contains 5 blue marbles, 8 green marbles, 4 red marbles and 7 yellow marbles. One marble is selected at random and find the probability of getting
   (i) a blue marble (ii) a green marble (iii) a red marble
   (iv) a yellow marble (v) not blue (vi) not red

4. In a class, there are 15 girls and 12 boys, one student is selected as a monitor. Find the probability that monitor is a (i) boy (ii) girl

5. One letter is selected from English alphabets. Find the probability that it is a (i) Vowel (ii) consonant

6. Following plane figures are divided into equal parts and one part is selected at random. Find the probability that selected part is (i) shaded (ii) unshaded part

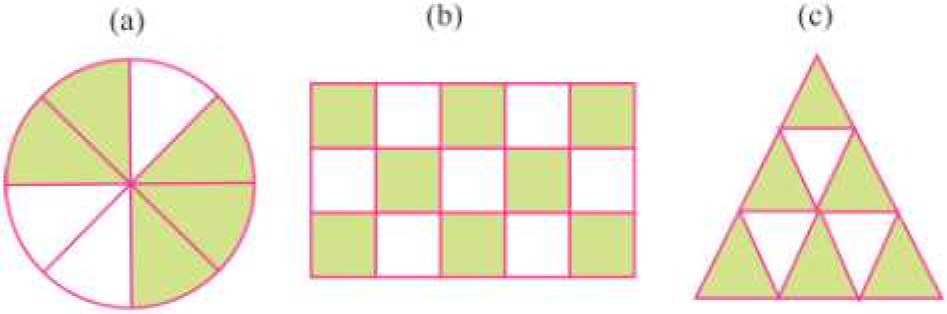


Figure 4.29

7. 25 cards numbered 1, 2, 3, ____, 25 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is :
   (i) an odd number (ii) an even number (iii) divisible by 5 (iv) a prime number

8. **Multiple Choice Questions :**

(i) The probability of getting a head in tossing of a coin is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{2}$

(ii) The probability of getting an even number in throwing a die is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

(iii) The probability of selecting a vowel from the English alphabets is

(a) $\frac{1}{5}$ (b) $\frac{5}{26}$ (c) $\frac{2}{5}$ (d) $\frac{5}{18}$

(iv) Which of the following is a random experiment?

(a) Rolling a die (b) Throwing a coin

(c) Choosing a marble from a Jar (d) All of them

(v) The probability of getting a red ball from a bag containing 10 blue balls.

(a) $\frac{1}{10}$ (b) 0 (c) 1 (d) $\frac{1}{5}$

Learning Outcomes

After completion of the chapter, the students are now able to

- *Collect and present data in various forms.*
- *Present the data in histograms and pie charts.*
- *Interpret the histograms and pie charts.*
- *Use histograms and pie charts in daily life situations.*
- *Understand the concept of probability.*
- *Relate chances and probability so real life problems.*

Answers

Exercise 4.1

1.

No. of Children	Tally Marks	Frequency
0	卌	5
1	卌 I	6
2	卌 卌 II	12
3	卌	5
4	卌 I	6
5	III	3
6	III	3
	Total	40

2.

Accident per day	Tally Marks	Frequency
0	II	2
1	III	3
2	卌 I	6
3	III	3
4	IIII	4
5	卌 I	6
6	卌 I	6
	Total	30

3.

Ages (in years)	Tally Marks	Frequency
12	III	3
13	𝍸 IIII	9
14	𝍸 III	8
15	II	2
16	II	2
17	I	1
	Total	25

4.

Shoppers	Tally Marks	Frequency
M	𝍸 𝍸 𝍸	15
W	𝍸 𝍸 𝍸 𝍸 𝍸 III	28
B	𝍸	5
G	𝍸 𝍸 II	12
	Total	60

5.

Interval	Tally Marks	Frequency
30-35	III	3
35-40	IIII	4
40-45	IIII	4
45-50	𝍸	5
50-55	I	1
55-60	I	1
60-65	I	1
65-70	I	1
	Total	20

6.

Marks obtained	Tally Mark	Frequency
0-10	II	2
10-20	𝍸 𝍸	10
20-30	𝍸 𝍸 𝍸 𝍸 I	21
30-40	𝍸 𝍸 𝍸 IIII	19
40-50	𝍸 II	7
50-60	I	1
	Total	60

7.

Tax Bills (in ₹)	**Tally Mark**	**Frequency**
10-20	III	3
20-30	I	1
30-40	IIII	4
40-50	IIII	4
50-60	I	1
60-70	II	2
70-80	𝍸	5
80-90	III	3
90-100	II	2
100-110	II	2
110-120	III	3
	Total	30

8.

Marks	**Tally Mark**	**Frequency**
0-5	I	1
5-10	𝍸	5
10-15	III	3
15-20	𝍸	5
20-25	IIII	4
25-30	III	3
30-35	II	2
35-40	II	2
40-45	II	2
45-50	II	2
50-55	I	1
	Total	30

9. (i) c (ii) a (iii) b (iv) d (v) a (vi) a

(vii) a (viii) a

(ix) (A) (a)

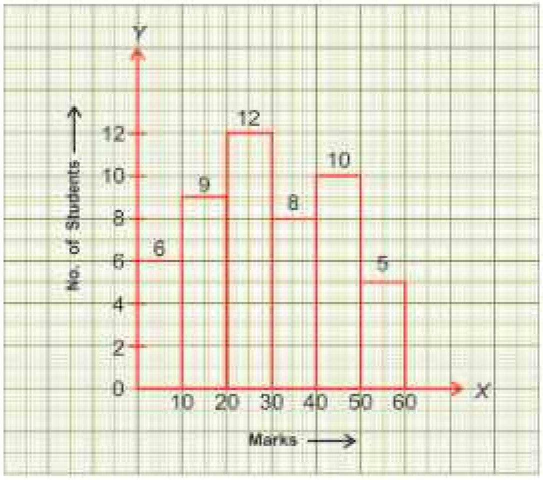
(B) (b)

(C) (c)

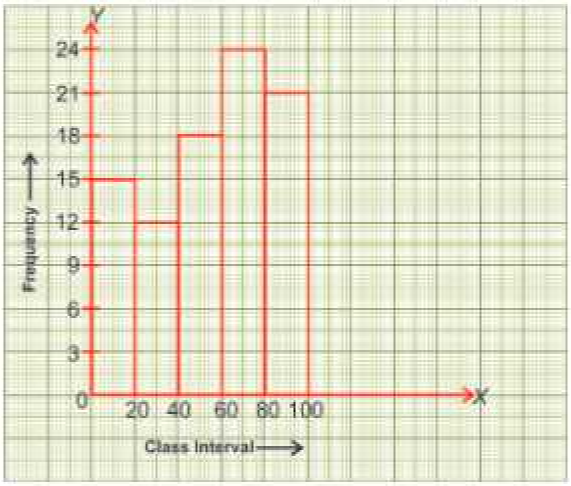
(D) (d)

Exercise 4.2

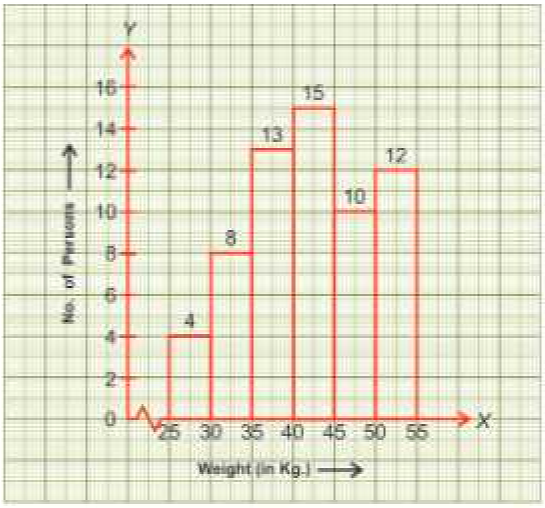
1.



2.



3.



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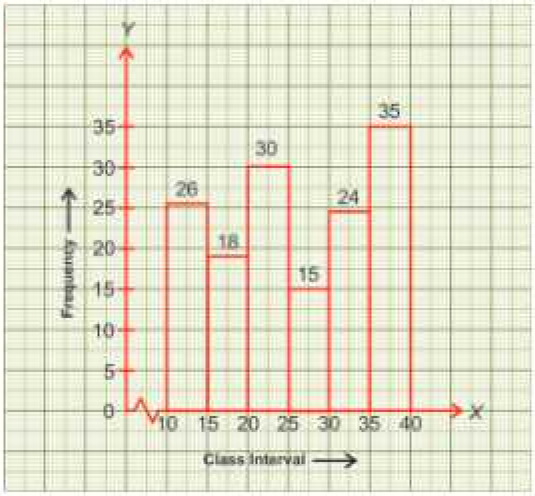

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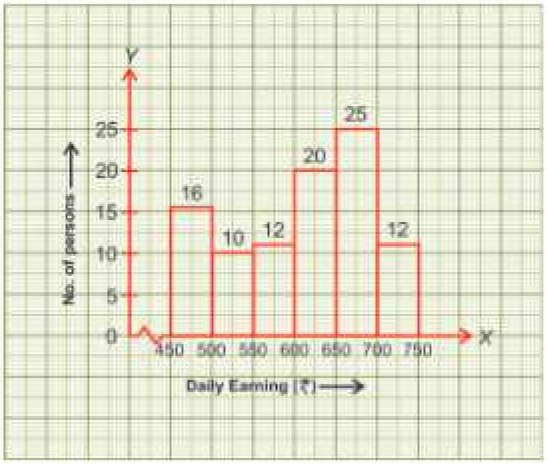
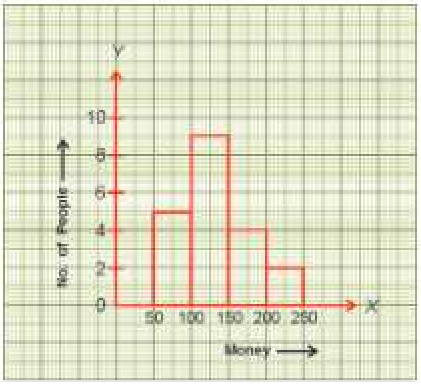


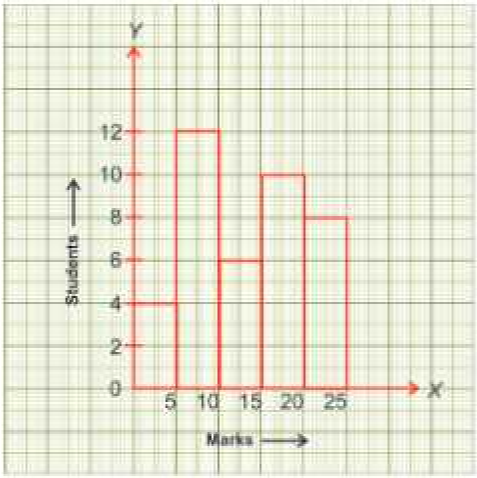
6. Frequency Distribution

Money	Tally Mark	No. of People
50-100	卌	5
100-150	卌 ||||	9
150-200	||||	4
200-250	||	2
	Total	20



7. Frequency Distribution

Marks	Tally Mark	Students
0-5	||||	4
5-10	卌 卌 ||	12
10-15	卌 |	6
15-20	卌 卌	10
20-25	卌 |||	8
	Total	40



8. (i) 10 (ii) 6 (iii) 15 (iv) 16

9. (i) Number of girls with different height (ii) 140-145 (iii) 8

10. (i) 2, 2 (ii) More age group 35-40, least 20-25, 50-55 (iii) 5

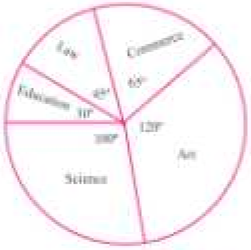
11. (A) (i) b (ii) d (iii) a (iv) d (v) c (B) d

12. (i) False (ii) False (iii) False

Exercise 4.3

1.

2.



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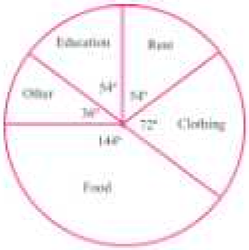

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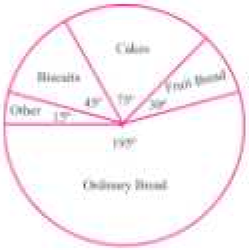


5.

6. Wheat = 2700 tonnes, Sugar = 2250 tonnes, Rice = 1350 tonnes, Gram = 1125 tonnes, Maize = 675 tonnes.

7. Mathematics = 264, English = 176, Science = 198, Hindi = 132, Social Science = 110

8. (i) ₹ 16500 (ii) Food = ₹ 4950, Rent = ₹ 4125, Saving = ₹ 2475, Clothing = ₹ 1650, others = ₹ 3300

9. (i) 540 (ii) English = 93, Hindi = 90, Social Science = 108, Science = 114, Mathematics = 135

10. (i) c (ii) c (iii) a (iv) b (v) a

11. (i) c (ii) a (iii) c (iv) c

Exercise 4.4

1. (i) $\frac{1}{3}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{6}$

2. (i) $\frac{7}{12}$ (ii) $\frac{5}{12}$

3. (i) $\frac{5}{24}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{6}$ (iv) $\frac{7}{24}$ (v) $\frac{19}{24}$ (vi) $\frac{5}{6}$

4. (i) $\frac{4}{9}$ (ii) $\frac{5}{9}$

5. (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$

6. (a) (i) $\frac{5}{8}$ (ii) $\frac{3}{8}$ (b) (i) $\frac{8}{15}$ (ii) $\frac{7}{15}$

(c) (i) $\frac{2}{3}$ (ii) $\frac{1}{3}$

7. (i) $\frac{13}{25}$ (ii) $\frac{12}{25}$ (iii) $\frac{1}{5}$ (iv) $\frac{9}{25}$

8. (i) d (ii) b (iii) b (iv) d (v) b

CHAPTER

5

Squares and Square Roots

Learning Objectives

In this chapter you will learn:

- *To find Square of a number.*
- *About perfect square number and different properties of squared numbers.*
- *To find square root of a number using different methods.*
- *To use concept of square and square roots in solving practical life problems.*

5.1 Introduction:-

In Geometry, you have learnt about square. You also know that area of a square = side × side (where side means, the length of a side)

Now, see the following numbers : 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, ... etc.

What is special in these numbers,

Since $1 = 1 \times 1$; $4 = 2 \times 2$; $9 = 3 \times 3$; $16 = 4 \times 4$; $25 = 5 \times 5$; $36 = 6 \times 6$ and so on, so all such numbers can be expressed as the product of some number with itself.

Such numbers like 1, 4, 9, 16, 25, 36, ... are known as **Perfect squares** or **Square numbers.**

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then, that number m is a perfect square or a square number.

Example **Is 49 a perfect square?**

Sol. First we shall express 49 as a product of two natural numbers.

i.e. $49 = 7 \times 7$

Here we see 49 can be expressed as the product of number 7 with itself.

So, 49 is a perfect square.

Example **Is 27 a perfect square number?**

Sol. First we shall express 27 as a product of two natural numbers.

i.e. $27 = 3 \times 9$

$= 1 \times 27$

Here we see that 27 cannot be expressed as the product of some number with itself.

Hence 27 is not a perfect square number, or we can say that 27 is not a square of any natural number.

Similarly 8, 10, 13, 58, 176 etc are not perfect square numbers.

So we can say that all the natural numbers are not Perfect Squares.

5.2 Properties of Square Numbers:

Study the following table, which shows the squares of first 30 natural numbers.

Natural Number	Square of the Number	Natural Numbers	Square of the Number	Natural Numbers	Square of the Number
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	289	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

Property 1 : Square numbers (perfect square numbers) always end with the digits 0, 1, 4, 5, 6 or 9.

From above table, we observe that digits at ones place of square number which are Red in colour, are 0, 1, 4, 5, 6 and 9. None of these have 2, 3, 7 or 8 at ones place.

Can we say that if a number ends with 0, 1, 4, 5, 6 and 9 then it must be a square number?

No, it is not always true.

For example, numbers 10 and 20 end with 0 but they are not perfect squares.

Similarly 11, 21, 31 etc end with 1 but they are also not perfect squares.

Number 14, 15, 24, 26, 29 etc. are some more examples whose ones digits are 4, 5, 6 and 9, but they are not perfect squares.

So, we can say that if a number ends with 0, 1, 4, 5, 6 or 9 then it may or may not be a perfect square.

A number ending in 2, 3, 7 or 8 can never be a perfect square.

For example 22, 33, 237, 2378, 3542, 15437 etc. are not perfect squares.

Example 5.1 Write five numbers on which you can decide by looking at their ones digit that they are not square numbers. (perfect square)

Sol. We know the numbers ending with 2, 3, 7 or 8, can never be perfect square.

So five numbers are 62, 93, 147, 228, 222 etc.

Example 5.2 Write five numbers on which you cannot decide just by looking at their ones digit that whether they are square numbers or not.

Sol. We know the numbers ending with 0, 1, 4, 5, 6 or 9 may or may not be a square number. So, we cannot decide just by looking at the numbers ending with these digits. Some examples are 120, 221, 534, 565, 216, 219 etc.

Property 2: The number of zeroes at the end of a square number is always even.

Observe the last row of the given table, which is in Green Colour.

We see that, all the perfect squares have even number of zeroes at the end.

For Example: $10^2 = 10\times10 = 100$
$20^2 = 20\times20 = 400$
$30^2 = 30\times30 = 900$
Some other example $60^2 = 60\times60 = 3600$
$100^2 = 100\times100 = 10000$

(Therefore, the number of zeroes at the end of the square of a number is twice the number of zeroes at the end of the number.)

Note: It is not always true that a number ending with even number of zeroes is always a perfect square. It may or may not be a perfect square.

For example, 400 is a perfect square but 300, 500 and 700 are not.

A number ending with odd number of zeroes is never a perfect square.

For example 10, 110, 1000, 5000, ends with odd number of zeroes. Hence none of these numbers is a perfect square.

Example 5.3. What will be the number of zeroes in the square of the following numbers.

(i) 60 (ii) 200 (iii) 8000

Sol. (i) As 60 has one zero at end, so its square will have two zeroes at end.

(ii) As 200 has two zeroes at end, so its square will have four zeroes at end.

(iii) As 8000 has 3 zeroes at end, so its squares will have 6 zeroes at end.

Property 3: Square of an even number is always even.

See the table, numbers marked with magenta colour.

$4^2 = 4\times4 = 16$
$8^2 = 8\times8 = 64$
$12^2 = 12\times12 = 144$
$24^2 = 24\times24 = 576$

Property 4: Square of an odd number is always odd.

See the table, numbers marked with blue colour.

$1^2 = 1\times1 = 1$

$5^2 = 5\times5 = 25$

$11^2 = 11\times11 = 121$

$19^2 = 19\times19 = 361$

Property 5: Ones digit of square of a number :

Observe the ones digit of the number and ones digit of square of that number. We have the following result :

Ones digit of Number	Ones digit of squared number (Square ends)
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	5

We can observe that from the ones digit of a number, we can find the ones digit of its square number.

Also from the ones digit of the square number, we can find the ones digit of the number.

Example 5.4 : What will be the ones digit in the square of following numbers?

(i) 211 (ii) 299 (iii) 1018 (iv) 1687 (v) 4204

Sol. (i) As ones digit of 211 is 1, so ones digit of its square will be 1.

(ii) As ones digit of 299 is 9, so ones digit of its square will be 1.

(iii) As ones digit of 1018 is 8, so ones digit of its square will be 4.

(iv) As ones digit of 1687 is 7, so ones digit of its square will be 9.

(v) As ones digit of 4204 is 4, so ones digit of its square will be 6.

Property 6: There are 2n natural numbers between the squares of two consecutive numbers i.e. n and n + 1.

For example, take $n = 1$ and $n + 1=2$. Now between $(1)^2$ and $(2)^2$ (between 1 and 4) there are $2 \times n = 2\times1=2$ natural numbers (i.e. 2 and 3)

Consider $n = 4$ and $n + 1 = 5$. Now between $(4)^2 = 16$ and $(5)^2=25$ there are $2 \times n = 2 \times 4 = 8$ natural numbers (i.e.17, 18, 19, 20, 21, 22, 23, 24)

Example 5.5 : How many natural numbers lie between

(i) 8^2 and 9^2 (ii) 11^2 and 12^2

Sol. As we know that there are 2n numbers between n^2 and $(n+1)^2$, So

(i) $8^2 = 64$ and $9^2 = 81$,

there are $2 \times 8 = 16$ natural numbers between them

[as $n = 8$, $n + 1 = 8 + 1 = 9$]

(ii) $11^2 = 121$ and $12^2 = 144$

there are $2 \times 11 = 22$ natural numbers between them

[as $n = 11$, $n+1 = 12$]

Property 7: Square of a number n can be expressed as the sum of the first n odd natural numbers.

i.e. sum of first n odd natural numbers = n^2

e.g. $2^2 = 4 = 1 + 3$ (Here 1 and 3 are the first two odd natural number)

$3^2 = 9 = 1 + 3 + 5$ (Here 1, 3 and 5 are the first 3 odd natural numbers)

$4^2 = 16 = 1 + 3 + 5 + 7$ (Here 1, 3, 5 and 7 are the first 4 odd natural numbers)

$5^2 = 25 = 1 + 3 + 5 + 7 + 9$ (Here 1, 3, 5, 7 and 9 are the first 5 odd natural numbers)

So we can say

$1 + 3 + 5 + \dots\dots\dots\dots + n = n^2$

i.e. sum of first n odd natural numbers = n^2

Example 5.6 : Find the sum of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 without actually adding them.

Sol. As we know that a sum of first *n* odd natural numbers can be expressed as a square of number n.

∴ Sum of first 8 odd natural numbers can be expressed as square of 8.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2 = 64$ (sum of first 8 odd natural numbers)

Example 5.7 : Write 100 as a sum of odd numbers.

Sol. We know that $100 = 10^2$

∴ 100 can be expressed as a sum of first 10 odd natural numbers.

∴ $100 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

Property 8: The square of an odd number (except 1) can be expressed as the sum of two consecutive natural numbers.

For example $3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

In general, for any odd number n,

$$n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$$

Example 5.8: Express the following as a sum of two consecutive numbers

(a) 19^2 **(b) 23^2**

Sol. (a) We know that $19^2 = 361$ [As $n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$

where n = 19 (odd natural number)

$$19^2 = \left(\frac{19^2 - 1}{2}\right) + \left(\frac{19^2 + 1}{2}\right)$$

$$= \left(\frac{361 - 1}{2}\right) + \left(\frac{361 + 1}{2}\right)$$

$$= \frac{360}{2} + \frac{362}{2}$$

$$= 180 + 181$$

(b) We know that $(23)^2 = 529$ [As $n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$

where n = 23 (odd natural number)

$$= \left(\frac{23^2 - 1}{2}\right) + \left(\frac{23^2 + 1}{2}\right)$$

$$= \left(\frac{529 - 1}{2}\right) + \left(\frac{529 + 1}{2}\right)$$

$$= \frac{528}{2} + \frac{530}{2}$$

$$= 264 + 265$$

Property 9: Product of two consecutive even or odd natural numbers.

If $n-1$ and $n+1$ are two consecutive even or odd natural numbers then

$(n-1) \times (n+1) = n^2-1$,

For example

$5 \times 7 = 35 = (6 - 1) \times (6 + 1)$ and $5 \times 7 = 35 = 6^2 - 1$

Here n is 6,

$10 \times 12 = 120 = (11 - 1) \times (11 + 1)$ and $10 \times 12 = 120 = (11^2-1)$

Here n is 11

Property 10: The difference of the squares of two consecutive natural numbers is equal to the sum of both the numbers.

For example $5^2 - 4^2 = 25 - 16 = 9 = 5 + 4$

$10^2 - 9^2 = 100 - 81 = 19 = 10 + 9$

Or we can say

$$\boxed{(n + 1)^2 - (n)^2 = (n + 1) + n \text{ or } 2n + 1}$$

Example 5.9 : Find the difference between $21^2 - 20^2$ without calculations.

Sol. We know that the difference of the squares of two consecutive natural numbers is equal to the sum of the two given numbers.

$\therefore \quad 21^2 - 20^2 = 21 + 20 = 41$

Property 11: The square of a negative integer is always positive.

For example $(-3)^2 = -3 \times -3 = 9$

$(-5)^2 = -5 \times -5 = 25$

$(-10)^2 = -10 \times -10 = 100$

Property 12: The square of a proper fraction is always less than the given fraction.

For example $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

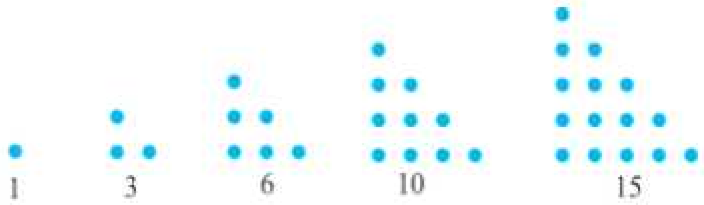
And $\frac{4}{9} < \frac{2}{3}$

$\Rightarrow 0.444... < 0.666......$

Some Interesting Patterns

1. Adding Triangular Numbers

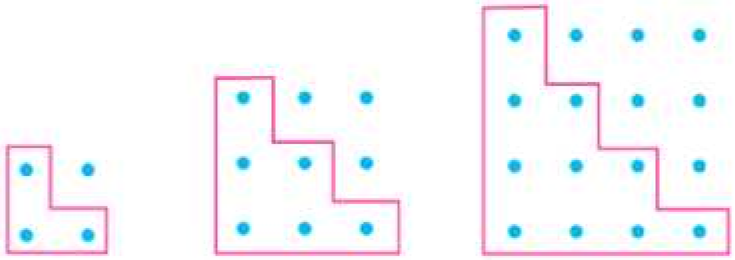
Triangular numbers are those numbers whose dot patterns can be arranged as triangles.



These numbers are 1, 3, 6, 10, 15, 21, 28...............

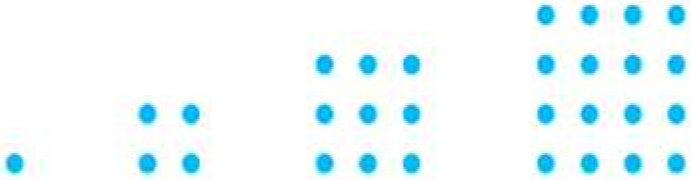
nth triangular number is given by $\frac{n(n+1)}{2}$

Note: Here we observe that, the sum of any two consecutive triangular numbers is a perfect square.



$3+1=4=2^2$ $6+3=9=3^2$ $10+6=16=4^2$

2. Square numbers of the form n^2 can be represented geometrically in the form of square with n rows and n columns of dots.



1^2 $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

3. Let us observe the pattern made by square of numbers such as 1^2, 11^2, 111^2 and so on

$1^2 = 1$

$11^2 = 1\ 2\ 1$

$111^2 = 1\ 2\ 3\ 2\ 1$

$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$

$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

On observing the above pattern, the square of any number of the type 1111111...... can be easily calculated.

Here we observe that

(i) The digits in the squares is a combination of ascending and descending order of digits.

(ii) There are odd numbers of digits in the squares.

(iii) The central digit in the square number is equal to the number of 1s in the number.

(iv) The first and last digits are always 1.

4. Observe the squares of 7, 67, 667....... . Here also observe a very intersting pattern.

$7^2 = 49$

$67^2 = 4\ 4\ 8\ 9$

$667^2 = 4\ 4\ 4\ 8\ 8\ 9$

$6667^2 = 4\ 4\ 4\ 4\ 8\ 8\ 8\ 9$

Here we observe that

(i) All the squares end with digit 9.

(ii) The numbes of 4s in the square of number is one more than the number of 6s in number.

(iii) The number of 8s is the square of number is same as the number of 6s in the number.

5. Observe the squares of 5, 15, 25, 35, 45............ i.e. the number having 5 as ones digit, it also follow a very interesting pattern.

$5^2 = 25 = 0 \times 1\text{(hundred)} + 5^2$

$15^2 = 225 = 1 \times 2 \text{ (hundred)} + 5^2$

$25^2 = 625 = 2 \times 3 \text{ (hundred)} + 5^2$

$35^2 = 1225 = 3 \times 4 \text{ (hundred)} + 5^2$

$105^2 = 11025 = 10 \times 11 \text{ (hundred)} + 5^2$

Or

$(a5)^2 = a\times (a+1) \text{ hundred} + 5^2$

6. Observe the following interesting patterns :

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + 20^2 = 21^2$ etc.

Here we observe, ignoring the squares, the first two numbers on the LHS are consecutive numbers and third number is the product of first two numbers. The number on the RHS is the successor of third number of LHS.

i.e. $a^2 + (a + 1)^2 + (a (a + 1))^2 = [a (a + 1) + 1]^2$

Example 5.10 : Express 21^2 as a sum of squares of 3 numbers.

Sol. Here we have to express 21^2 as a sum of squares of 3 numbers.

$\therefore \quad a^2 + b^2 + c^2 = 21^2$

If we ignore the square, c will be the predecessor of 21 i.e. 20.

Now 20 is equal to product of two consecutives numbers i.e 4 and 5.

So $21^2 = (4)^2 + (5)^2 + (20)^2$

Pythagorean Triplets

We know that, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides, which is the Pythagoras Theorem.

So Pythagorean triplet refers to a group of 3 numbers which follows the Pythagoras Theorem. 3 numbers, named a, b and c are known as Pythagorean triplet if $a^2 + b^2 = c^2$

Consider three numbers 3, 4 and 5

Now $(3)^2 = 9$, $(4)^2 = 16$ and $(5)^2 = 25$

Here $3^2 + 4^2 = 5^2$

i.e. $9 + 16 = 25$

$25 = 25$

Hence 3, 4 and 5 forms a Pythagorean triplet.

There are uncountable number of Pythagorean triplets.

Some of them are (6, 8, 10), (5, 12, 13) etc.

For any natural number n>1, we have

$(2n)^2 + (n^2-1)^2 = (n^2+1)^2$

So $(2n, n^2-1, n^2+1)$ forms a Pythagorean triplet.

By this we can easily found a Pythagorean triplet.

Note: All Pythagorean triplets may not be obtained using this method.

Example 5.11: Write a Pythagorean triplet with 6 as one of the numbers of the triplet.

Sol. We can get Pythagorean triplet by using the general form $2n, n^2-1, n^2+1$

Let us take $2n = 6 \Rightarrow n = 3$

So $n^2 - 1 = 3^2 - 1 = 8$ and $n^2 + 1 = 3^2 + 1 = 10$

So Pythagorean triplet is 6, 8 and 10.

Example 5.12: Find a Pythagorean triplet whose one of the number is 15.

Sol. We can get Pythagorean triplet by using the general form $2n, n^2-1, n^2+1$

Let us take $2n = 15 \Rightarrow n = \frac{15}{2}$, which is not a natural number

So we cannot takes $2n = 15$

So, let us take $n^2 - 1 = 15$

$\Rightarrow n^2 = 16$

$\Rightarrow n = 4$

$\therefore \quad 2n = 2 \times 4 = 8$

also $n^2 - 1 = (4)^2 - 1 = 16 - 1 = 15$

also $n^2 + 1 = (4)^2 + 1 = 16 + 1 = 17$

So required triplet is (8, 15, 17)

Exercise 5.1

1. Find the square of the following numbers :

(i) 19 (ii) 41 (iii) –11 (iv) $\frac{3}{7}$ (v) $1\frac{2}{3}$

(vi) 1.7 (vii) 0.02 (viii) 0.014

2. The following numbers are not perfect squares, give reasons.

(i) 177 (ii) 1058 (iii) 7928 (iv) 23453 (v) 42222

(vi) 64000 (vii) 222000 (viii) 42977 (ix) 5000 (x) 100000

3. What will be the number of zeroes in the square of following numbers?

(i) 90 (ii) 120 (iii) 400 (iv) 6000 (v) 80000

(vi) 1600

4. The square of which of the following would be an odd number or an even number?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004 (v) 473
(vi)4096 (vii) 9267 (viii) 27916

5. What will be the ones digit in the squares of following numbers?

(i) 41 (ii) 321 (iii) 89 (iv) 439 (v) 62
(vi)4012 (vii) 88 (viii) 348 (ix) 93 (x) 703
(xi)57 (xii) 1327 (xiii) 44 (xiv) 1024 (xv) 26
(xvi) 2226 (xvii) 55 (xviii) 125

6. How many natural numbers lie between squares of the following numbers?

(i) 14 and 15 (ii) 21 and 22 (iii) 30 and 31 (iv) 10 and 11

7. Express the following as indicated

(i) 81 as the sum of first 9 odd numbers.

(ii) 144 as the sum of first 12 odd numbers.

(iii)256 as the sum of first 16 odd numbers.

8. Without actually adding, find the sum of:

(i) $1 + 3 + 5 + 7 + 9$

(ii) $1+ 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$

9. Express the following as the sum of two consecutive numbers:

(i) $(15)^2$ (ii) $(21)^2$ (iii) $(33)^2$ (iv) $(37)^2$

10. Solve following without actually calculating:

(i) $8^2 - 7^2$ (ii) $13^2 - 12^2$ (iii) $25^2 - 24^2$ (iv) $80^2 - 79^2$
(v) $110^2 - 109^2$

11. Observe the following patterns and find the missing terms:

(i) $1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + \ldots.. = 13^2$
$\ldots. + 5^2 + \ldots.. = 21^2$
$5^2 + \ldots. + 30^2 = \ldots\ldots.$
$6^2 + 7^2 + \ldots\ldots. = \ldots\ldots$

(ii) $1^2 = 1 = 1$
$2^2 = 4 = 1 + 2 +1$
$3^2 = 9 = 1 + 2 + 3 + 2 + 1$
$\ldots. = \ldots.. = 1 + 2 + 3 + 4 + 3 + 2 + 1$
$\ldots. = \ldots.. = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$

(iii) $21^2 = 441$
$201^2 = 40401$
$2001^2 = 4004001$
$\boxed{\quad}^2 = 400040001$
$(2000001)^2 = 4000004000001$
$(20000001)^2 = \boxed{\qquad\qquad}$

12. Using the pattern $1^2 = 1$, $11^2 = 121$, $111^2 = 12321$, $1111^2 = 1234321$ find $(1111111)^2$.

13. Find the squares of the following numbers having 5 at ones digit place?

(i) 45 (ii) 75 (iii) 95 (iv) 125 (v) 205

14. Which of the following are Pythagorean triplets?

(i) 3, 4, 5 (ii) 6, 8, 10 (iii) 8, 15, 17 (iv) 13, 17, 19

15. Write a Pythagorean triplet whose one of the number is:

(i) 8 (ii) 12 (iii) 16 (iv) 18 (v) 20

16. Multiple Choice Questions :

(i) Square of an odd number is always ....

(a) Even (b) Odd (c) Even or Odd (d) None of these

(ii) The number of zeros in the square of 600 will be

(a) 1 (b) 2 (c) 3 (d) 4

(iii) The ones digit in the square of 52698 is

(a) 1 (b) 4 (c) 6 (d) 9

(iv) How many natural numbers lie between 6^2 and 7^2 ?

(a) 6 (b) 8 (c) 10 (d) 12

(v) 1 + 3 + 5 + 7 + ... upto n terms is equal to

(a) $n^2 - 1$ (b) $n^2 + 1$ (c) $(n + 1)^2$ (d) n^2

(vi) The square of a proper fraction is :

(a) Less than the fraction (b) Greater than the fraction

(c) Equal to the fraction (d) None of these

(vii) The value of $(111111)^2$ is

(a) 1234564321 (b) 1234455321

(c) 12345654321 (d) 1234554321

(viii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = ...

(a) $(6)^2$ (b) $(5)^2$ (c) $(7)^2$ (d) $(8)^2$

(ix) Which of the following is a perfect square ?

(a) 4000 (b) 40000 (c) 40 (d) 400000

(x) The nth triangular number is given by

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n}{2}$

5.3 Square Root

We have already learnt how to find square of a number. Now suppose we have a squared number (perfect square). Now how can we find the original number whose square we have ? For this purpose we have concept of square root.

Let we are given a number K, we need to find a number which when squared gives K. In other words we are simply going to reverse the process.

∴ Square root can be defined as inverse of doing square:

Symbol used for square root is $\sqrt{\ }$. Now observe the table and apply above definition.

$2^2 = 4$	Square Root of 4 = 2	i.e. $\sqrt{4} = 2$
$6^2 = 36$	Square Root of 36 = 6	i.e. $\sqrt{36} = 6$
$13^2 = 169$	Square Root of 169 = 13	i.e. $\sqrt{169} = 13$
$21^2 = 441$	Square Root of 441 = 21	i.e. $\sqrt{441} = 21$

As $4^2 = 16$

And $(-4)^2 = 16$

∴ $\sqrt{16}$ can have both 4 and –4 as its answers but here, in this chapter we will only discuss to positive square root of a number.

5.3.1 Finding Square Roots

Before we learn how to find square root of a number we must know why we need to find square root of a number? Consider the following situations:

1. We know that area of square = $(\text{Side})^2$

   If you are given area of square as 3125cm² how will you find its side?
2. Suppose you are given with sides of a rectangle, how will you find the diagonal?
3. Suppose you have a right triangle whose adjacent sides are given, how will you calculate its hypotenuse?

In all above situations and many other similar situations we need to calculate square root at some stage.

Before we learn to calculate square root of a number. Let's connect square and square root of a number through properties already learnt.

1. Square root of even number is even and that of odd number is odd.

   i.e. $\sqrt{196} = 14$ as $14^2 = 196$

   and $\sqrt{225} = 15$ as $15^2 = 225$
2. Ones place digit of square root of any perfect square number ending with 1 is either 1 or 9.

   i.e. $\sqrt{121} = 11$ as $11^2 = 121$

   and $\sqrt{361} = 19$ as $19^2 = 361$
3. Ones place digit of square root of any perfect square number ending with 4 is either 2 or 8.

   i.e. $\sqrt{144} = 12$ as $12^2 = 144$

   and $\sqrt{324} = 18$ as $18^2 = 324$

4. Ones place digit of square root of a perfect square ending with 9 is either 3 or 7.

i.e. $\sqrt{169} = 13$ as $13^2 = 169$

and $\sqrt{729} = 27$ as $27^2 = 729$

5. Ones place digit of square root of any perfect square ending with 5 is 5.

i.e. $\sqrt{225} = 15$ as $15^2 = 225$

and $\sqrt{625} = 25$ as $25^2 = 625$

6. Any number which ends with 2, 3, 7, 8 or have odd number of zeros at its end cannot be a perfect square. Square root of these type numbers will not be a natural number.

e.g. 232, 407, 1603, 1008 and 1690 can never be a perfect square numbers.

Example 5.13 Fill in the blanks

(i) $11^2 = 121$ So $\sqrt{121}$ = ... (ii) $14^2 = 196$ So $\sqrt{......}$ = 14

Sol. (i) As $11^2 = 121$ So $\sqrt{121} = 11$

(ii) As $14^2 = 196$ So $\sqrt{196} = 14$

5.3.2 Finding square root by repeated subtraction

As we have learnt earlier that the sum of the first n odd natural numbers is n^2. That is, every square number can be expressed as sum of consecutive odd natural number starting from 1. consider $\sqrt{49}$

Step (i) $49 - 1 = 48$ Step (ii) $48 - 3 = 45$ Step (iii) $45 - 5 = 40$

Step (iv) $40 - 7 = 33$ Step (v) $33 - 9 = 24$ Step (vi) $24 - 11 = 13$

Step (vii) $13 - 13 = 0$

We can write $49 = 1 + 3 + 5 + 7 + 5 + 9 + 11 + 13$

Se here we have started subtracting successive odd natural numbers starting from 1 and obtained 0 at 7th step. So $\sqrt{49} = 7$

Example 5.14 By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect square or not? if number is perfect square, find its square root.

(a) 36 (b) 55 (c) 121

Sol. (a) The given number is 36, Start subtracting the odd numbers starting from 1

(i) $36 - 1 = 35$ (ii) $35 - 3 = 32$ (iii) $32 - 5 = 27$

(iv) $27 - 7 = 20$ (v) $20 - 9 = 11$ (vi) $11 - 11 = 0$

As we have obtained 0, so 36 is a perfect square and we obtained 0 at 6th step

so $\sqrt{36} = 6$

(b) (i) 55 – 1 = 54 (ii) 54 – 3 = 51 (iii) 51 – 5 = 46

(iv) 46 – 7 = 39 (v) 39 – 9 = 30 (vi) 30 – 11 = 19

(vii) 19 – 13 = 06 (viii) 6 – 15 = –9

As we did not get zero, hence 55 is not a perfect square number.

(c) (i) 121 – 1 = 120 (ii) 120 – 3 = 117 (iii) 117 – 5 = 112

(iv) 112 – 7 = 105 (v) 105 – 9 = 96 (vi) 96 – 11 = 85

(vii) 85 – 13 = 72 (viii) 72 – 15 = 57 (ix) 57 – 17 = 40

(x) 40 – 19 = 21 (xi) 21 – 21 = 0

As we got zero, so 121 is a perfect square number.

and we obtained 0 at 11th step so $\sqrt{121} - 11$

Now can you find $\sqrt{625}$ using this method? Yes, but it will be time consuming. We have another methods for finding the square root. We will discuss in next section.

5.3.3 Finding square root through prime factorisation :

Consider the prime factorisation of some numbers and their squares, as shown in table 6.7

Number	**Prime factorisation of number**	**Square of number**	**Prime factorization of square of number**
4	2 × 2	16	2 × 2 × 2 × 2
6	2 × 3	36	2 × 2 × 3 × 3
8	2 × 2 × 2	64	2 × 2 × 2 × 2 × 2 × 2
9	3 × 3	81	3 × 3 × 3 × 3
10	2 × 5	100	2 × 2 × 5 × 5
12	2 × 2 × 3	144	2 × 2 × 2 × 2 × 3 × 3
15	3 × 5	225	3 × 3 × 5 × 5

On observing given table, you will see that 2 occurs twice in prime factorisation of 4 and 2 occurs 4 times in the prime factorisation of 16. Similarly, How many times 2 occur in prime factorisation of 6? It is one time. Now how many times does 2 occur in the prime factorisation of 36? It is two times. Similarly observe the occurrence of 3 in 6 and 36.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given number say 100.

The prime factorisation of 100 is as $100 = 2 \times 2 \times 5 \times 5$

By pairing the prime factors,we get

$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2 = (2 \times 5)^2$

2	100
2	50
5	25
5	5
	1

So $\sqrt{100} = 2 \times 5 = 10$

Similarly, we can find the square root of 144.

The Prime Factorisation of 144 is

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 2^2 \times 2^2 \times 3^2 = (2 \times 2 \times 3)^2$

So $\sqrt{144} = 2 \times 2 \times 3 = 12$

2	144
2	72
2	36
2	18
3	9
3	3
	1

Is 48 a perfect square?

We know $48 = 2 \times 2 \times 2 \times 2 \times 3$

Since one factor (3) of 48 is not in pair, so 48 is not a perfect square.

Suppose we want to find the smallest number which will make 48 a perfect square, how should we proceed? In the prime factorisation of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply 48 by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Now can you tell by which smallest number should we divide 48 to get a perfect square?

As we have seen the factor 3 is not in pair, so if we divide 48 by 3,

we get $48 \div 3 = 16$ which is a perfect square

Example 5.15 Find the square root of the following numbers by method of Prime factorisation (i) 729 (ii) 9604

Sol. Let us find the prime factoisation of the given numbers is :

(i)

3	729
3	243
3	81
3	27
3	9
3	3
	1

So $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

i.e. $729 = 3^2 \times 3^2 \times 3^2$

$729 = (3 \times 3 \times 3)^2$

Hence $\sqrt{729} = 3 \times 3 \times 3 = 27$

(ii)

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

So $9604 = 2 \times 2 \times 7 \times 7 \times 7 \times 7$

i.e. $9604 = 2^2 \times 7^2 \times 7^2$

$9604 = (2 \times 7 \times 7)^2$

Hence $\sqrt{9604} = 2 \times 7 \times 7 = 98$

Example 5.16 For each of the following numbers, find the smallest number by which it should be multiplied to get a perfect square number. Also, find the square root of the perfect square number so obtained. (i) 180 (ii) 768

Sol. (i) Let us find the prime factorisation of 180.

Now 180 $= 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5^1$

The prime factor 5, does not occur in pair. So we need to multiply 180 by 5 to get a perfect square number.

Now $180 \times 5 = 900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^2 \times 5^2$

i.e. $900 = (2 \times 3 \times 5)^2 = (30)^2$

so $\sqrt{900} = 30$

2	180
2	90
3	45
3	15
5	5
	1

(ii) Let us find the Prime factorisation of 768.

$768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here 3 is the only factor that does not occur in pair

So we need to multiply 768 by 3 to complete the pair. So new number is

$768 \times 3 = 2304$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

i.e. $2304 = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2$

$2304 = (2 \times 2 \times 2 \times 2 \times 3)^2 = (48)^2$

Hence $\sqrt{2304} = 48$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Example 5.17 Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Also find the square root of the quotient.

Sol. Let us find the Prime factorisation of 9408

$9408 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 \times 7$

Here except 3, all factors are in pairs.

So if we divide 9408 by 3, then

$9408 \div 3 = 3136$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 2^2 \times 2^2 \times 2^2 \times 7^2$

$= (2 \times 2 \times 2 \times 7)^2$

which is a perfect square and

$\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Example 5.18 2025 Plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and number of plants in each row.

Sol. As we have to plant as many plants in each row as the number of rows. So number of plants will be a squared number.

Let number of plants in each row = number of rows = x

As per question x × x = 2025

i.e. $x^2 = 2025$

To find x, we have to find a number whose square is 2025.

i.e. x is square root of 2025

Now $2025 = 3\times3\times3\times3\times5\times5 = 3^2 \times 3^2 \times 5^2$

i.e. $2025 = (3 \times 3 \times 5)^2$

So $\sqrt{2025} = 3 \times 3 \times 5 = 45$

3	2025
3	675
3	225
3	75
5	25
5	5
	1

Hence number of rows and number of plants in each row are 45.

Example 5.19: Find the smallest square number which is divisible by each of numbers 8, 12, 50.

Sol. As required number is divisible by each of 8, 12 and 50

∴ We have to find L.C.M. of 8, 12 and 50

L.C.M (8, 12, 50) $= 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 600$

But 600 is not a perfect square.

So we have to make perfect square:

We have $600 = 2\times2\times2\times3\times5\times5$

2	8 - 12 - 50
2	4 - 6 - 25
2	2 - 3 - 25
3	1 - 3 - 25
5	1 - 1 - 25
5	1 - 1 - 5
	1 - 1 - 1

Since, to make 600 a perfect square, we have to multiply it with 2 and 3.

i.e. $600 \times 2 \times 3 = 3600$

∴ 3600 is a perfect square which is divisible by 8, 12 and 50.

Exercise 5.2

1. Tell the ones place digit of square root of following numbers:

   (i) 121 (ii) 729 (iii) 676 (iv) 1936

   (v) 484 (vi) 2401 (vii) 1600 (viii) 3025

2. From the following numbers find the number which cannot be a perfect square number.

   100, 512, 1728, 529, 1024, 441, 1320, 3617

3. Find the square root of following numbers by method of repeated subtraction.

   (i) 64 (ii) 49 (iii) 121 (iv) 100

4. Find square root of following numbers using method of prime factorisation:

   (i) 3600 (ii) 676 (iii) 9216 (iv) 2916

   (v) 6400 (vi) 1764 (vii) 12100 (viii) 1024

5. Find the smallest number by which following number must be multiplied so that the resultant is a perfect square. Also find square root of that number:

   (i) 243 (ii) 240 (iii) 2662 (iv) 972

   (v) 3087 (vi) 5000

6. Find the smallest number by which following numbers must be divided so that quotient is a perfect square. Also find square root of the number obtained:

   (i) 108 (ii) 3125 (iii) 2400 (iv) 5103

   (v) 2205 (vi) 12168

7. Find the smallest perfect square which is divisible by:

   (i) 8, 12, 28 (ii) 27, 24, 15

8. During a plantation drive 256 plants were planted in a such a way that each row contains as many plants as number of rows. Find number of rows.

9. Area of a square is equal to the area of rectangle whose sides are 27cm and 12cm. Find the side of square.

10. Some students raised funds to help an orphange. If total funds collected was 3136 and each student contributed amount equal to number of students. Find amount contributed by each student.

11. **Multiple Choice Questions :**

   (i) What will be ones place digit of $\sqrt{961}$?

   (a) 1 or 7 (b) 1 or 9 (c) 3 or 6 (d) 7 or 8

   (ii) Guess ones place digit of square root of 625.

   (a) 1 (b) 4 (c) 9 (d) 5

   (iii) Which of the following cannot be perfect square ?

   (a) 625 (b) 728 (c) 729 (d) 144

   (iv) What will be square root of 144?

   (a) 10 (b) 12 (c) 18 (d) 22

   (v) By what number 32 should be multiplied to make it a perfect square?

   (a) 2 (b) 3 (c) 4 (d) 5

   (vi) By what number 288 should be divided to make it a perfect square?

   (a) 5 (b) 4 (c) 3 (d) 2

5.3.4 Finding Square Root by Long Division Method

Consider a number 16777216. If we find its square root by prime factorisation it involve division by 2, 24 times which makes it quite lengthy that is why we use long division method for such problems. Before solving such question let us learn to find number of digits in square root of a number. Observe the following table.

Number	No. of digits	Square root of number	No. of digits in square root
64	2	8	1
144	3	12	2
961	3	31	2
1024	4	32	2
262144	6	512	3
16777216	8	4096	4

If you observe carefully we can draw following inference from above table.

(a) If number of digits in squared number are even, then number of digits in square root of number $= \frac{n}{2}$

(b) If number of digits in number are odd, then number of digits in its square root of number $= \frac{n+1}{2}$

Estimating the number :

We know, in the square root of a perfect square having n digits, the number of digits are $\frac{n}{2}$ (if n is even) and $\frac{n+1}{2}$ (if n is odd). As we use bar from unit digit of the number by taking two digits at a time to find the number of digits in the square root of a perfect square number

$$\sqrt{\overline{6}\,\overline{25}} = 25 \;\; ; \;\; \sqrt{\overline{12}\,\overline{96}} = 36$$

Both the numbers $\overline{6}\,\overline{25}$ and $\overline{12}\,\overline{96}$ have two bars and the number of digits in their square root are 2. Can you tell the number of digits in the square root of 14400? By Placing bars we get $\overline{1}\,\overline{44}\,\overline{00}$. Since there are 3 bars, the square root will be a 3 digit number.

Example 5.20: Find number of digits in square roots of following

(i) 7744 (ii) 15625 (iii) 25600

Sol. (i) Number of digits in 7744 = 4 (Even)

Number of digits in its square root $= \frac{4}{2} = 2$

(ii) Number of digits in 15625 = 5 (odd)

Number of digits in its square root $= \frac{5+1}{2} = \frac{6}{2} = 3$

(iii) Number of digits in 25600 = 5 (odd)

Number of digits in square root of 25600 $= \frac{5+1}{2} = \frac{6}{2} = 3$

Now we can move further to find square root of a number by method of long division.

It is very useful method to find the square root of a given square number. Consider the following steps to find the square root of 625.

Step 1 Place a bar over every Pair of digits starting from the digit at ones place. If the number of digits are odd, then the left most single digit too will have a bar. So we have $\overline{6}\ \overline{25}$

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 6 < 3^2$). Take this number as divisor and the quotient with the number under extreme left bar (here 6). Divide and get the remainder (2 in this case)

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline & 2 \end{array}$$

Step 3 Bring down the number under the next bar (25 in this case) to the right of remainder

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline & 2\ 25 \end{array}$$

Setp 4 Double the quotient and enter it with a blank on its right

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 4\_ & 2\ 25 \end{array}$$

Step 5 Guess a largest possible digit to fill in the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new digit in the quotient, the product is less than or equal to the dividend. In this case 45 × 5 = 225, so we choose the new digit as 5, to get the remainder.

$$\begin{array}{r|l} & 2\ 5 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 4\underline{5} & 225 \\ & -225 \\ \hline & 0 \end{array}$$

Step 6 Since the remainder is 0 and no digit left in the given number so $\sqrt{625} = 25$

Example 5.21 Find the square root of 1296 using long division method

Sol. Following the above steps

$$\begin{array}{r|l} & 3\ 6 \\ \hline 3 & \overline{12}\ \overline{96} \\ & -9 \\ \hline 6\underline{6} & 396 \\ & -396 \\ \hline & 0 \end{array}$$

Therefore, $\sqrt{1296} = 36$

Example 5.22 **Find the least number that must be subtracted from 1308 so as to get a perfect square. Also find the square root of the perfect square.**

Sol. Let us try to find the $\sqrt{1308}$ by long division method

$$\begin{array}{r|l} & \;\;3\;\;6 \\ \hline 3 & \overline{13}\;\overline{08} \\ & -9 \\ \hline 66 & \;\;408 \\ & -396 \\ \hline & \;\;\;12 \end{array}$$

we get the remainder 12.

It shows that $(36)^2$ is less than 1308 by 12.

This means if we subtract 12 from the number, we get a perfect square. so the required perfect square number is

$1308 - 12 = 1296$ and $\sqrt{1296} = 36$

Example 5.23 **Find the greatest four digits number which is a perfect square.**

Sol. Greatest 4 digit number is 9999.

$$\begin{array}{r|l} & \;\;9\;\;9 \\ \hline 9 & \overline{99}\;\overline{99} \\ & -81 \\ \hline 189 & \;1899 \\ & -1701 \\ \hline & \;\;198 \end{array}$$

Now we will check whether this number is a perfect square. Otherwise we have to find the nearest number less than 9999, which is a perfect square. Here 9999 is not a perfect square. As as we get remainder 198. So If we subtract 198 from 9999, the new number will be a perfect square. So largest four digit perfect square number = 9999 – 198 = 9801 and $\sqrt{9801} = 99$

Example 5.24 **Find the least number that must be added to 5615 so as the get a perfect square. Also, find the square root of the perfect square.**

Sol. We find $\sqrt{5615}$ by long division method,

$$\begin{array}{r|l} & \;\;7\;\;4 \\ \hline 7 & \overline{56}\;\overline{15} \\ & -49 \\ \hline 144 & \;\;715 \\ & \;\;576 \\ \hline & \;\;139 \end{array}$$

This shows $(74)^2 < 5615$

next perfect square is $(75)^2 = 5625$

Hence the number to be added is = $(75)^2 - 5615$

$= 5625 - 5615 = 10$

So number is 5625 and its square root is 75

Example 5.25 **Find the smallest four digit number which is a perfect square?**

Sol. The smallest four digit number is 1000. We will check whether this number is a perfect square. Otherwise we have to find its nearest four digit number which is a perfect square. By long division method, we get remainder 39. This show that 1000 is not a perfect square also.

$$\begin{array}{r|l} & \;\;3\;\;1 \\ \hline 3 & \overline{10}\;\overline{00} \\ & -9 \\ \hline 61 & \;\;100 \\ & -\;\;61 \\ \hline & \;\;\;39 \end{array}$$

$(31)^2 < 1000$

Next perfect square is $(32)^2 = 1024$

Hence the number to be added is $= 1024 - 1000 = 24$

and square root of 1024 = $\sqrt{1024} = 32$

Example 5.26 **Area of a square Plot is 3136m². Find the side of square**

Sol. We know that area of a square = (side)²

Let side of Plot is = x m

So $x^2 = 3136$ i.e $x = \sqrt{3136}$

So, side of square = 56m

$$\begin{array}{r|l} & \quad 56 \\ \hline 5 & \overline{31}\;\overline{36} \\ & -25 \\ \hline 106 & \;\;636 \\ & -636 \\ \hline & \quad\;\; 0 \end{array}$$

Example 5.27 **There are 505 students in a school. For a P.T. drill, they have to stand in such a manner that the number of rows is equal to number of columns. How many children will be left out in the arrangement.**

Sol. As number of rows and columns are equal, so it is a square arrangement. we have to find nearest number (smaller) to 505, which is a perfect square

By using long division method, we get the remainder 21.

It shows that (22)² is less than 505 by 21.

This means if we subtract 21 from 505, we get a perfect square.

Hence 505 – 21 = 484 students can be arranged in equal number of rows and columns. 21 students will be left out in the arrangement.

$$\begin{array}{r|l} & \quad 22 \\ \hline 2 & \overline{5}\;\overline{05} \\ & -4 \\ \hline 42 & \;105 \\ & -84 \\ \hline & \;\;21 \end{array}$$

Exercise 5.3

1. Find number of digits in square root of following numbers.

   (i) 12996 (ii) 6084 (iii) 698896 (iv) 72900 (v) 1806336

2. Using Long division method, find the square root of following :

   (i) 9216 (ii) 8100 (iii) 50176 (iv) 4761

   (v) 421201 (vi) 16900 (vii) 5184 (viii) 86436

   (ix) 16777216 (x) 46656

3. Find the least number that must be added to the following numbers to get a perfect square. Also; find the square root of new number.

   (i) 540 (ii) 1765 (iii) 3260 (iv) 4000 (v) 5200 (vi) 790

4. Find the least number that must be subtracted from the following numbers so as to get a perfect square number. Also find the square root of the perfect square number.

   (i) 696 (ii) 1140 (iii) 6021 (iv) 10204 (v) 126441 (vi) 788501

5. Find the greatest five digits number which is a perfect square. Also find the square root.

6. Find the smallest four digits number which is a perfect square. Also find the square root.

7. Find the length of side of the following square field whose area is

   (i) 3136m² (ii) 7225m² (iii) 12100m² (iv) 18225m²

8. Find the length of hypotenuse of a right angle triangle whose other two sides are 6cm and 8cm.

9. A gardener has 1100 Plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

10. Choose the correct answer:

(i) What will be number of digits in square root of 676.

(a) 1 (b) 2 (c) 3 (d) 4

(ii) What will be number of digits in square root of 186624.

(a) 1 (b) 3 (c) 2 (d) 4

(iii) What smallest number must be added to 140 to make it a perfect square?

(a) 4 (b) 8 (c) 12 (d) 16

(iv) What smallest number must be subtracted from 750 to make it a perfect square?

(a) 11 (b) 21 (c) 31 (d) 41

(v) If area of square is 384cm². Find its side.

(a) 12cm (b) 14cm (c) 16cm (d) 18cm

(vi) If 404 children are arranged in rows and columns such that number of rows and number of columns are equal. Find how many children are left out.

(a) 10 (b) 4 (c) 8 (d) 6

5.4 Square Root of Decimal Numbers :

To find the square root of a decimal number, we will follow the steps given below. Consider the number is 51.84

Step 1 We put bars on the integral part (here 51) of the number in the usual manner and place bars on the decimal part (here 84) on every pair of digits beginning with the first decimal place. proceed as usual, we get $\overline{51}.\overline{84}$

Step 2 Now proceed in similar manner. The left most bar is 51 and $7^2 < 51 < 8^2$, take 7 as divisor and the number under the left most bar as the divided i.e. 51.

$$\begin{array}{r|l} & 7 \\ \hline 7 & 51.84 \\ & -49 \\ \hline & 2 \end{array}$$

Divide and get the remainder

Step 3 The remainder is 2. Write the number under the next bar (i.e. 84) to the right of remainder, we get 284

$$\begin{array}{r|l} & 7 \\ \hline 7 & 51.84 \\ & -49 \\ \hline 14\_ & 284 \end{array}$$

Step 4 Double the quotient (i.e. 7) and enter a blank on its right. Since 84 is the decimal part so put a decimal point in the quotient i.e (after 7)

$$\begin{array}{r|l} & \quad 7. \\ \hline 7 & \overline{51}.\overline{84} \\ & -49 \\ \hline 14\_ & \quad 284 \end{array}$$

$$\begin{array}{r|l} & \quad 7.2 \\ \hline 7 & \overline{51}.\overline{84} \\ & -49 \\ \hline 142 & \quad 284 \\ & \; -284 \\ \hline & \quad\; 0 \end{array}$$

Step 5 We know 142 × 2 = 284, So the new digit is 2, Divide and get the remainder

Step 6 **Since the remainder is zero and no bar left, therefore $\sqrt{51.84} = 7.2$**

Example 6.28: **Find the square root of 31.36**

Sol.

$$\begin{array}{r|l} & \quad 5.6 \\ \hline 5 & \overline{31}.\overline{36} \\ & -25 \\ \hline 106 & \quad 636 \\ & \; -636 \\ \hline & \quad\; 0 \end{array}$$

$\sqrt{31.36} = 5.6$

Note : Let us learn how to put bars in a decimal numbers. Consider a number say 325.732, it has two parts : integral and decimal parts. For integral part 325, we start from the unit's place (here 5) and move towards left. The first bar is over 25 and the second bar is over 3. For decimal part 732, we start from decimal and move towards right. First bar is over 73 and for second bar we put 0 after 2 and make $.\overline{73}\,\overline{20}$

Exercise 5.4

1. Find the square root of the following decimal numbers

   (i) 9.61 (ii) 11.56 (iii) 466.56 (iv) 1.4641 (v) 1354.24

   (vi) 1.218816

2. Find the square root of the followings:

   (i) $\frac{64}{169}$ (ii) $\frac{144}{441}$ (iii) $\frac{81}{784}$ (iv) $\frac{196}{625}$

3. Find the square root of 2, 3 and 5 upto three digits of decimals.

4. Multiple choice questions :

(i) Choose the correct way of placing bars from following:

(a) $\sqrt{15625}$ (b) $\sqrt{15625}$ (c) $\sqrt{15625}$ (d) $\sqrt{15625}$

(ii) After how many places decimal will appear in square root of 24.01.

(a) 1 (b) 2 (c) 3 (d) 4

(iii) Find square root of 39.0625

(a) 6.25 (b) 62.5 (c) 0.625 (d) 6.6251

(iv) Find the length of hypotenuse of a right triangle having other two sides as 6cm and 8cm.

(a) 6cm (b) 8cm (c) 10cm (d) $10cm^2$

Activity

AIM :- To observe some given number pattern and write their next three steps/rows.

Objectives : To understand number pattern and to generalise them

Previous knowledge :- Knowledge of number patterns.

Material Required : Some patterns involving numbers.

Procedure :

1. Observe the following number pattern:

(a)
$1^2 = 1$
$(11)^2 = 121$
$(111)^2 = 12321$

(b)
$1^2 = 1$
$2^2 = 1 + 3$
$3^2 = 1 + 3 + 5$

2. Identify the rule involved in each pattern.
3. Complete the next three rows of each pattern on the basis of rule described in step 2.

Observations:

(i) 4th row in pattern (a) is $(1111)^2$ = 1234321

5th row in pattern (a) is $(11111)^2$ = ...

6th row in pattern (a) is .... = 123456....................

(ii) 4th row in pattern (b) is 4^2 = 1+3+5+7

5th row in pattern (b) is .... = 1+3+5+7+9

6th row in pattern (b) is .... = 1+3+5+7+.............

VIVA VOCE

Q1. What is the value of $(1111111)^2$?

Ans: 1234567654321

Q2. Express 12^2 as a sum of first 12 odd natural numbers.

Ans: $12^2 = 1+3+5+7+9+11+13+15+17+19+21+23$

Q3. What is value of 1+3+5+7+9+11+13+15 ?

Ans: $(8)^2 = 64$

Learning Outcomes

After completion of the chapter, students are now able to:

- *Find square of a number.*
- *Understand different properties of square numbers.*
- *Find square root of a number using different methods.*
- *Understand perfect square numbers.*
- *Use concept of square and square root in solving practical life problems.*

Answers

Exercise 5.1

1. (i) 361 (ii) 1681 (iii) 121 (iv) $\frac{9}{49}$

(v) $\frac{25}{9}$ (vi) 2.89 (vii) 0.004 (viii) 0.000196

2. (i) Ones digit is 7 (ii) Ones digit in 8 (iii) Ones digit is 8

(iv) Ones digit is 3 (v) Ones digit is 2 (vi) No of zeros are odd

(vii) No of zeros are odd (viii) Ones digit is 7 (ix) No. of zeros are odd

(x) No. of zeros are odd

3. (i) Two zeros (ii) Two zeros (iii) Four zeros

(iv) Six zeros (v) Eight zeros (vi) Four zeros

4. (i), (iii), (v), (vii) would be odd numbers

(ii), (iv), (vi), (viii) would be even numbers.

5. (i) 1 (ii) 1 (iii) 1 (iv) 1 (v) 4 (vi) 4 (vii) 4

(viii)4 (ix) 9 (x) 9 (xi) 9 (xii) 9 (xiii) 6 (xiv) 6

(xv) 6 (xvi) 6 (xvii) 5 (xviii) 5

6. (i) 28 (ii) 42 (iii) 60 (iv) 20

7. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17

(ii) 1 + 3 + 5 + 7 + 9 + 11+13+15+17+19+21+23

(iii) 1+3+5+7+9+11+13+15+17+19+21+23+25+27+29+31

8. (i) $(5)^2$ (ii) $(13)^2$

9. (i) (112, 113) (ii) (220, 221) (iii) (544, 545)

(iv) (684, 685)

10. (i) 15 (ii) 25 (iii) 49 (iv) 159 (v) 219

11. (i) $1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + \underline{12}^2 = 13^2$

$\underline{4}^2 + 5^2 + \underline{20}^2 = 21^2$

$5^2 + \underline{6}^2 + 30^2 = \underline{31}^2$

$6^2 + 7^2 + \underline{42}^2 = \underline{43}^2$

(ii) $1^2 = 1 = 1$

$2^2 = 4 = 1 + 2 + 1$

$3^2 = 9 = 1 + 2 + 3 + 2 + 1$

$\underline{4}^2 = \underline{16} = 1 + 2 + 3 + 4 + 3 + 2 + 1$

$\underline{5}^2 = \underline{25} = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$

(iii) $21^2 = 441$

$201^2 = 40401$

$2001^2 = 4004001$

$\underline{(20001)}^2 = 400040001$

$(2000001)^2 = 4000004000001$

$(20000001)^2 = \underline{400000040000001}$

12. 1234567654321'

13. (i) 2025 (ii) 5625 (iii) 9025 (iv) 15625 (v) 42025

14. a, b, c

15. (i) 8, 15, 17 (ii) 12, 35, 37 (iii) 16, 63, 65 (iv) 18, 80, 82

(v) 20, 99, 101

16. (i) b (ii) d (iii) b (iv) d (v) d

(vi) a (vii) c (viii) d (ix) b (x) a

Exercise 5.2

1. (i) 1 or 9 (ii) 3 or 7 (iii) 4 or 6 (iv) 4 or 6

(v) 2 or 8 (vi) 1 or 9 (vii) 0 (viii) 5

2. 512, 1320, 3617
3. (i) 8 (ii) 7 (iii) 11 (iv) 10
4. (i) 60 (ii) 26 (iii) 96 (iv) 54
(v) 80 (vi) 42 (vii) 110 (viii) 32
5. (i) 3, 27 (ii) 15, 60 (iii) 22, 242 (iv) 3, 54
(v) 7, 147 (vi) 2, 100
6. (i) 3, 6 (ii) 5, 25 (iii) 6, 20 (iv) 7, 27
(v) 5, 21 (vi) 2, 78
7. (i) 7056 (ii) 32400
8. 16 rows **9.** 18cm **10.**56
11. (i) b (ii) d (iii) b (iv) b (v) a (vi) d

Exercise 5.3

1. (i) 3 (ii) 2 (iii) 3 (iv) 3 (v) 4
2. (i) 96 (ii) 90 (iii) 224 (iv) 69 (v) 649
(vi) 130 (vii) 72 (viii) 294 (ix) 4096 (x) 216
3. (i) 36, 24 (ii) 84, 43 (iii) 104, 58 (iv) 96, 64 (v) 129, 73
(vi) 51, 29
4. (i) 20, 26 (ii) 51, 33 (iii) 92, 77 (iv) 3, 101 (v) 416, 355
(vi) 1732, 887
5. 99856, 316 **6.** 1024, 32
7. (i) 56cm (ii) 85m (iii) 110m (iv) 135m
8. 10cm **9.** 56 plants
10. (i) b (ii) b (iii) a (iv) b (v) d (vi) b

Exercise 5.4

1. (i) 3.1 (ii) 3.4 (iii) 21.6 (iv) 1.21
(v) 36.8 (vi) 1.104
2. (i) $\frac{8}{13}$ (ii) $\frac{12}{21}$ (iii) $\frac{9}{28}$ (iv) $\frac{14}{25}$
3. (i) 1.414, 1.732, 2.236
4. (i) a (ii) a (iii) a (iv) c

CHAPTER

6

Cube and Cube Roots

Learning Objectives

In this Chapter you will learn

- *To find cube of a number.*
- *To learn about different properties of cubes.*
- *To find cube root of a number using different methods.*
- *To use concept of cube and cube roots in solving practical life problems.*

6.1 Introduction :-

The greatest Indian Mathematician **S. Ramanujan** experimented with numbers throughout his life. He loved the numbers. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi, whose number was 1729. While talking to Ramanujan, he described the taxi number as "a dull number". But Ramanujan quickly pointed out that the number 1729 is a very interesting number. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways, as :

$1729 = 1728 + 1 = 12^3 + 1^3$ and $1729 = 1000 + 729 = 10^3 + 9^3$

1729 has since been known as Hardy-Ramanujan number. There are infinitaly many such numbers. Few of them are 4104 {(2, 16); (9, 15)}, 13832 {(18, 20); (2, 24)} etc.

6.2 Cube :-

Cube of any number is when its exponent is 3 i.e. when some number is raised to power three. For example cube of 2 is written as 2^3 which is equal to $2 \times 2 \times 2$.

We can say that cube of any number is a number when that number is mutliplied three times by itself.

For Example: $8^3 = 8\times8\times8 = 512$

$12^3 = 12\times12\times12 = 1728$

$7^3 = 7\times7\times7 = 343$

In Geometry, we use the word cube. A cube is a solid figure whose all sides are equal. (Fig. 6.1). It has six faces, all are squares, for example, dice is a form of cube.

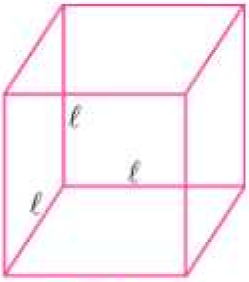


Fig. 6.1

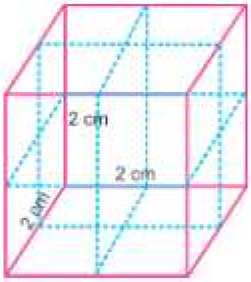


Fig. 6.2

How many cubes of side 1 cm will make a cube of side 2 cm ? Observe fig. 6.2, 8 cubes of side 1cm will make a cube of side 2cm.

Now consider the numbers 1, 8, 27, 64, 125, ...

Now observe :-

$1 = 1 \times 1 \times 1$; $8 = 2 \times 2 \times 2$; $27 = 3 \times 3 \times 3$
$64 = 4 \times 4 \times 4$; $125 = 5 \times 5 \times 5$; and so on.

Each of these is obtained when a number is multiplied three times by itself.

These types of numbers are known as **Perfect Cube or Cube numbers.**

Is 9 a perfect cube ?

As $9 = 3 \times 3$ and there is no natural number which when multiplied three times by itself gives 9 So 9 is not a Perfect cube number.

The following are the cubes of numbers form 1 to 20 (the table for cube of a number.)

Number	1	2	3	4	5	6	7	8	9	10	...	20
Cube	1^3 $=1\times1\times1$ $= 1$	2^3 $=2\times2\times2$ $= 8$	3^3 $=3\times3\times3$ $= 27$	4^3 $=4\times4\times4$ $= 64$	5^3 $=5\times5\times5$ $= 125$	6^3 $=6\times6\times6$ $= 216$	7^3 $=7\times7\times7$ $= 343$	8^3 $=8\times8\times8$ $= 512$	9^3 $=9\times9\times9$ $= 729$	10^3 $=10\times10\times10$ $= 1000$	...	20^3 $=20\times20\times20$ $= 8000$

(Table 6.1)

Now observe, whether cube of even numbers are even ? and cube of odd numbers are odd ? From table you can observe that cube of even number is even and odd number is odd. Also there are only few perfect cube numbers from 1 to 1000. How many Perfect cubes are there from 1 to 200 ?

Consider few numbers having 1 as the digit at Unit's (ones) place. (Find the cube of them for example 1, 11, 21, 31, 41, ..., 111, etc.)

Number	1	11	21	31	41	...	111	...
Cube	1^3 $=1\times1\times1$ $= 1$	11^3 $=11\times11\times11$ $= 1331$	21^3 $=21\times21\times21$ $= 9261$	31^3 $=31\times31\times31$ $= 29791$	41^3 $=41\times41\times41$ $= 68921$	...	111^3 $=111\times111\times111$ $= 1367631$	...

Table 6.2

What can you say about the ones digit of the cube of a number having 1 as the Unit's (ones) place number ? The unit's place digits of cube of a number having 1 as ones digit is 1.

Similarly, explore the ones digit of cubes of numbers ending in 2, 3, 4, ... etc. You will observe that the ones place digit of the cube of number having ones place 2 is 8; having ones place 3 is 7 and having ones place 4 is 4.

Example 6.1 : What should be the ones digit of the cube of each of the following numbers, tell without actual calculation?

(i) 2561 (ii) 342 (iii) 463 (iv) 1264

Sol. As we observe that, the ones digit of the cube of number :

(i) 2561 will be 1
(ii) 342 will be 8
(iii) 463 will be 7
(iv) 1264 will be 4

Example 6.2: Find cube of (i) 13 (ii) –4 (iii) $\frac{3}{8}$ (iv) 2.1

Sol. (i) Cube of 13 = $13^3 = 13 \times 13 \times 13 = 2197$

(ii) Cube of –4 = $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(iii) Cube of $\frac{3}{8} = \left(\frac{3}{8}\right)^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{27}{512}$

(iv) Cube of 2.1 = $(2.1)^3 = 2.1 \times 2.1 \times 2.1 = 9.261$

Example 6.3: Find the volume of cube having side 3 cm.

Sol. Here, side of cube = 3 cm; As volume of cube = $(\text{side})^3$

So volume of cube having side 3cm = $(3 \text{ cm})^3 = 27 \text{ cm}^3$

Exercise 6.1

1. What should be the ones digit of the cube of the each of the following numbers tell without actual calculation?

(i) 231 (ii) 4584 (iii) 6259 (iv) 105 (v) 17 (vi) 120

2. Find the cube of following numbers.

(i) –9 (ii) 16 (iii) –14 (iv) $\frac{1}{13}$ (v) $\frac{8}{7}$

(vi) 2.4 (vii) 0.002 (viii) 9.9 (ix) 1.01

3. Find volume of cube having side :

(i) 4 cm (ii) 15cm (iii) 17cm (iv) 2,3 cm (v) 7,2m

4. **Multiple Choice Questions :**

(i) Ones digit of cube of 7 is :

(a) 7 (b) 3 (c) 5 (d) 6

(ii) Ones digit of cube of a number having 2 at ones place is :

(a) 2 (b) 4 (c) 6 (d) 8

(iii) Volume of a cube of side 5cm is:

(a) 15cm (b) 125cm^3 (c) 45cm^3 (d) 50cm

(iv) Ones digit of 1823^3 is :

(a) 3 (b) 9 (c) 7 (d) 6

(v) How many cubes of side 1cm will form a cube of side 2cm.

(a) 2 (b) 4 (c) 6 (d) 8

(vi) What is ones place digit in 626^3.

(a) 2 (b) 3 (c) 4 (d) 6

6.2.1 Some Interesting Patterns :

1. (a) Adding consecutive odd numbers

Observe the following patterns of sums of odd number :

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$
$$31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$$

Is it not interesting ? What do you observe?

(b) Subtracting cubes of consecutive numbers.

Observe the following Pattern :

$2^3 - 1^3 = 1 + 2 \times 1 \times 3 = 1 + 6 = 7$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3 = 1 + 18 = 19$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3 = 1 + 36 = 37$

$5^3 - 4^3 = 1 + 5 \times 4 \times 3 = 1 + 60 = 61$

Is it not interesting? What do you observe?

2. Cubes and their Prime factors

Consider the following Prime factorisation of some numbers and their cubes.

Prime factorisation of a number	Prime factorisation of its cube
$10 = 2 \times 5$	$10^3 = 1000 = 2\times2\times2\times5\times5\times5 = 2^3 \times 5^3$
$18 = 2 \times 3 \times 3$	$18^3 = 5832 = 2\times2\times2\times3\times3\times3\times3\times3\times3 = 2^3\times3^3\times3^3$
$14 = 2 \times 7$	$14^3 = 2744 = 2\times2\times2\times7\times7\times7 = 2^3 \times 7^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3\times3\times3\times5\times5\times5 = 3^3 \times 5^3$
$20 = 2\times2\times5$	$20^3 = 8000 = 2\times2\times2\times2\times2\times2\times5\times5\times5 = 2^3\times2^3\times5^3$

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

In the prime factorisation of any number, if each factor appears three times, then the number is perfect cube.

Example 6.4: By Prime Factorisation check whether following are perfect cube or not ?

(i) 512 (ii) 5000 (iii) 1372 (iv) 1331

Sol. (i) 512

Prime factorisation of 512 is =

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

As we can group prime factors in triplets

$\therefore$ It is a perfect cube.

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) 5000

Prime factorisation of 5000 = $\underline{2\times2\times2}\times\underline{5\times5\times5}\times5$

As prime factors of 5000 cannot be grouped in triplets.

$\therefore$ It is not a perfect cube.

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

(iii) 1372

Prime factorisation of 1372 = $2 \times 2 \times \underline{7 \times 7 \times 7}$

As prime factors of 1372 cannot be grouped in triplets.

$\therefore$ It is not a perfect cube.

2	1372
2	686
7	343
7	49
7	7
	1

(iv) 1331

Prime Factorisation of 1331 = $\underline{11 \times 11 \times 11}$

As prime factors of 1331 can be grouped in triplets.

$\therefore$ it is a perfect cube.

11	1331
11	121
11	11
	1

Exercise 6.2

1. Express the following numbers as the sum of consecutive odd natural numbers.

(i) 7^3 (ii) 8^3 (iii) 9^3

2. Find the value of following by using suitable pattern?

(i) $12^3 - 11^3$ (ii) $20^3 - 19^3$ (iii) $51^3 - 50^3$

3. Which of the following are perfect cubes ?

(i) 225 (ii) 10648 (iii) 1125 (iv) 2744

6.2.2 Smallest multiple that is Perfect Cube :-

In last section we have observed that every number is not a perfect cube. If a number is not a perfect cube, we can find the smallest natural number by which the given number must be multiplied, so that the product is a perfect cube. We can also find the smallest number to divide the given number, so that quotient is a perfect cube.

Example 6.5 : Is 243 a perfect cube ? If not, find the smallest number by which 243 must be multiplied so that the product is a perfect cube. Also find the number.

Sol. The prime factorisation of 243 is

$$243 = \underbrace{3 \times 3 \times 3} \times 3 \times 3 = 3^3 \times 3^2$$

The prime factor 3 does not appear in a group of three. So 243 is not a perfect cube. To make it perfect cube, we need one more 3, so to make 243 a perfect cube. we have to multiply it by 3.

$$243 \times 3 = 729 = \underbrace{3 \times 3 \times 3} \times \underbrace{3 \times 3 \times 3} = 729$$

3	243
3	81
3	27
3	9
3	3
	1

Now, it is a perfect cube number. Hence, the smallest number to be multiplied is 3 and 729 is the number which is perfect cube.

Example 6.6 : Is 675 a perfect cube ? If not, find the smallest natural number by which 675 must be multiplied so that the product is a perfect cube.

Sol. Let us find the prime factorisation of

$$675 = \underbrace{3 \times 3 \times 3} \times 5 \times 5$$

The prime factor 5 does not appear in group of three.

So, 675 is not a perfect cube. To make it a perfect cube, we need one more 5. In that case

$$675 \times 5 = \underbrace{3 \times 3 \times 3} \times \underbrace{5 \times 5 \times 5} = 3375$$

3	675
3	225
3	75
5	25
5	5
	1

which is a perfect cube. Hence, the smallest natural number by which 675 should be multiplied to make it a perfect cube is 5.

Example 6.7 : Is 31944 a perfect cube ? If not, then by which smallest natural number should 31944 be divided so that quotient is a perfect cube ?

Sol. Let us first find the prime factorisation of 31944.

Now $31944 = \underbrace{2 \times 2 \times 2} \times 3 \times \underbrace{11 \times 11 \times 11}$

The prime factor 3 does not appear in a group of three, so 31994 is not a perfect cube. In the factorisation 3 appears only once.

So, if we divide the number by 3, then the prime factorisation of quotient will not contain 3.

So, $31944 \div 3 = 10648$

2	31944
2	15972
2	7986
3	3993
11	1331
11	121
11	11
	1

Hence the smallest number by which 31944 should be divided to make it a perfect cube is 3.

The perfect cube in that case is 10648.

Exercise 6.3

1. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

   (i) 81 (ii) 100 (iii) 72 (iv) 625 (v 2916 (vi) 41503

2. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

   (i) 2187 (ii) 78125 (iii) 16384 (iv) 19773 (v) 36501 (vi) 23625

3. Check which of the following are perfect cubes :

   (i) 2700 (ii) 16000 (iii) 8000 (iv) 27000 (v) 125000 (vi) 15125

   Which pattern do you observe in these Perfect cubes ?

4. **Multiple Choice Questions :**

   (i) By what number 108 be multiplied to make it a perfect cube.

   (a) 2 (b) 3 (c) 4 (d) 6

   (ii) By what number 625 be divided so as to make it a perfect cube?

   (a) 5 (b) 8 (c) 6 (d) 9

   (iii) Which of following is not a perfect cube ?

   (a) 16 (b) 27 (c) 64 (d) 125

   (iv) Find the number which when multiplied with 500 makes it a perfect cube?

   (a) 5 (b) 2 (c) 3 (d) 6

   (v) Find $7^3 - 6^3$:

   (a) 127 (b) 397 (c) 1141 (d) 200

6.3 Cube Roots:-

Sometime, we have to find the number whose cube is given. For example, if the volume of a cube is 64 cm³, what should be the length of side of cube ? Now, here we need to find a number whose cube is 64.

In last chapter we have already studied about square and square root. As you know, finding the square root is inverse operation of squaring. Similarly, finding the cube root is inverse operation of finding cube.

We know that $4^3 = 64$. So we say that cube root of 64 is 4. We write $\sqrt[3]{64} = 4$. The symbol

$\sqrt[3]{\ }$ denotes cube root. In terms of power, we write it as $(\ )^{1/3}$. Study the following (table 6.3) :

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$
$\sqrt[3]{1}$ $= 1$	$\sqrt[3]{8} = (2^3)^{1/3}$ $= 2$	$\sqrt[3]{27} = (3^3)^{1/3}$ $= 3$	$\sqrt[3]{64} = (4^3)^{1/3}$ $= 4$	$\sqrt[3]{125} = (5^3)^{1/3}$ $= 5$	$\sqrt[3]{216} = (6^3)^{1/3}$ $= 6$

Table 6.3

6.3.1 Cube Root through Prime factorisation :-

We can find the cube root of a given cube number by its Prime factorisation. Study the following examples :

Example 6.8 : Find the cube root of 42875.

Sol. Let us find Prime factorisation of the number 42875.

Now $42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$$\sqrt[3]{42875} = \sqrt[3]{5\times5\times5\times7\times7\times7}$$
$$= 5 \times 7 = 35$$

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

Example 6.9 : Find cube root of 175616 by Prime factorisation.

Sol. Prime factorisation of

$175616 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{7\times7\times7}$

$$\text{So}\quad \sqrt[3]{175616} = \sqrt[3]{\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{7\times7\times7}}$$
$$= 2 \times 2 \times 2 \times 7 = 56$$

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Exercise 6.4

1. Cube of a number is 64. Find the number.
2. Cube of a number is 3375. Find the number.
3. Find the cube root of each of the following numbers by prime factorisation :

   (i) 5832 (ii) 216000 (iii) 456533 (iv) 729000

   (v) 85184 (vi) 328509

4. **Multiple Choice Questions :**

   (i) What is cube root of 512 ?

   (a) 2 (b) 4 (c) 6 (d) 8

   (ii) Find $\sqrt[3]{1728}$.

   (a) 10 (b) 12 (c) 14 (d) 16

   (iii) Find cube root of 1331.

   (a) 11 (b) 21 (c) 31 (d) 23

   (iv) A perfect cube ends with digit 2 what will be ones digit of its cube root.

   (a) 4 (b) 2 (c) 6 (d) 8

Learning Outcomes

After completion of the chapter, the students are now able to:

- *Find cube of a number.*
- *Understand about different properties of cubes.*
- *Find cube root of a number using different methods.*
- *Use concept of cube and cube roots in solving practical life problems.*

Answers

Exercise 6.1

1. (i) 1 (ii) 4 (iii) 9 (iv) 5 (v) 3 (vi) 0

2. (i) –729 (ii) 4096 (iii) –2744 (iv) $\frac{1}{2197}$ (v) $\frac{512}{343}$

   (vi) 13.824 (vii) 0.000000008 (viii) 970.299 (ix) 1.030301

3. (i) 64 cm^3 (ii) 3375cm^3 (iii) 4913cm^3 (iv) 12.167 cm^3 (vi) 373.248 cm^3

4. (i) b (ii) d (iii) b (iv) c (v) d (vi) d

Exercise 6.2

1. (i) $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

(ii) $8^3 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

(iii) $9^3 = 73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89$

2. (i) $12^3 - 11^3 = 1 + 12 \times 11 \times 3$; (ii) $20^3 - 19^3 = 1 + 20 \times 19 \times 3$

(iii) $51^3 - 50^3 = 1 + 51 \times 50 \times 3$

3. (ii) and (iv)

Exercise 6.3

1. (i) 9 (ii) 10 (iii) 3 (iv) 25 (v) 2 (vi) 11

2. (i) 3 (ii) 5 (iii) 4 (iv) 9 (v) 3 (vi) 7

3. (i) No (ii) No (iii) Yes (iv) Yes (v) Yes (vi) No

[**Hints :** Pattern : As $(a \times b)^m = a^m \times b^m$ $\sqrt[3]{ab} = (a)^{1/3} \times b^{1/3}$]

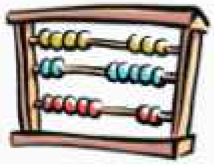
4. (i) a (ii) a (iii) a (iv) b (v) a

Exercise 6.4

1. 4 **2.** 15

3. (i) 18 (ii) 60 (iii) 77 (iv) 90 (v) 44 (vi) 69

4. (i) d (ii) b (iii) a (iv) d



CHAPTER

7

Comparing Quantities

Learning Objectives

In this chapter, you will learn

- *About concept of ratio and percentage.*
- *About concept of marked price. (M.P.) and discount, their percentage and their application in daily life.*
- *About sale tax and its application in daily life.*
- *About simple and compound interest and their application in our daily life.*
- *About bank passbook and how to fill deposit and withdrawl slip.*

7.1 Recalling Ratios and Percentage :-

In Class VII we have already studied about concept of ratio and proportion. We know, the meaning of ratio is to compare two quantities. Let us take a basket having two types of fruits say, apples and bananas (fig.). Let number of apples be 40 and number of bananas be 10, then the ratio of number of apples to the number of bananas is 40 : 10.

We can also do the comparison by using fractions as $\frac{40}{10}=\frac{4}{1}$.

From this we can say that in the basket, the number of apples are four times the number of bananas. Similarly the ratio of number of bananas to the number of apples is 10 : 40.

In fraction form, we can write it as $\frac{10}{40}=\frac{1}{4}$ i.e. 1:4 So We can say that in the basket the number of bananas are $\frac{1}{4}$th the number of apples.

We read it as 1 is to 4.

The comparison can also be done by using **Percentage**. There are two different methods to find the percentage.

There are 40 apples out of 50 fruits in the basket. So out of total fruits, Ratio of apples in the baske1t is $\frac{40}{50}$, to find the percentage we have to make the denominator 100

So $\frac{40}{50} = \frac{40}{50} \times \frac{2}{2} = \frac{80}{100}$

Hence we can say that in the basket out of total number of fruits, 80% are apples.

Or

we can also find it with the help of Unitary method.

Out of 50 fruits in the basket the number of apples are 40.

So out of 100 fruits in basket the

number of apples $= \frac{40}{50} \times 100$

$= 80.$

As basket contains only two fruits apples and bananas, so Percentage of apples + Percentage of bananas = 100

i.e. 80 + Percentage of bananas = 100

So, Percentage of bananas = 100 – 80 = 20

So basket has 80% apples and 20% bananas.

Aliter : We can also find it as out of total 50 fruits in the basket, the number of bananas are 10, so out of 100 fruits, the number of bananas are $= \frac{10}{50} \times 100 = 20$. Hence in basket, bananas are 20%.

We often ask a student that how much percent of marks he/she got in his/her previous exam.

To further clear the concept, study the following examples :

Example 7.1 : Find the ratio of the following:

(i) Speed of cycle 20km/hour to speed of car 60 km/hour

(ii) 5 m to 20 m **(iii) 50 paise to ₹ 5**

Sol. (i) Speed of cycle = 20km/h

Speed of car = 60km/h

Speed of cycle to the speed of car is 20 : 60.

In fraction, we can write it as $\frac{20}{60} = \frac{1}{3}$ i.e. 1:3

So, we can say that speed of cycle per hour is $\frac{1}{3}$rd the speed of car per hour.

(ii) 5 m to 20 m is 5 : 20.

In fraction form it is $\frac{5}{20} = \frac{1}{4}$ *i.e.* 1 : 4. We read it as 1 is to 4.

(iii) 50 paise to ₹ 5

50 paise to 500 paise. (As ₹1 = 100 Paise, So, ₹ 5 = 500 Paise)

= 50 : 500

In fraction form it is $\frac{50}{500} = \frac{1}{10}$ *i.e.* 1 : 10

We read it as 1 is to 10.

Example 7.2: Convert the following ratios into percentage :

(i) 1 : 4 **(ii) 3 : 4** **(iii) 2 : 5**

Sol. (i) $1 : 4 = \frac{1}{4} = \frac{1}{4} \times \frac{25}{25} = \frac{25}{100}$ *i.e.* 25%

(To find percentage, make the denominator 100)

(ii) $3 : 4 = \frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$ *i.e.* 75%

(iii) $2 : 5 = \frac{2}{5} = \frac{2}{5} \times \frac{20}{20} = \frac{40}{100}$ *i.e.* 40%

Example 7.3 : In a class, out of total students, 40% are boys. If boys are 12 in number then find (i) Total students in the class (ii) number of girls in the class (iii) Ratio of girls to boys of the class.

Sol. Let total number of students be x

So, as per question, 40% of $x = 12$

$$\Rightarrow x \times \frac{40}{100} = 12$$

$$\text{So, } x = \frac{12 \times 100}{40} = 30$$

(i) So total number of students in the class are 30.

(ii) Number of girls = 30 – Number of boys = 30 – 12 = 18

(iii) Ratio of girls to boys = $18 : 12 = \frac{18}{12} = \frac{3}{2}$

We can write it as 3 : 2 and read as 3 is to 2.

Aliter : As we know that percent means out of 100. So 40% boys means, there are 40 boys out of 100 students. As boys are 12, so the total number of students are = $\frac{100}{40} \times 12$ = 30.

From the total number of students, we can find number of girls and the ratio of girls and boys as we have done above.

Example 7.4 : A school organised a picnic for a class having 38 students. The picnic spot is 60 km away from the school. The transport company is charging at the rate of ₹ 8 per km. If two teachers are also going with the class and cost of refreshment is ₹3840. Find (i) Cost per head (ii) If their first stop is 18 km from the school, then up to first stop. what percent of total trip they have covered and what percent is left behind ?

Sol. (i) To find the cost per head, first, we have to find the total cost of trip.

$\therefore$ Total cost of trip = Transport Charges + Refreshment Cost

$= (60 \times 2) \times 8 + 3840$

$= 960 + 3840 = ₹\ 4800$

Total Persons = Total Students + Teachers

$= 38 + 2 = 40$

So Cost per head $= 4800 \div 40 =$ ₹120

(ii) Their first stop is at 18 km from school.

So percentage of distance covered out of total distance is $= \dfrac{18}{120} \times 100 = 15\%$

and percentage of distance left = 100 – 15 = 85%
For remaining distance, we can also find it as
Remaining distance = 120 – 18 = 102km

So percentage $= \dfrac{102}{120} \times 100 = 85\%$

Exercise 7.1

1. Find the ratio :

   (i) Speed of cycle 12 km/hr to the speed of car 36 km/hr.

   (ii) 10 m to 10 km (iii) 1.5 m to 10 cm (iv) 1 hr to 300 seconds

   (v) 80 paise to ₹4 (vi) 200g to 8kg

2. Out of 20 students in a class, 50% of students are good in science . Find the number of students good in science.

3. 35% of 40 students are good in statistics. How many students are not good in statistics?

4. What percent of numbers from 1 to 50 are prime?

5. Convert the following ratios to percentage :

   (i) 1 : 3 (ii) 4 : 5 (iii) 1 : 2 (iv) 2:5 (v) 5:4 (vi) 1:5

6. A man spent 87% of his salary. If he saved ₹325, find his salary.

7. A Kabbadi team played 15 matches and won 60% of the matches. How many matches did they lose?

8. From a class of 60 students, 40% students like chess, 15% like carrom and remaining students like other games. Find number of students who like carrom, chess and other games.

9. **Multiple Choice Questions :**

   (i) The ratio of 6 km to 600 m is

   (a) 1:100 (b) 10:1 (c) 1:10 (d) 100:1

   (ii) Percentage of 3:4 is

   (a) 75% (b) 50% (c) 25% (d) 100%

   (iii) Ratio of 200 paise to ₹3 is

   (a) 2 : 3 (b) 3 : 2 (c) 200 : 3 (d) 3 : 200

(iv) There are 48 girls out of 80 students. Percentage of girls is

(a) 50% (b) 80% (c) 75% (d) 60%

(v) Conversion of 3:5 into percentage is

(a) 30% (b) 50% (c) 60% (d) 80%

7.2 Finding Discount :-

To attract customers or to promote the sale of goods, the companies often give discount. The discount is reduction given on the **Marked Price** (M.P.). Marked Price is also known as List Price and Discount = Marked Price (M.P.) – Selling Price (S.P.)

$$\text{Discount } \% = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

Example 7.5 : A shop gives 20% discount on the Marked Price. What would be selling price of each of the following :

(i) A dress marked at ₹ 300 (ii) A pair of shoes marked at ₹ 750

Sol. (i) Marked Price of dress = ₹300

$$\text{Discount} = 20\% \text{ of ₹ } 300 = \left(300 \times \frac{20}{100}\right)$$

$$= \text{₹ } 60$$

So Selling price of dress = Marked Price – Discount

= ₹ 300 – ₹ 60 = ₹ 240

(ii) Marked Price of pair of Shoes = ₹ 750

$$\text{Discount} = 20\% \text{ of ₹ } 750 = \text{₹ } \left(750 \times \frac{20}{100}\right)$$

$$= \text{₹ } 150$$

So Selling price of pair of shoes = Marked Price – Discount

= ₹ 750 – ₹ 150

= ₹ 600

Example 7.6 : A Photoframe is marked at ₹ 600 is sold for ₹ 450. What is the discount and discount percentage ?

Sol. We know, Discount = Marked Price (M.P.) – Selling Price (S.P.)

So Discount = ₹(600 – 450)

= ₹150

$$\text{Discount } \% = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$ (As Discount is always on Marked Price.)

$$\text{Now} \quad \text{Discount } \% = \frac{150}{600} \times 100$$

∴ Discount percentage = 25%

Exercise 7.2

1. An article marked at ₹1920 is sold for ₹1840, what is discount and discount percentage?
2. A book marked at ₹791 is sold for 678 ₹. Find discount and discount percent.
3. The list price (M.P.) of bag is ₹220. A discount of 15% is announced on sale. What is its sale price ?
4. The marked price of a ceiling fan is ₹720. During off season it is sold for ₹684. Determine the discount percentage.
5. A shop offers 4% discount on all cash purchases. What cash amount do we need to pay for an item whose marked price is ₹650 ?
6. A saree is sold for ₹720 after giving a 20% discount on Marked Price. What is the Marked Price ?
7. If Ankush is getting discount of 8% on an item with marked price of ₹400. Find the discount and cost price of item for ankush.
8. Rachna is getting discount of 10%, 15% and 20% on 3 books each with marked price ₹100. Find total amount Rachna has to pay.
9. **Multiple Choice Questions :**
   (i) Discount is calculated on:
   (a) S.P. (b) M.P. (c) C.P. (d) None of these
   (ii) Discount percent is equal to:
   (a) $\frac{\text{Discount}}{\text{M.P}}\times 100$ (b) $\frac{\text{Discount}}{\text{S.P}}\times 100$
   (c) S.P.– M.P (d) M.P– C.P.
   (iii) A table marked at ₹15000 is available for ₹14400. The discount percent is:
   (a) 2% (b) 4% (c) 5% (d) 7%
   (iv) A book marked at ₹900 is sold for ₹873. The discount is
   (a) 72 (b) 27 (c) 29 (d) 24
   (v) A Chair is sold at 4% discount and marked price of chair is ₹450. What is selling price of chair?
   (a) ₹ 412 (b) ₹ 425 (c) ₹ 432 (d) ₹ 440

7.2.1 Estimation in Percentage

Suppose your total purchase is of ₹1157.80 and you are offered a discount of 30% on total bill value. Then how will you estimate the final amount to be paid ?

Step I : Round off the bill of 1157.80 to nearest tens i.e. ₹ 1160.

Step II: Find 30% of 1160.

$$= 1160\times\frac{30}{100} = 348$$

When estimated to nearest tens will be ₹350.

Amount to be paid = 1160 – 350 = ₹810

Example 7.7 : Estimate the bill amount to be paid, if

(i) bill is ₹669.70 and discount is 10%

(ii) bill is ₹1008, discount 10%

Sol. (i) Round the bill to nearest tens, here bill is ₹669.70, so its nearest tens is ₹670.

$$\text{Discount} = 10\% \text{ of } 670$$

$$= 670 \times \frac{10}{100} = 67, \text{ Its nearest tens is ₹}70$$

So estimated bill amount to pay = ₹670 – ₹ 70 = ₹600

(ii) Bill amount is ₹1008, its nearest tens is ₹ 1010.

$$\text{Discount} = 10\% \text{ of } 1010$$

$$= 1010 \times \frac{10}{100}$$

$$= 101, \text{ Its nearest tens is ₹}100.$$

So Estimated bill amount to pay is ₹1010 – ₹100 = ₹910.

Exercise 7.3

1. Let your bill in a shop is ₹598.80 and the shopkeeper gives a discount of 20%. Estimate your bill amount.
2. Estimate the bill amount, if bill is ₹378 and shopkeeper gives a discount of 15%.

7.3 Sales Tax

For the development of a country, Government requires money. Government collect this money by implementing various types of taxes.

The sale tax is charged by the Government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. Therefore, it is always on the selling price of an item and is added to the value of the bill. Now a days Goods and Services Tax (GST) is being levied on supply of goods and services.

Example 7.8 : Find the buying price of each of the following when 5% sales tax is added to the purchase of

(i) A towel at ₹120

(ii) Pair of roller skates at ₹450

Sol. (i) Price of Towel = ₹120

Sales tax = 5%

On ₹ 100 the sales tax = ₹5

$$\text{On ₹ 1 the sales tax} = ₹\frac{5}{100}$$

On ₹120 the sales tax = ₹$\left(\frac{5}{100}\times 120\right)$ = ₹6

So Bill amount (Selling Price) = Cost of Item + Sales tax

= ₹120 + ₹ 6 = ₹126

(ii) Price of Pair of Roller Skates = ₹450

Sales tax = 5%

On ₹100 the sales tax = ₹5

So on ₹450 the sales tax = ₹$\left(\frac{5}{100}\times 450\right)$ = ₹22.50

So bill amount (Selling amount) = Cost of item + Sale Tax

= ₹450 + ₹22.50

= ₹472.50

Example 7.9 : A person purchased a LED TV for ₹5400 including 8% sales tax. Find the price before sales tax was added.

Sol. The price of LED TV included the 8% sales tax is ₹5400. It means if the price without sales tax is ₹100 then price including sales tax is ₹108.

So, when price including sales tax is ₹108, then original price = ₹100

Hence when price including Sales Tax is ₹5400, then original price = ₹$\left(\frac{100}{108}\times 5400\right)$

= ₹5000

Exercise 7.4

1. The cost of a TV set at a showroom is ₹36500. The sales tax is 8%. Find the bill amount.
2. A LED TV is available for ₹26880 including sales tax. If the original cost of LED TV is ₹24000. Find the rate of sales tax.
3. The sales tax rate is 8%. If Rahul bought a washing machine and paid a sales tax of ₹1920. What is the cost of washing machine before sales tax ?
4. Seema bought a box of biscuits for ₹904 which includes a sale tax of 13%. What is the price of biscuit box without sale tax ?
5. The cost of pair of shoes at a shop is ₹440. The sales tax is 5%. Find bill amount.
6. **Multiple Choice Questions :**

(i) Sales Tax on an item at 5% with marked price of ₹200.

(a) ₹5 (b) ₹10 (c) ₹15 (d) ₹20

(ii) If sales tax of 15% is levied on shoes marked at ₹2000, find the final price after adding sales tax.

(a) ₹2500 (b) ₹2015 (c) ₹2300 (d) ₹2500

(iii) A book after adding sales tax at 10% is sold for ₹165. What was its price before adding sales tax.

(a) ₹100 (b) ₹150 (c) ₹160 (d) ₹140

(iv) A Cricket bat with list price of ₹5000 is sold after adding sales tax of 8%. Find selling price of bat.

(a) ₹5200 (b) ₹5600 (c) ₹6000 (d) ₹5400

7.4 Simple Interest

Interest is the extra money paid by banks or post offices etc. on money deposited with them. If people borrow money then also they have to pay interest. In class 7, you have already learnt how to calculate simple interest. In this section, we will discuss about compound interest.

During your bank visit, you might have come across the statements like "one year interest for FD (Fixed Deposit) in the bank @ 8% per annum or saving account with interest @ 4% per annum. Let's first revise concept of simple interest with following examples.

Example 7.10 : A sum of ₹5,000 is borrowed at a rate of 8% per annum for 2 years. Find the simple interest and the amount to be paid at the end of 2 years.

Sol. You know how to find the simple interest, we have the formula

$$\text{S.I.} = \frac{\text{P} \times \text{R} \times \text{T}}{100}$$

Here, P = ₹5000, R = 8% per annum and T = 2 years.
where P means Principal (sum borrowed), R means Rate percent per year, T means time

$$\text{So S.I.} = ₹\left(\frac{5000 \times 8 \times 2}{100}\right) = ₹800$$

Amount to be paid at the end of two years = Principal + S.I. = ₹5000 + ₹800
$\therefore$ Amount = ₹5800

Aliter We can also find interest using Unitary Method, as under.
On ₹100, interest charged for one year = ₹8

$$\text{So on ₹5,000 interest charged for one year} = ₹\left(\frac{8}{100} \times 5000\right) = ₹400$$

Interest for two years = ₹400 × 2 = ₹800

So amount to be paid at the end of two years

= Principal + Interest

= ₹5000 + ₹800 = ₹5800

Exercise 7.5

1. In what time ₹1600 will amount to ₹1760 at rate 5% per annum simple interest.
2. At what rate of simple interest will a sum double itself in two years.
3. Find simple interest and amount to be paid on ₹15000 at 5% per annum after two years.

7.5 Calculating Compound Interest

Normally the interest paid or charged is never simple. The interest is calculated on the amount of previous year. This is known as interest compound or compound interest. If we have some money in our bank account, every year some interest is added to it, which is shown in the pass book. This interest is not the same, each year it increases (if we don't withdraw any amount from our account.)

Note that Principal remains same under simple interest while it changes year after year under compound interest. In compound interest, amount at the end of first year becomes the principal for second year, if it's compounded annually.

Let us take an example and find the interest year by year. Each year our principal changes.

Example 7.11 : A sum of ₹5000 was borrowed by Sham for 2 years at an interest of 4% compounded annually. Find the compound interest (C.I.) and the amount Sham has to pay at the end of 2 years.

Sol. Let us find simple interest for first year.

Here P = ₹5000, R = 4% p.a., T = 1 year

So simple interest after one year $= ₹\left(\frac{5000 \times 4 \times 1}{100}\right) = ₹200$

At the end of first year

Amount = Principal + Interest = ₹5000 + ₹200 = ₹5200

In compound interest, the amount at the end of first year becomes the principal for second year.

Now interest for second year $= ₹\left(\frac{5200 \times 4 \times 1}{100}\right) = ₹208$

Now amount which has to be paid at the end of second year

= ₹5200 + ₹208

= ₹5408

Compound Interest = Amount – Principal

= ₹5408– ₹5000 = ₹408

Study the table given below, to find the difference between Simple and Compound Interest.

We start with Principal ₹200 and Rate 20%.

		Under S.I.	Under C.I.
First year	Principal	₹200.00	₹200.00
	Rate 20% p.a.	₹40.00	₹40.00
	Year end amount	₹240.00	₹240.00
Second year	Principal	₹200.00	₹240.00
	Rate 20% p.a.	₹40.00	₹48.00
	Year end amount	₹240.00 + ₹40.00 = ₹280.00	₹288.00
Third year	Principal	₹200.00	₹288.00
	Rate 20% p.a.	₹40.00	₹57.60
	Year end amount	₹280 + ₹40.00 = ₹320	₹345.60

Table 7.1

Note that in 3 years

Interest earned as simple interest = ₹320 – ₹200 = ₹120

whereas, Interest earned as Compound Interest = ₹345.60 – ₹200 = ₹145.60

Note that the principal remains same under simple interest, whereas it changes year after year under compound interest. At the end of year, first year amount becomes the second year principal and so on.

7.6 Deducing a formula for Compound Interest

Under this section, we shall deduce a formula to find compound interest. You know already a formula to find simple interest.

Suppose P_1 is the principal on which interest is compounded annually at a rate of R% per annum. After one year we have to find amount, we will denote the amount as A_1, which will become the principal for second year, that will be denoted as P_2 and so on.

$SI_1 = \text{₹}\ \frac{P_1 \times R \times 1}{100}$ (Here SI_1 means S.I. for first year)

$$A_1 = P_1 + SI_1 = \text{₹}\left(P_1 + \frac{P_1R}{100}\right) = \text{₹}\ P_1\left(1+\frac{R}{100}\right) = P_2 \quad ...(1)$$

(As amount at the end of first year will be Principal for second year)

Now $SI_2 = \frac{P_2 \times R \times 1}{100} = P_1\left(1+\frac{R}{100}\right) \times \frac{R}{100}$ $\left[\because P_2 = P_1\left(1+\frac{R}{100}\right) \text{ from (1)}\right]$

$$= \frac{P_1R}{100}\left(1+\frac{R}{100}\right) \qquad ...(2)$$

Now $A_2 = P_2 + SI_2$

$$= P_1\left(1+\frac{R}{100}\right) + P_1\,\frac{R}{100}\left(1+\frac{R}{100}\right) \qquad \text{[Using (1) \& (2)]}$$

$$= P_1\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right) = P_1\left(1+\frac{R}{100}\right)^2 = P_3$$

Proceeding in this way, the amount at the end of T years will be

$$A_T = P_1\left(1+\frac{R}{100}\right)^T$$

or We can say $A = P\left(1+\frac{R}{100}\right)^T$

Where A is amount, P is principal, R is rate of interest and T is time.

From this, we can find compound interest as C.I. = A – P

Example 7.12 : Find the compound interest for 2 years at 5% per annum compounded annually where principal is ₹10,500.

Sol. For compound interest we will calculate Amount first of all by using formula,

$$A = P\left(1+\frac{R}{100}\right)^T$$

where A = Amount, P = Principal, R = rate, T = time

we have, P = ₹10,500, R = 5% p.a., T = 2 years

$$\text{So } A = ₹10{,}500\left(1+\frac{5}{100}\right)^2 = ₹10500 \times \left(\frac{21}{20}\right)^2$$

$$= ₹10500 \times \frac{21}{20} \times \frac{21}{20} = ₹11576.25$$

C.I. = A – P

= ₹11576.25 – ₹10,500.00

= ₹1076.25

Exercise 7.6

1. Find compound interest on Rs 14,000 for 2 years at 10% per annum compounded annually.
2. Find compound interest on Rs 1000 for 3 years at 20% per annum compounded annually.
3. Multiple choice Question :

(i) $S.I = \frac{P \times ...... \times ....}{100}$

(a) R, S (b) R, T (c) A, T (d) A, R

(ii) S.I. on ₹2000 for 1 year at 10% p.a. is

(a) ₹2000 (b) ₹200 (c) ₹20 (d) ₹2

(iii) Compound interest = Amount – ....

(a) S.I. (b) Profit (c) Rate of interest (d) Principal

(iv) Formula for calculating amount when compounded annually is

(a) $P\left(1+\frac{T}{100}\right)^R$ (b) $R\left(1+\frac{P}{100}\right)^T$ (c) $P\left(1+\frac{R}{100}\right)^T$ (d) $R\left(1+\frac{T}{100}\right)^P$

(v) In case of simple and compound interest for a period more than one year.

(a) S.I. < C.I. (b) C.I > S.I (c) S.I = C.I. (d) None of these

7.7 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in compound interest.

(i) Growth and depriciation of value of an article.

(ii) Increase or decrease in population.

(iii) The growth of bacteria if rate of growth is known.

Example 7.13. The population of a town is 15,000. If it increases at the rate of 4% per annum, then what will be the population after two years ?

Sol. As population increases at the rate of 4% per year, so every new year has new population. Thus we can say it is increasing in compounded form.

$$\text{So Population after two years} = 15000\left(1+\frac{4}{100}\right)^2$$

$$= 15000\left(1+\frac{1}{25}\right)^2$$

$$= 15000\left(\frac{26}{25}\right)^2$$

$$= 15000\times\frac{26}{25}\times\frac{26}{25} - 16224$$

Example 7.14. A second hand scooter was bought at a price of ₹24000. Its rate was depreciated by 5% per year. Find the value of scooter after two years. (Depreciation means reduction of value due to use and age of item.)

Sol. Here Price of Scooter (P) = ₹24000

Time = 2 years

Rate of depreciation = 5% per year

As rate is depreciating every year by 5%, So we can use amount formula of compound interest. But note it as value is decreasing so rate will be –5%.

So Value of Scooter after two years $= ₹24000\left(1-\frac{5}{100}\right)^2$

$$\left[\textbf{Note : } A = P\left(1+\frac{r}{100}\right)^t, \text{ Here } r = -5\right]$$

$$= ₹24000\left(1-\frac{1}{20}\right)^2 \quad = ₹24000\left(\frac{19}{20}\right)^2$$

$$= ₹24000 \times \frac{19}{20} \times \frac{19}{20} = ₹21660$$

Exercise 7.7

1. The value of a machine depreciates at the rate of 10% per annum. If its present value is ₹10,00,000. What will be its value after two years ? Also find the depreciation.
2. The cost of a plot is ₹6,40,000. It increases at a rate of 5% of its previous value after every year. What will be its value after two years ?
3. A person purchased a second hand bike for ₹16,000. If its rate depreciates at 5% per year. What will be its value after 2 years ?
4. The cost of LED TV was ₹16,000 during 2018. In next year (2019), the price was hiked by 5%. In next year (2020), the cost was reduced by 4%. What is cost of LED TV in 2020 ?
5. Population of town is 1,50,000. The annual birth rate is 5% and mortality rate is 3%. Find the population after 2 years.

7.8 Financial Awareness

Mr. Pritpal planned a dinner for his family. After having dinner, his younger daughter Tavleen, a student of 8th class,anxiously asked her father that why had he paid more money than the actual amount of food. Then Mr. Pritpal told her that we had to pay some taxes to the government which is used for developing roads, infrastructure and educational institutions for us. There are mainly two types of taxes - Direct Tax (levied on Income i.e. Income Tax, Wealth Tax)and Indirect Tax (levied on items other than income). Tavleen was curious to know about these in detail. Mr. Pritpal informed her that previously, we were mainly dealing with various indirect taxes such as Excise, VAT and Sales Tax, which are different forms of consumer tax. However, the ways and form in which they were levied on consumer, differed. Presently, many of these taxes have been merged into a single tax called as GST.

GST (Goods & Services Tax)- GST came into effect from 1 July, 2017 through the 101th amendment in the constitution of India by Government of India. The GST replaced the erstwhile multiple taxes levied by the Central and State Governments. It is an indirect tax or consumption based tax used in India on the supply of goods and service. It is levied at every step in the production process, but refunded to all the parties engaged in various stages of production except the final consumer who actually bears the GST.

For collection, GST rates have been divided into five different tax slabs for collection of tax: 0%, 5%, 12%, 18% and 28%. However, some products like petroleum, alcoholic drinks and electricity are presently outside the purview of GST.

After explaining about GST, Mr. Pritpal showed her the restaurant bill in which 5% GST was imposed on food ordered for dinner. Thereafter, Tavleen was quite happy to learn new information.

Exercise 7.8

Multiple Choice Questions :

1. GST stands for ....
   - (a) Goods and Sales Tax   (b) Gross Sales Tax
   - (c) Goods and Service Tax   (d) Gross Service Tax
2. GST is effective in India from..................
   - (a) 1 July 2010   (b) 1 July 2017
   - (c) 1 July 2019   (d) 1 July 2018
3. How many different tax slabs are there in GST?
   - (a) 1   (b) 8
   - (c) 3   (d) 5
4. Which of the following is not a tax slab under GST?
   - (a) 0%   (b) 6%
   - (c) 5%   (d) 12%
5. Which amendment of the constitution is related to GST?
   - (a) 91st   (b) 102nd
   - (c) 101st   (d) 100th
6. Which of the following is taxed under GST?
   - (a) Food   (b) Petroleum Products
   - (c) Alcoholic drink   (d) Electricity

Operating Your Account

Keeping money in bank account provides safety to your hard-earned money. Apart from this, bank gives some interest on your deposit and generates income on your savings. Banks also facilitates transfer of funds from one account to another as per the instructions of the account holder. Through bank, we can manage our expenditure. Banks offer various type of accounts such as Savings and Current Accounts.

- Saving Accounts can be opened by an individual or jointly by two our more people with an aim to save money.
- Current Account is mostly opened by businessmen and institutions.

In this section, we shall learn how to deposit and withdraw money from a bank account.

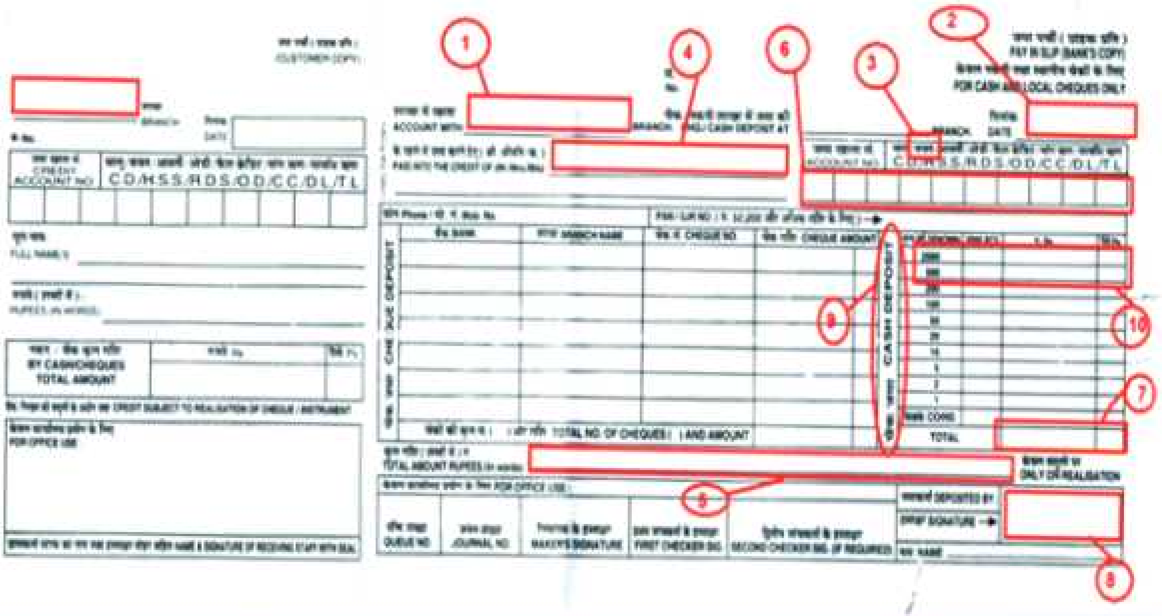
Depositing and Withdrawing Money :-

Operating a Current or Savings account is very easy. We can deposit and withdraw money either by going to the bank or using an ATM. The same can also be done through internet and mobile banking. Today we will learn the basic steps of depositing and withdrawing money at branch of a bank.

Making a deposit :- If we want to deposit money into our bank account, we can use Cheque or deposit slip. If we receive a cheque, we can deposit it with our bank through the clearing system or we can make a deposit of money by bank deposit slip.

Filling deposit slip - Shweta went to a bank with her son Aryan,who studies in 8th class, for depositing some money. Aryan was very excited as he was going first time in a bank. After reaching at the bank, Shweta showed her some bank slips like deposit slips and withdrawal slips and explained him about them in detail.

Bank Deposit Slip



Circled Number	Detail
1	Name of the Bank's Branch
2	Date (of deposit)
3	Paid in to the credit of (Nature of account)
4	A/c holder's name
5	Amount of deposit in words
6	A/c No. (Account Number)
7	Amount of deposit in figures
8	Signature of depositor
9	Cash/Cheque (method of deposit)
10	Denomination (breakup of amount in different currency notes)

Firstly, she showed him a bank deposit slip and told him that a bank deposit slip is a small piece of paper form that a person has to submit while depositing money into a bank account.

Bank deposit slip has two parts - the right side is for bank's use and left side is for depositor's record.

Let us understand the details to be filled up on the Bank's copy. Most of the information is filled on right side.

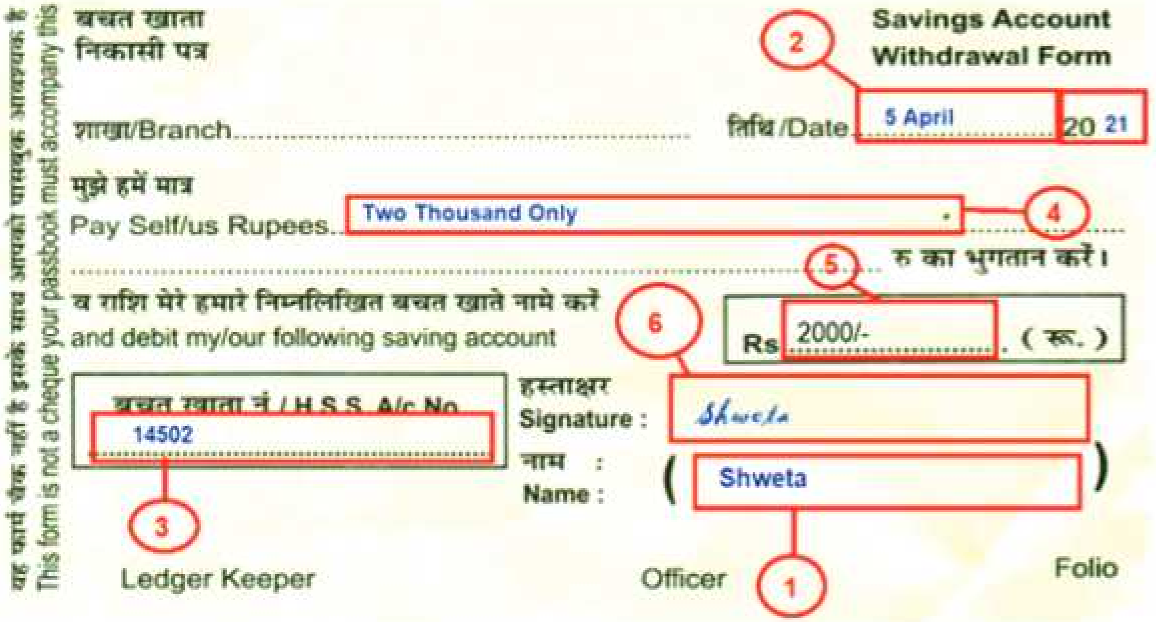
Note:- No authorization is required for depositing money, anyone can deposit money into anyone's account.

The main purpose of a deposit slip is that it tells the cashier that we have to deposit the money in which bank account number.

Making a Withdrawal : To withdraw money we can write a cheque made out to 'cash' and then put date and sign it, we would deposit endorse the cheque and give it to the bank employee, who will give us cash.

Bank Withdrawal Slips:-

After telling about bank deposit slips, she showed her bank withdrawal slips.



यह फार्म चैक नहीं है इसके साथ आपकी पासबुक आवश्यक है
This form is not a cheque your passbook must accompany this

बचत खाता
निकासी पत्र

Savings Account
Withdrawal Form

शाखा/Branch..

तिथि /Date 5 April 20 21

मुझे हमें मात्र
Pay Self/us Rupees Two Thousand Only

रु का भुगतान करें।

व राशि मेरे हमारे निम्नलिखित बचत खाते नामे करें
and debit my/our following saving account

Rs 2000/- (रू.)

बचत खाता नं / H S S A/c No 14502

हस्ताक्षर
Signature : Shweta

नाम :
Name : (Shweta)

Ledger Keeper Officer Folio

- A withdrawal slip is a bank document on which a person writes the date, account number and amount of money to withdraw from a bank."

Here are some restrictions on the use of a withdrawal slip.These are:

- Only the account holder can use this slip to withdraw the amount for one self.
- This slip / form cannot be used to make payment to others.

Circled Number	Detail
1	Name of the Account Holders
2	Date (of withdrawal)

Circled Number	Detail
3	Account Number in the Bank
4	Amount of withdrawal in words
5	Amount of withdrawal in figures
6	Signature of Account Holder

We can also withdraw the money through an ATM (Automated Teller Machine). To withdraw money,put your ATM card into the machine, enter your PIN (Personal Identification Number) and specify the amount of cash you would like. Shweta further explained that nowadays many people are using mobile phones and they can transfer funds from one account to another and make payments.

Passbook :All these deposit and withdrawal entries are printed by the bank on a "Bank Passbook" which is provided to us at the time of opening the account.

Exercise 7.9

Multiple Choice Questions :

1. When an account is opened in two names, the account is known as...........
   a) Two Accounts b) Dual Account
   c) Duo Account d) Joint Account
2. For taking cash out of an account, we have to fill a .... slip.
   a) Passbook b) Cheque
   c) Withdrawal d) Deposit
3. ATM stands for.............
   a) Automated Teller Machine b) Auto Telling Machine
   c) Auto Teller Machine d) Automated Telling Machine
4. PIN stands for.............
   a) Personal Identity Number b) Personal Identification Number
   c) Person Identity Number d) Personal identity Number
5. For putting cash into an account, we have to fill a ....slip.
   a) Passbook b) Cheque
   c) Withdrawl d) Deposit
6. .... lists the transactions carried out in the account.
   a) Deposit b) Passbook
   c) Withdrawl d) Cheque

Learning Outcomes

After completion of this chapter, students are now able to:

- ***Understand concept of ratio and percentage.***
- ***Understand concept of marked price (M.P.)***
- ***Understand about discount their percentage and their application in daily life.***
- ***Understand sales tax and its applications in daily life.***
- ***Understand simple and compoud interest and their applications in daily life.***
- ***Understand about bank passbook and how to fill deposit and withdrawl slips.***

Answers

Exercise 7.1

1. (i) 1 : 3 (ii) 1 : 1000 (iii) 15 : 1 (iv) 12 : 1 (v) 1:5 (vi) 1:40
2. 10 **3.** 26 **4.** 30%
5. (i) $33\frac{1}{3}\%$ (ii) 80% (iii) 50% (iv) 40% (v) 125% (vi) 20%
6. ₹2500 **7.** 6 matches **8.** (i) Carrom = 24 (ii) Chess : 9, other games = 27
9. (i) b (ii) a (iii) a (iv) d (v) c

Exercise 7.2

1. ₹80, $41\frac{2}{3}\%$ **2.** ₹113, 14.29% **3.** ₹32, ₹188 **4.** 5%
5. ₹624 **6.** ₹900 **7.** Discount = ₹32, C.P = ₹368
8. ₹255
9. (i) b (ii) a (iii) b (iv) b (v) c

Exercise 7.3

1. ₹480 **2.** ₹32

Exercise 7.4

1. ₹39420 **2.** 12% **3.** ₹24000 **4.** ₹800
5. ₹462
6. (i) b (ii) c (iii) b (iv) d

Exercise 7.5

1. 2 years
2. 50%
3. ₹1500, ₹16500

Exercise 7.6

1. ₹2940
2. ₹728
3. (i) b (ii) b (iii) d (iv) c (v) b

Exercise 7.7

1. ₹ 8,10,000; ₹1,90,000
2. ₹705600
3. 14440
4. ₹16128
5. 156060

Exercise 7.8

1. c
2. b
3. d
4. c
5. c
6. a

Exercise 7.9

1. d
2. c
3. a
4. b
5. d
6. b

CHAPTER

8

Algebraic Expression and Identities

Learning Objectives

In this chapter you will learn:

- *To Identify the Algebraic expression.*
- *To know about terms and coefficients in algebraic expressions.*
- *To define variable, factors of a term.*
- *To define a polynomial.*
- *To differentiate between an expression and a polynomial.*
- *To define monomial, binomial and trinomial etc.*
- *To identify like and unlike terms.*
- *To solve addition, subtraction and multiplication of algebraic expression and polynomials.*
- *To use multiplication in their life for practical use to find area of a rectangle and volume of rectangular box etc.*
- *To understand about identities and uses of identities in daily life.*

8.1 Meaning of Expressions (Introduction)

In earlier classes, we have learnt about algebraic expressions (or simply expressions). Algebraic expressions are formed by using variables and constants. Some examples of expressions are

$2x + 7$, $7xy - 8$, $\sqrt{x+5}$, $y + 8$, $x^2 + 7$ etc.

The expression $2x + 7$ is formed with variable x and constants 2 and 7, where as the expression $7xy - 8$ is formed with variables x and y and constants 7 and 8. Similarly we can say about other expressions.

8.1.1 Value of an Algebraic Expression

In expression, we can give any value to the variable or variables. The value of the expression changes with the chosen value of the variable or variables, it contains. For example, in expression $2x + 7$ if $x = 2$ then $2x + 7 = 2 \times 2 + 7 = 11$ and if $x = 0$ then $2x + 7 = 2 \times 0 + 7 = 7$ and so on. So we can find different values of expression $2x + 7$ for different values of the variable x.

8.1.2 Number line and an expression (in variable X)

Consider an expression $x + 3$. Let the position of variable x is X on number line (considering x to be +ve). So X may be any where on the number line to the right hand side of the origin. Now place of $x + 3$ will be a point (say A) three units to the right of X.

Similarly, the place of $x - 2$ is two unit left of X.

Now if we want to find the position of $3x + 2$ (taking x positive). The position of 3x (three times x) will be at point B.

So, position of $3x + 2$ is two units right of B i.e at point C.

8.2 Terms, Factors and Coefficients:-

Term is either a single number or variable, or numbers and variables multiplied together. So 4, x, 4x and 4xy all are terms. Terms are added to form expressions.

Let us take three terms 4x, 3y and 8. From these three terms expression is $4x + 3y + 8$. The term 4x is product of 4 and x. 4 and x are **factors** of 4x. Whereas 3y is product of **factors** 3 and y. The term 8 is made from single number 8.

The expression $9xy - 3x$ has two terms 9xy and –3x. The term 9xy is product of **factors** 9, x and y. The term –3x is product of factors –3 and x. The numerical factor of a term is called its numerical coefficient or simply coefficient. The coefficient in term 9xy is 9 and in –3x is **–3**.

8.3 Monomials,Binomials,Trinomial

An expression containing one or more terms with real coefficients and variables having number whole as exponents is called a **polynomial.**

Examples of polynomials : $3x$, $3x + 2y$, $x^2 + 3x + 5$, $ax + by + cz + d$

- Polynomial that contains: only one term is known as **monomial.**

  e.g. 4, $3x$, $4y$, $7xy$, $8x^2y$, $-4xy^2$

- Polynomial that contains two terms is called a **binomial**.

  e.g. $3x + 4y$, $x - 2y$, $ax + by$

- Polynomial having three terms is **trinomial** and so on.

  e.g. $x^2 - 3x + 5$, $ax + by + cz$

8.4 Like and Unlike terms

Like terms are terms whose variables and their exponents are same (equal). The coefficients can be different. So **3y, –4y, $\frac{21}{8}$y** are like terms. Similarly $3t^2$ and $-11t^2$ are like terms. Also, 4ab, –21ab and 11ab are like terms.

The terms which are not like are known as unlike terms. Here 7x and 4y are unlike because variables are different. Similarly $7x^2$ and 4x are unlike term because exponent are unequal.

8.5 Addition and Subtraction of Algebraic expressions

In earlier classes, we have learnt about addition and subtraction of algebraic expressions. Recall that in addition we write each expression to be added in a separate row. While doing so, we write like terms one below the other and add them.

Also subtraction of numbers is the same as addition of its additive inverse. Therefore subtracting -4 is same as adding $+4$. Similarly, subtracting $5y$ is same as adding $-5y$. Subtracting $-3x^2$ is same as adding $3x^2$ and so on. So in subtraction the sign of each term of the expression, to be subtracted, will be changed. The signs in the third row written below each term in the second row help us in knowing which operation has to be performed. Observe the following examples to clear the concept.

Example 8.1. Add the following expression

(i) $x + y - 2z$ and $2x - 2y + 3z$

(ii) $2x + 3y - 4z$ and $x + y - 4$

(iii) $7xy + 5yz - 3zx$, $4xy + 7zx$ and $3yz + 4$

Sol. Write the expression in separate rows with like terms one below the other, we have

(i)
$$\begin{array}{rrr} x & +\ y & -\ 2z \\ 2x & -\ 2y & +\ 3z \\ \hline 3x & -\ y & +\ z \\ \hline \end{array}$$

(ii)
$$\begin{array}{rrrr} 2x & +\ 3y & -\ 4z & \\ x & +\ y & & -\ 4 \\ \hline 3x & +\ 4y & -\ 4z & -\ 4 \\ \hline \end{array}$$

there is no like terms of $-4z$ and -4

(iii)
$$\begin{array}{rrrr} 7xy & +\ 5yz & -\ 3zx & \\ 4xy & & +\ 7zx & \\ & 3yz & & +\ 4 \\ \hline 11xy & +\ 8yz & +\ 4zx & +\ 4 \\ \hline \end{array}$$

Example 8.2. Subtract

(i) $5a^2 - 3ab + 4b - 7$ from $8a^2 - 3b^2 - 8ab + 9a - 7b$

(ii) $x + 3y - 4z + x^2 - y^2$ from $8x + 5z - x^2 - y^2 + 7$

Sol. Like addition, we will write the expression in separate rows with like terms one below the other and then we will subtract

(i)
$$\begin{array}{rrrrrr} 8a^2 & -\ 3b^2 & -\ 8ab & +\ 9a & -\ 7b & \\ 5a^2 & & -\ 3ab & & +\ 4b & -\ 7 \\ - & & + & & - & + \\ \hline 3a^2 & -\ 3b^2 & -\ 5ab & +\ 9a & -\ 11b & +\ 7 \\ \hline \end{array}$$

(ii)
$$\begin{array}{rrrrrr} 8x & +\ 5z & -\ x^2 & -\ y^2 & +\ 7 & \\ x & -\ 4z & +\ x^2 & -\ y^2 & & +\ 3y \\ - & + & - & + & & - \\ \hline 7x & +\ 9z & -\ 2x^2 & & +\ 7 & -\ 3y \\ \hline \end{array}$$

Example 8.3. Subtract $x + 3y - 5z + 7$ from the sum of the expressions $2x - 3y + 4z - 2$ and $-3x + 8y + 12z - 4$

Sol. First, we will add the expressions $2x - 3y + 4z - 2$ and $-3x + 8y + 12z - 4$, as we did earlier

$$\begin{array}{rrrr} 2x & -\ 3y & +\ 4z & -\ 2 \\ -\ 3x & +\ 8y & +\ 12z & -\ 4 \\ \hline -\ x & +\ 5y & +\ 16z & -\ 6 \\ \hline \end{array}$$

Now subtract $x + 3y - 5z + 7$ from $-x + 5y + 16z - 6$

$$\begin{array}{rrrr} -\ x & +\ 5y & +\ 16z & -\ 6 \\ x & +\ 3y & -\ 5z & +\ 7 \\ - & - & + & - \\ \hline -\ 2x & +\ 2y & +\ 21z & -\ 13 \\ \hline \end{array}$$

Exercise 8.1

1. **Give five examples of expressions having one variable and having two variables.**
2. **Construct :-**
   - (i) Three polynomials with only x as variable
   - (ii) Three binomials with x and y as variables
   - (iii) Three monomials with x and y as variables
   - (iv) Three polynomials with four or more terms
3. **Write two terms which are like to**

   (i) $7x$ (ii) $3ab$ (iii) $7x^2y$ (iv) $2lm$
4. **Identify the terms, their coefficients for each of the following expressions:**

   (i) $5xy - 3zy$ (ii) $2 + 2x - 3x^2$ (iii) $4x^2y^2 - 4z^2 + 3xy$

   (iv) $ab + bc + abc + 7$ (v) $\frac{x}{6} + \frac{y}{6} + 2xz$ (vi) $0.3a - 0.5ab$

   (vii) $\frac{xy}{2} + 7x + \frac{3}{2}y$ (viii) $0.4a - 0.6ab + 3b^2$ (ix) $3xy^2 + 5xyz - 6y^2$
5. **Classify the following polynomials as monomials, binomials and trinomials. Which polynomials do not fit in any of these three categories? and why?**

   (i) $3x$ (ii) y (iii) 4

   (iv) $3x - 2y$ (v) $\frac{y}{2} + z$ (vi) $x + y + 2z$

   (vii) $2x - y + 7$ (viii) $a + b + c$ (ix) $x - y + 2z$

   (x) $14x^2yz$ (xi) $x^2 - y^2$ (xii) $a^2 + b^2 + c^2$
6. **Add the following**
   - (i) $ab + a^2b - 3abc$ and $4abc - 7a^2b + 2ab + 3$
   - (ii) $x + y + 3z - 2xyz$ and $-2x + 3y + 4z - 8$
   - (iii) $x^2 - y^2, y^2 - z^2, z^2 - x^2$
   - (iv) $x - y, -y + z, z - x$
   - (v) $2x^2y^2 - 3xy + 4$ and $5 + 7xy - 3x^2y^2$
   - (vi) $x^2 + y^2 - z^2, x^2 - y^2 + z^2, -x^2 + y^2 + z^2$
7. **Subtract**
   - (i) $5x - 3xy + 7y + 18$ from $13x - 7xy - 6y + 8$
   - (ii) $2\ell m + 3mn - 8n\ell$ from $9\ell m + 7mn + 13n\ell$
   - (iii) $ab + bc + ca + abc$ from $3ab - 2bc - 4abc$
   - (iv) $2x + 3y + 4z + 3xyz$ from $4x - 7xyz$
   - (v) $0.3x + 0.2y + 2xyz$ from $0.7x + 0.8y - 9xyz$
   - (vi) $ab + bc - cd + abc$ from $2ab - 2bc + 2cd - 2abc$

8. Subtract the third expression from the sum of first two expressions.

(i) $2ab + bc - cd$, $abc + ab - 2bc$, $-2bc + 3ab$

(ii) $2x + 3y - 2z$, $x - y + 3xyz$, $4x + 3y - 4z + 7xyz$

(iii) $0.2x + 0.3y + 0.4xy$, $0.8x + 0.7y$, $x + y - 0.6xy$

(iv) $7xy + 3x + 2y - 3z$, $x + y + 2z$, $4xy - x - y + 4z$

(v) $0.3xy + 0.2yz$, $0.4xy + 0.3zx$, $0.2xy + 0.2yz$

(vi) $0.4xyz + 0.3xy^2$, $0.7xyz + 0.2xy^2$, $xyz + 0.4xy^2$

9. If sides of a triangle are given by expressions, $x^2 - 5x + 6$, $3 - 3x^2 + 7x$ and $11x^2 + 8x - 11$. Find the perimeter of triangle.

10. Multiple Choice Questions :

(i) Identify coefficient of y in $7y - 5$.

(a) 7 (b) -5 (c) 5 (d) 12

(ii) Which of following is a monomial?

(a) $7x + 5$ (b) $x + y + z$ (c) $3x^3$ (d) $5x^2 - 7x + 6$

(iii) Identify the binomial.

(a) $5x + 2$ (b) $x + x + 1$ (c) $6z$ (d) $\sqrt{t}$

(iv) Find the trinomial from following expressions.

(a) $5xy - 3zy$ (b) $2x - y + 7$ (c) $x - y + 2z + 4$ (d) $x^3 + 3$

(v) Out of given expression which are like terms?

(a) $7x$ and $7y$ (b) $3x$ and $3x^2$ (c) x^2 and $3x^2$ (d) $x^3 + 3$

(vi) Addition of $2a - b$ and $a - 2b$ will give:

(a) $a - b$ (b) $2a - 2b$ (c) $3a - 3b$ (d) $a + b$

(vii) What does given diagram represents.

(a) $x + 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $x^2 + 3$

(viii) The expression $3x - 5$ is a:

(a) Monomial (b) Binomial (c) Trinomial (d) None of these

(ix) Identify the terms in expression $-5x + 7xy$.

(a) -5 and 7 (b) $-5x$ and $7x$ (c) $-5x$ and $7xy$ (d) $-5x$ and $7y$

(x) Add $ab - bc$, $bc - ac$, $ac - ab$

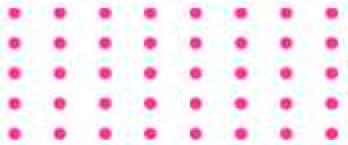
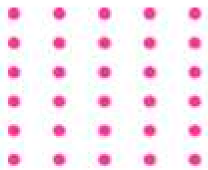
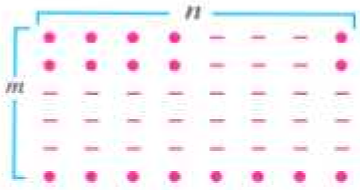
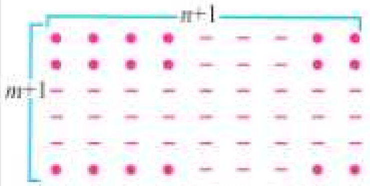
(a) 0 (b) $ab + bc + ac$ (c) abc (d) $a + b + c$

(xi) Find the value of expression $3x - 5$ at $x = 5$.

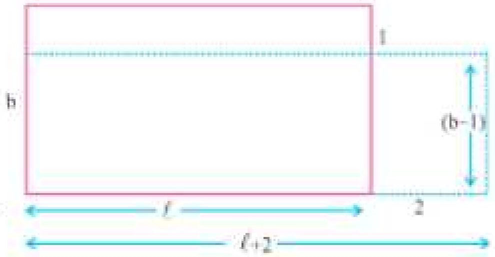
(a) 5 (b) 10 (c) 15 (d) 20

8.6 Multiplication of Algebraic Expressions:-

Introduction :- (i) Look at the followoing pattern of dots

Pattern	Number of rows	Number of Columns	Total Number of dots
	5	8	5×8 To find total number of dots we have to multiply number of rows to the number of columns
	6	5	6×5
	m	n	$m \times n$ Here number of rows are m and number of columns are n
	m + 1	n + 1	$(m + 1)(n + 1)$ Here number of rows are $(m + 1)$ and number of columns are $(n + 1)$

(ii) Let us think of some other situations in which two algebraic expressions have to be multiplied? We know the area of rectangle is $\ell \times b$ where ℓ is length and b is breadth of rectangle. If length of rectangle is increased by 2 units and breadth is reduced by 1 unit then area of new rectangle will be $(\ell + 2) \times (b - 1)$



(iii) For buying things we have to multiply number of things with their unit price.

e.g. let price of one note book = ₹ p

Number of note books required = q

then he has to pay = ₹ $(p \times q)$

Now suppose the price of one notebook is increased by ₹ 1 and number of note books required is 2 more, then

Price of one note book = ₹ $(p + 1)$

Required number of same note books = $q + 2$

So he has to pay = ₹ $(p + 1)(q + 2)$

In all the examples discussed above we need to multiply quantities in the form of algebeic expressions, so now we will learn, how to multiply algebraic expressions. First we will learn multiplication of monomial with another monomial.

8.7 Multiplying a Monomial by a Monomial

8.7.1 Multiplying two Monomials

We know multiplication is repeated addition

as 4×3 means 4 times 3

i.e. $4 \times 3 = 3 + 3 + 3 + 3 = 12$

Similarly $4 \times (5y) = 5y + 5y + 5y + 5y = 20y$

and $5 \times (3x) = 3x + 3x + 3x + 3x + 3x = 15x$

Now observe some following products:-

(i) $y \times 3x = y \times 3 \times x = 3 \times x \times y = 3xy$

(ii) $5x \times 4y = 5 \times x \times 4 \times y = 5 \times 4 \times x \times y = 20xy$

(iii) $3x \times (-2y) = 3 \times x \times (-2) \times y = 3 \times (-2) \times x \times y = -6xy$

Note that product of two monomials is a monomial

Now observe some following examples

(iv) $5x \times 3x^2 = 5 \times x \times 3 \times x^2$

$= (5 \times 3) \times (x \times x^2) = 15 \times x^3 = 15x^3$

Here we will use the rules of exponents and powers that for any non zero integer a, $a^m \times a^n = a^{m+n}$

(v) $5x^3 \times (-4x^4yz) = (5 \times -4) \times (x^3 \times x^4) \times (yz)$

$= -20\, x^7yz$

8.7.2 Multiplying three or more monomials

Observe the following examples

(i) $2x \times 3y \times 4z = (2x \times 3y) \times 4z = 6xy \times 4z = 24xyz$

(ii) $2xy \times 5x^2y^2 \times 6xy^2$

$= (2xy \times 5x^2y^2) \times 6xy^2$

$= 10x^3y^3 \times 6xy^2$

$= (10 \times 6)\, x^3y^3 \times xy^2$

$= 60\, (x^3 \times x) \times (y^3 \times y^2) = 60x^4y^5$

It is evident that first of all we multiply first two monomials and answer obtained is multiplied with third monomial to get the final answer.

Note: We can multiply the monomials in any order, result will be same.

Example 8.4. Complete the table to find the area of rectangle with given length and breadth

Length	Breadth	Area
3x	5y	
4x	2x	
2xy	3x	

Sol.

Length	Breadth	Area
$3x$	$5y$	$3x \times 5y = (3 \times 5) \times x \times y = 15xy$
$4x$	$2x$	$4x \times 2x = (4 \times 2) \times (x \times x) = 8x^2$
$2xy$	$3x$	$2xy \times 3x = (2 \times 3) \times (x \times x) \times y = 6x^2y$

Example 8.5. Find the volume of cubiod (rectangular box) whose length, breadth and height are respectively

(i) $2x, 3y, 4z$

(ii) $2ax, 3by, 7cz$

(iii) $2pq, 3qr, 4rp$

Sol. We know volume of cuboid $= l \times b \times h$

So volumes of rectangular boxes (cuboid) are

(i) $2x \times 3y \times 4z = (2 \times 3 \times 4) \times (x) \times (y) \times (z) = 24xyz$

(ii) $2ax \times 3by \times 7cz = (2 \times 3 \times 7) \times (ax) \times (by) \times (cz) = 42abcxyz$

(iii) $2pq \times 3qr \times 4rp = (2 \times 3 \times 4) \times (p \times p) \times (q \times q) \times (r \times r) = 24p^2q^2r^2$

Exercise 8.2

1. (i) The product of two monomials is ....

(ii) The product of three monomials is ....

2. Find the product of following pairs of monomials

(i) $8x, 3y$ (ii) $4, 2x$ (iii) $-4p, 3q$ (iv) $8p, -3pq$

(v) $3xy, 0$ (vi) $p^2, 2pq$ (vii) $2p, 3pr$ (viii) $r, 2p$

3. Find the area of rectangles with following pairs as their length and breadth respectively

(x, y), $(2\ell, 4m)$, $(10m, 6n)$, $(3mn, 4n)$, $(9a^2b, 13abc)$

$(2ax, 3pr)$, $(3mn, 4np)$, $(2p, pqr)$, $(3x^3y, 7xy^2)$

4. Complete the table of Products

First Monomial → **Second Monomial ↓**	**$2x$**	**$-5y$**	**$2x^2$**	**$-3xy$**	**$7x^2y$**	**$-9x^2y^2$**
$-2y$						
$3x$						
y^2						
$-4xy$						
$2x^2y^2$						

5. Find the Product of

(i) $3x, 4x^2, -7x^3$ (ii) $2zx, 3y, 4z$ (iii) $\frac{a}{2}, \frac{b}{3}, \frac{c}{4}$

(iv) $ab, abc, abcd$ (v) $\frac{x^2y}{3}, 9y^2z, -8z^2x$ (vi) $-3pq, 4p^2x^2$

6. **Find the volume of rectangular box having length, breadth and height respectively as**

(i) x, y, z (ii) 2x, 3y, 3z (iii) 2a, 7b, c

(iv) 4*l*, 5m, 6n (v) ab^2, bc^2, ca^2 (vi) $\frac{a}{2}, \frac{b}{3}, \frac{c}{4}$

7. **Multiple Choice Questions :**

(i) Multiplying a monomial by a monomial will give you a:

(a) Monomial (b) Binomial (c) Trinomial (d) None of these

(ii) Multiplying a monomial with a binomial will give you a:

(a) Monomial (b) Binomial (c) Trinomial (d) None of these

(iii) Find the product of 3x and 5y.

(a) 3xy (b) 15x (c) 15xy (d) 15y

(iv) Find the product of 3a and 7ab.

(a) $21a^2+b$ (b) 15a+21ab (c) $21a^2b$ (d) 21ab

(v) If sides of a rectangle are 2ab and 3bc respectively. Then its area is:

(a) 6abc (b) $6ab^2c$ (c) 2ab+3bc (d) 6+ab+bc

(vi) Find volume of a cuboid with sides a^2b, b^2c and c^2a.

(a) abc (b) $a^2b^2c^2$ (c) $a^3b^3c^3$ (d) $a^2b + b^2c + c^2a$

8.8 Multiplying a monomial by a polynomial

8.8.1 Multiplying a monomial by a binomial

Let us multiply monomial 4x by binomial (5x + 2y)

i.e. Find 4x × (5x + 2y)

Here we will use the distributive law for this multiplication.

So $4x \times (5x + 2y) = (4x \times 5x) + (4x \times 2y)$

$= 20x^2 + 8xy$

Here observe that product of monomial and binomial is a binomial

Similarly $(-4x) \times (-5y + 2x) = (-4x \times -5y) + (-4x \times 2x)$

$= 20xy - 8x^2$

Note:- If we multiply a binomial with a monomial, we will again get a binomial. We can use commutative law for this multiplication.

For example $(a-7b) \times 2b$

$= 2b \times (a-7b)$

$= 2b \times a + 2b \times (-7b) = 2ba - 14b^2$

or $2ab - 14b^2$ $[\because ab = ba]$

8.8.2 Multiplying a monomial by a trinomial

Consider $5a\,(a^2 + 2a + 3)$ We use distributive property of multiplication over addition.

$$5a\,(a^2 + 2a + 3) = (5a \times a^2) + (5a \times 2a) + (5a \times 3)$$
$$= 5a^3 + 10a^2 + 15a$$

Example 8.6. Simplify the expressions and evaluate them with given value of variable

(i) $x\,(x + 3) - 2$ **for** $x = 2$ **(ii)** $2y\,(3y - 7) - 2\,(y + 4) + 5$ **for** $y = -3$

Sol. (i) $x\,(x + 3) - 2 = x^2 + 3x - 2$

for $x = 2$; We have $= (2)^2 + 3 \times 2 - 2 = 4 + 6 - 2 = 8$

(ii) $2y\,(3y - 7) - 2\,(y + 4) + 5 = 6y^2 - 14y - 2y - 8 + 5$

$= 6y^2 - 16y - 3$ (Combine like terms)

Now for $y = -3$; We have $= 6\,(-3)^2 - 16 \times (-3) - 3$

$= 6 \times 9 + 48 - 3$

$= 54 + 48 - 3 = 99$

Example 8.7. Add

(i) $2y\,(5 - y)$ **and** $6y^2 + 14y + 7$

(ii) $3x\,(x^2 + 2x - 5)$ **and** $2\,(x^2 + 7x - 2)$

Sol. (i) First expression $= 2y\,(5 - y) = 2y \times 5 - 2y \times y = 10y - 2y^2 = -2y^2 + 10y$

Now adding first and second expression

$$\begin{array}{rrr} -2y^2 & +10y & \\ +6y^2 & +14y & +7 \\ \hline 4y^2 & +24y & +7 \end{array}$$

(ii) First expression $= 3x\,(x^2 + 2x - 5) = 3x \times x^2 + 3x \times 2x + 3x \times (-5)$

$= 3x^3 + 6x^2 - 15x$

Second expression $= 2(x^2 + 7x - 2) = 2 \times x^2 + 2 \times 7x + 2 \times (-2)$

$= 2x^2 + 14x - 4$

Now adding first and second expression

$$\begin{array}{rrrr} 3x^3 & +6x^2 & -15x & \\ & +2x^2 & +14x & -4 \\ \hline 3x^3 & +8x^2 & -x & -4 \end{array}$$

Example 8.8. Subtract $2pq\,(3p - 2q)$ **from** $3pq\,(p + q)$.

Sol. Here, $2pq\,(3p - 2q) = 2pq \times 3p + 2pq \times (-2q)$

$= 6p^2q - 4pq^2$ ________(i)

and $3pq\,(p + q) = 3pq \times p + 3pq \times q$

$= 3p^2q + 3pq^2$ ________(ii)

Subtracting (i) from (ii), we have

$$\begin{array}{rr} 3p^2q & +\ 3pq^2 \\ 6p^2q & -\ 4pq^2 \\ - & + \\ \hline -3p^2q & +\ 7pq^2 \end{array}$$

Exercise 8.3

1. Multiply each of the following pairs

(i) $4x, x + y$ (ii) $(x–3y), x^2$ (iii) $(x + y), 7xy$

(iv) $(x^2 – 9x), 4x$ (v) $(a + b), 0$ (vi) $(ab + bc), ab$

2. Complete the table

First expression	Second expression	Product
(i) $a^2b^2c^2$	$ab + bc + ca$	
(ii) $x + y + z$	$2xy$	
(iii) $p + q –2r$	$2p$	
(iv) $b + c – a$	abc	

3. Find the Product of :

(i) a^2 and $(a^2 – b^2)$ (ii) $4xy$ and $(– 2x – 3y)$

(iii) a and $(a^2 – 2ab + b^2)$ (iv) $4x^2$ and $(– x^2 – y^2 + 2x)$

4. Simplify the following and find its value with the given value of the variable

(i) $x (3x + 2) – 7$ if $x = 1$ and $x = \frac{1}{2}$ (iii) $xy (x^2y – xy^2)$ if $x = 1, y = 2$

(ii) $y (2y^2 – 7y) + 8$ if $y = 0$ and $y = –1$ (iv) $ab (a + ab + abc)$ for $a = 2, b = 1, c = 0$

5. Add: (i) $x (x — y)$, $y (y – z)$ and $z (z – x)$

(ii) $2x (x – y – z)$ and $2y (z – y – x)$

6. Subtract : (i) $8l (l –4m + 5n)$ from $9l (10n – 3m + 2l)$

(ii) $2a (a + b – c) – 2c (a + b – c)$ from $2c (– a + b + c)$

7. Subtract sum of $x(2x+7) –2$ and $3x (x–2) + 7$ from $7x – 1$

8. Add $2xy (x + y + z)$ and $3y (x^2 – xy + xz)$ then subtract from $5x (xy + y^2 – 4yz)$.

9. Multiple Choice Questions :

(i) Product of pqr and $p + q + r$ will be:

(a) pqr (b) $p^2qr + pq^2r + pqr^2$

(c) $pq + qr + pr$ (d) $p^2qr + pqr^2$

(ii) Find value of $x^2 + x$ at $x = 2$.

(a) 4 (b) 6 (c) 8 (d) 10

(iii) Find $y \times y^2 \times y^3 \times y^4$

(a) y (b) y^6 (c) y^{10} (d) y^{25}

(iv) Find the product of $xy + 4z + 3x$ with 0.

(a) $xy+yz+3x$ (b) xyz (c) 0 (d) $x^2y^2z^2$

8.9 Multiplying a Polynomial by a Polynomial

8.9.1 Multiplying a binomial by a binomial

Let us multiply one binomial $(3a + 3b)$ with another binomial $(7a + 4b)$. As we did in earlier cases, we will use distributive law to find the multiplication

$$(3a + 3b) \times (7a + 4b) = 3a \times (7a + 4b) + 3b \times (7a + 4b)$$
$$= (3a \times 7a) + (3a \times 4b) + (3b \times 7a) + (3b \times 4b)$$
$$= 21a^2 + 12ab + 21ba + 12b^2$$
$$= 21a^2 + 12ab + 21ab + 12b^2$$
$$= 21a^2 + 33ab + 12b^2$$

(As ab is same as ba, so by combining like terms)

[After doing product of 2 polynomials, we need to look for like terms and must combine them to get result.]

Example 8.9. **Multiply**

(i) **$(7a + b)$ and $(a + 3b)$**

(ii) **$(2x - y)$ and $(x + 3y)$**

Sol. (i) $(7a + b)(a + 3b)$

$= 7a(a + 3b) + b(a + 3b)$

$= (7a \times a) + (7a \times 3b) + (b \times a) + (b \times 3b)$

$= 7a^2 + 21ab + ba + 3b^2$

$= 7a^2 + 21ab + ab + 3b^2$ $[\because ba = ab]$

$= 7a^2 + 22ab + 3b^2$

(ii) $(2x - y) \times (x + 3y) = 2x \times (x + 3y) - y(x + 3y)$

$= (2x \times x) + (2x \times 3y) - (y \times x) - (y \times 3y)$

$= 2x^2 + 6xy - yx - 3y^2$

$= 2x^2 + 6xy - xy - 3y^2$ $[\because yx = xy]$

$= 2x^2 + 5xy - 3y^2$ (combining like terms)

Example 8.10. **Multiply**

(i) **$(a + 6)$ and $(b - 8)$**

(ii) **$(2a^2 + 3b)$ and $(5a - 3b)$**

Sol. (i) $(a + 6) \times (b - 8) = a \times (b - 8) + 6 \times (b - 8)$

$= ab - 8a + 6b - 48$

(ii) $(2a^2 + 3b) \times (5a - 3b) = 2a^2 \times (5a - 3b) + 3b \times (5a - 3b)$

$= (2a^2 \times 5a) + (2a^2 \times (-3b)) + (3b \times 5a) + (3b \times (-3b))$

$= 10a^3 - 6a^2b + 15ba - 9b^2$

8.8.2 Multiplying a Binomial by a trinomial

In this multiplication, we shall have to multiply each of the two terms of binomials by each of the three terms of trinomial. We will get total 6 terms. which may reduce to 5 or less, if like terms appears in multiplication. Consider

$(2x + 3y) \times (x + 2y + 5) = 2x \times (x + 2y + 5) + 3y\,(x + 2y + 5)$ (using distributive law)

$= 2x^2 + 4xy + 10x + 3yx + 6y^2 + 15y$

$= 2x^2 + 7xy + 10x + 6y^2 + 15y$ (as $xy = yx$)

Example 8.11. Simplify $(a + b)\,(2a + 3b - 2c)$

Sol. $(a + b)\,(2a + 3b - 2c)$

$= a\,(2a + 3b - 2c) + b\,(2a + 3b - 2c)$

$= 2a^2 + 3ab - 2ac + 2ab + 3b^2 - 2cb$

$= 2a^2 + 5ab - 2ac - 2bc + 3b^2$

Exercise 8.4

1. **Find the product of**

(i) $(x + 5)$ and $(x + 4)$

(ii) $(2x + 3)$ and $(x - 7)$

(iii) $(x - 8)$ and $(x + 3)$

(iv) $(2x - 3)$ and $(x - 4)$

(v) $(2x + 3y)$ and $(x + 2y)$

(vi) $(x + y)$ and $(x - 3y)$

(vii) $(p - q)$ and $(p + 3q)$

(viii) $(2p - 3q)$ and $(4p - 3q)$

(ix) $(a^2 - b)$ and $(a + b^2)$

(x) $\left(\frac{7}{2}x + y^2\right)$ and $\left(x^2 - \frac{2}{7}y\right)$

(xi) $(0.2x + 0.5y)$ and $(3xy - 5y^2)$

(xii) $(p^2 - q)$ and $(p^2 + q)$

2. **Simplify:-**

(i) $(y - 3)\,(y + 3) + 28$

(ii) $(a^2 - 3)\,(b^2 + 5) - 8$

(iii) $(y^2 - 7)\,(x + y) + 13y$

(iv) $(3x - y)\,(x + 5y) - 14xy$

(v) $(a + b)\,(a - b) + (b + c)\,(b - c) + (c + a)\,(c - a)$

(vi) $\left(\frac{3}{2}x + y\right)\left(x + \frac{1}{2}y\right) - \left(\frac{1}{2}x + y\right)\left(x + \frac{3}{2}y\right)$

(vii) $(p - q)(p + q) + (p + q + r)(p + q - r)$

(viii) $(x+y)(x-y + xy) - 3xy(x+y)$ (ix) $(\ell + m)(\ell - m + n) - (\ell^2 + m^2)$

(x) $(2x^2 - 5x + 7)(x - 6) + 42$

8.10 What is an Identity ?

Consider the equality $(x-5)(x+1) = x^2 - 4x - 5$ We will solve both sides of equality for same value of x say $x = 7$.

for $x = 7$, LHS $= (7 - 5)(7 + 1) = 2 \times 8 = 16$

RHS $= (7)^2 - 4\times 7 - 5 = 49 - 28 - 5$

$= 49 - 33$

$= 16$

So for $x = 7$ the values of the two sides of equality are same

Let us take a new value $x = -3$

LHS $= (-3 - 5)(-3 + 1) = -8 \times -2 = 16$

RHS $= (-3)^2 - 4(-3) - 5 = 9 + 12 - 5 = 16$

Thus for $x = -3$; LHS = RHS

We find that for any value of x, LHS = RHS.

Any equality which is true for every value of variable in it, is called an identity.

So $(x - 5)(x + 1) = x^2 - 4x - 5$ is an identity.

Difference between an identity and equation

Consider an equation $x^2 - 5x + 6 = 0$

Now for $x = 1$

LHS $= (1)^2 - 5 \times 1 + 6 = 1 - 5 + 6 = 7 - 5 = 2$

RHS $= 0$

LHS $\neq$ RHS

For $x = 2$

LHS $= (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 10 - 10 = 0$

RHS $= 0$

LHS = RHS

For $x = 3$

LHS $= (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 15 - 15 = 0$

RHS $= 0$

LHS = RHS

For $x = 0$

$$\text{L.H.S} = (0)^2 - 5 \times 0 + 6 = 6$$
$$\text{R.H.S} = 0$$
$$\text{L.H.S} \neq \text{R.H.S}$$

We observe that equation $x^2-5x+6=0$ is true for $x = 2$ and 3 only and is not true for any other value of x.

We can say that equation is true for some specific values of variables only i.e. it is not true for all values of variable involved in it. Therefore it is not an identity.

8.11 Standard Identities

Now we shall study three identities which are very useful. The identities are obtained by actual multiplication of a binomial with same or another binomial.

(a) Let us first consider $(a + b)(a + b)$ or $(a + b)^2$

$$\begin{aligned} \text{Now } (a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2 \ (\because ab = ba) \end{aligned}$$

So $(a + b)^2 = a^2 + 2ab + b^2$ __________ (I)

We can verify the above identity with different values of a and b. The values of two sides will be same.

(b) Now consider $(a - b)^2$

$$\begin{aligned} (a - b)^2 &= (a - b)(a - b) = a(a - b) - b(a - b) \\ &= a^2 - ab - ba + b^2 = a^2 - 2ab + b^2 \end{aligned}$$

So $(a - b)^2 = a^2 - 2ab + b^2$ __________ (II)

(c) Now consider $(a + b)(a - b)$

$$\begin{aligned} \text{Now } (a + b)(a - b) &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \end{aligned}$$

So $(a + b)(a - b) = a^2 - b^2$ _________ (III)

The above three identities (I), (II) and (III) are known as **standard Identities**.

(d) Now we shall discuss one more useful identity

$$\begin{aligned} (x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \\ &= x^2 + (a + b)x + ab \end{aligned}$$

or $(x + a)(x + b) = x^2 + (a + b)x + ab$ ____________ (IV)

Activity: To verify $(a+b)^2 = a^2 + 2ab + b^2$ by paper cutting activity.

Material Required : Coloured paper, glue, scissor, geometry box, card board.

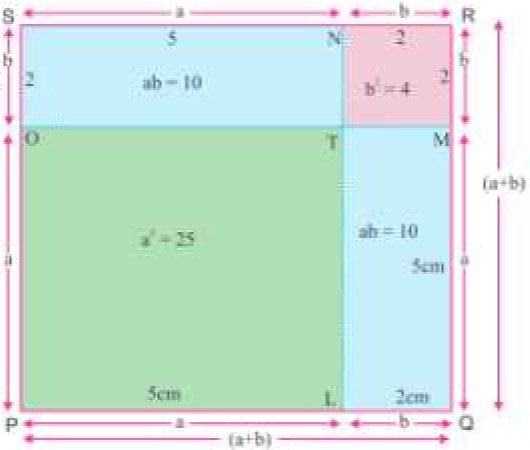
Procedure: 1. From coloured paper cut out figures of following dimensions.

(i) A square of side a = 5cm.

(ii) A square of side b = 2cm

(iii) Two rectangles of length 5cm and breadth 2cm

2. Paste these squares and rectangles on card board as shown.
3. As it is evident from figure that:



Area of square PQRS = Sum of areas of 4 figures.

Area of square PLTO = a × a = a^2

Area of 2 rectangles = 2 ×a × b = 2ab

Area of square MTNR = b × b = b^2

Sum of areas = $a^2 + 2ab + b^2$

Side of square PQRS =a+b

Area of square PQRS =(a+b) (a+b) = $(a+b)^2$

∴ As area of square PQRS = Sum of area of 4 figures

∴ $(a + b)^2 = a^2 + 2ab + b^2$

Result : $(a + b)^2 = a^2 + 2ab + b^2$

VIVA VOCE:

Q.1. Write the expression for $(a+b)^2$.

Ans : $a^2 + 2ab + b^2$

Q.2. Expand $(3x+2y)^2$ using the identity $(a+b)^2 = a^2 + 2ab + b^2$

Ans: $9x^2 + 12xy + 4y^2$

Q.3. Find the value of 101^2 using identity $(a+b)^2 = a^2 + 2ab + b^2$

Ans: 10201

8.12 Applying Identities

Now we will see how these identities are useful in calculating multiplication of various expression in simple and efficient manner.

Example 8.12. Using identity (I) Find

(i) $(x + 2y)^2$ (ii) $(107)^2$ (iii) $(2x + 2y)^2$ (iv) $(10.1)^2$

Sol. (i) $(x + 2y)^2 = (x)^2 + 2(x)(2y) + (2y)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= x^2 + 4xy + 4y^2$

(ii) $(107)^2 = (100 + 7)^2 = (100)^2 + 2 \times 100 \times 7 + (7)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= 10000 + 1400 + 49$

$= 11449$

(iii) $(2x + 2y)^2 = (2x)^2 + 2 \times (2x)(2y) + (2y)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= 4x^2 + 8xy + 4y^2$

(iv) $(10.1)^2 = (10+0.1)^2 = (10)^2 + 2 \times 10 \times 0.1 + (0.1)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= 100 + 2 + 0.01$

$= 102.01$

Note: See that method of using identity is more convenient than the direct multiplication.

Example 8.13. Using identity find :

(i) $(2p - 3q)^2$ **(ii)** $(x - 3y)^2$ **(iii)** $(98)^2$ **(iv)** $(9.9)^2$

Sol. (i) $(2p - 3q)^2 = (2p)^2 - 2(2p)(3q) + (3q)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 4p^2 - 12pq + 9q^2$

(ii) $(x - 3y)^2 = (x)^2 - 2(x)(3y) + (3y)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= x^2 - 6xy + 9y^2$

(iii) $(98)^2 = (100-2)^2 = (100)^2 - 2(100)(2) + (2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 10000 - 400 + 4$

$= 9604$

(iv) $(9.9)^2 = (10-0.1)^2 = (10)^2 - 2 \times 10 \times 0.1 + (0.1)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= 100 - 2 + 0.01$

$= 98.01$

Example 8.14. Using identity find :

(i) $991^2 - 9^2$ **(ii)** 198×202

(iii) $(3 + 2x)(3 - 2x)$ **(iv)** $\left(\frac{3}{4}m + \frac{3}{2}n\right)\left(\frac{3}{4}m - \frac{3}{2}n\right)$

Sol. (i) $991^2 - 9^2 = (991 + 9)(991 - 9)$ $[(a+b)(a-b) = a^2 - b^2]$

$= 1000 \times 982$

$= 982000$

(ii) $198 \times 202 = (200 - 2)(200 + 2)$

$= (200)^2 - (2)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= 40000 - 4$

$= 39996$

(iii) $(3 + 2n)(3 - 2n) = (3)^2 - (2n)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= 9 - 4n^2$

(iv) $\left(\frac{3}{4}m + \frac{3}{2}n\right)\left(\frac{3}{4}m - \frac{3}{2}n\right)$

$= \left(\frac{3}{4}m\right)^2 - \left(\frac{3}{2}n\right)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= \frac{9}{16}m^2 - \frac{9}{4}n^2$

Example 8.15. Use identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following :

(i) 104×107 **(ii)** 501×503 **(iii)** 97×104 **(iv)** $(2y + 3)(2y + 6)$

(v) $(3p - 7)(3p + 8)$

Sol. (i) $104 \times 107 = (100 + 4)(100 + 7)$

$= (100)^2 + (4 + 7) \times 100 + 4 \times 7$ [Here $x = 100, a = 4, b = 7$]

$= 10000 + 1100 + 28$

$= 11128$

(ii) $501 \times 503 = (500 + 1) \times (500 + 3)$

$= (500)^2 + (1 + 3) \times 500 + 1 \times 3$ [Here $x = 500, a = 1, b = 3$]

$= 250000 + 2000 + 3$

$= 252003$

(iii) $97 \times 104 = (100 - 3) \times (100 + 4)$

$= (100)^2 + (-3 + 4) \times 100 + (-3)(4)$ [Here $x = 100, a = -3, b = 4$]

$= 10000 + 100 - 12$

$= 10088$

(iv) $(2y+3)(2y+6) = (2y)^2 + (3+6)(2y) + 3 \times 6$ [Here $x = 2y, a = 3, b = 6$]

$= 4y^2 + 9 \times 2y + 18$

$= 4y^2 + 18y + 18$

(v) $(3p-7)(3p+8) = (3p)^2 + (-7+8)(3p) + (-7) \times (8)$ [Here $x = 3p, a = -7, b = 8$]

$= 9p^2 + 1 \times 3p - 56$

$= 9p^2 + 3p - 56$

Example 8.16. Simplify:

(i) $(3p+2q)^2 - (3p-2q)^2$ (ii) $(2ab + 3bc)^2 - 12ab^2c$ (iii) $(x+5y)^2 - (x+y)(x-y)$

Sol. (i) $(3p+2q)^2 - (3p-2q)^2$

$= [(3p)^2 + (2q)^2 + 2 \times 3p \times 2q] - [(3p)^2 + (2q)^2 - 2 \times 3p \times 2q]$

$= [9p^2 + 4q^2 + 12pq] - [9p^2 + 4q^2 - 12pq]$

$= 9p^2 + 4q^2 + 12pq - 9p^2 - 4q^2 + 12pq$

$= 24pq$

(ii) $(2ab + 3bc)^2 - 12ab^2c$

$= (2ab)^2 + (3bc)^2 + 2 \times 2ab \times 3bc - 12ab^2c$

$= 4a^2b^2 + 9b^2c^2 + 12ab^2c - 12ab^2c$

$= 4a^2b^2 + 9b^2c^2$

(iii) $(x+5y)^2 - (x+y)(x-y)$

$= [x^2 + (5y)^2 + 2 \times x \times 5y] - [x^2 - y^2]$

$= x^2 + 25y^2 + 10xy - x^2 + y^2$

$= 26y^2 + 10xy$

Exercise 8.5

1. Use suitable identity to get each of the following products :

(i) $(x + y)(x + y)$ (ii) $(y + 2x)(y + 2x)$ (iii) $(a + 7b)(a + 7b)$

(iv) $(2a - b)(2a - b)$ (v) $(2x - 3y)(2x - 3y)$ (vi) $\left(x - \frac{1}{2}y\right)\left(x - \frac{1}{2}y\right)$

(vii) $(2x + 3y)(2x + 3y)$ (viii) 101×99 (ix) $\left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

(x) $61^2 - 39^2$ (xi) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$ (xii) 54×46

(xiii) $(q + p)(p - q)$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following :

(i) $(x + 2)(x + 3)$ (ii) $(x + 2)(x - 5)$ (ii) $(x - 7)(x + 3)$

(iv) $(4x + 5)(4x + 1)$ (v) $(7p + 6)(7p - 3)$ (vi) $(5y^2 - 1)(5y^2 + 2)$

3. Solve the following squares by using the identities :

(i) $(xy + 3z)^2$ (ii) $\left(\frac{2}{3}x - \frac{3}{2}y\right)^2$

(iii) $(-a + c)(-a + c) = (-a + c)^2$ (iv) $(1.2p - 1.5q)^2$

(v) $(x^2 + 3y^2)^2$ (vi) $(x - y^2z)^2$

4. Simplify :

(i) $(x^2 + 3y)^2 + (3 + x^2y)^2$ (ii) $(2m + 5n)^2 + (2n + 5m)^2$

(iii) $(ab + bc)^2 - 2ab^2c$ (iv) $(9p - 5q)^2 - (9p + 5q)^2$

5. Prove that :

(i) $(a + b)^2 - (a - b)^2 = 4ab$

(ii) $(2x + 3y)(2x - 3y) + (3y - 5z)(3y + 5z) + (5z - 2x)(5z + 2x) = 0$

(iii) $(2x + 5)^2 - 40x = (2x - 5)^2$

(iv) $(x - y)^2 + (x + y)^2 = 2(x^2 + y^2)$

6. Using identities, evaluate :

(i) 99^2 (ii) 103^2 (ii) 5.1^2

(iv) 9.8^2 (v) 71×69 (vi) 1.02×0.98

7. Using $a^2 - b^2 = (a + b)(a - b)$ evaluate

(i) $153^2 - 147^2$ (ii) $64^2 - 36^2$

(iii) $(1.05)^2 - (.95)^2$ (iv) $12.1^2 - 7.9^2$

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find :

(i) 105 × 102 (ii) 5.1 × 5.2 (iii) 46 × 49

(iv) 103 × 94 (v) 9.3 × 9.2 (vi) 10.3 × 9.8

9. Multiple Choice Questions :

(i) Complete the identity $(a + b)^2 =$

(a) $a^2 - b^2$ (b) $a^2 + b^2 + 2ab$ (c) $a^2 + b^2 - 2ab$ (d) $a^2 + b^2$

(ii) Complete the identity: $a^2 - 2ab + b^2 =$ ....

(a) $(a-b)^2$ (b) $a-b^2$ (c) $a-b$ (d) a^2-b^2

(iii) Complete the identity : $(a+b)\ (a-b)$:

(a) a^2+b^2 (b) a^2-b (c) a^2-b^2 (d) $a-b$

(iv) Complete the identity: $(x+a)\ (x+b) = x^2 +$ .... $x +$ ....

(a) $a^2b,\ a+b$ (b) $(a+b),\ ab$ (c) $a^2+b^2,\ a^2b^2$ (d) $a-b,\ ab$

(v) To solve $(y+5)\ (y-5)$ identify the suitable identity:

(a) $(a + b)^2 = a^2 + 2ab + b^2$ (b) $(a - b)^2 = (a^2 - 2ab + b^2)$

(c) $(a + b)\ (a - b) = a^2 - b^2$ (d) $a^2 + b^2 = ab$

(vi) Solve: $\left(\frac{3}{2}p + \frac{2}{3}q\right)\left(\frac{3}{2}p - \frac{2}{3}q\right)$

(a) $\frac{3}{2}p^2 - \frac{2}{3}q^2$ (b) $\frac{9}{4}p^2 - \frac{4}{9}q^2$ (c) $\frac{3}{2}p^2 - \frac{2}{3}q$ (d) $\frac{9}{4}p^2 + \frac{4}{9}q^2$

(vii) To multiply $(2x-3)\ (2x+5)$, identify the identity that should be used:

(a) $(a + b)\ (a - b) = a^2 - b^2$ (b) $(a + b)^2 = a^2 + 2ab + b^2$

(c) $(x + a)\ (x + b) = x^2 + (a+b)\ x + ab$ (d) $(a-b)^2 = a^2 - 2ab + b^2$

(viii) If $(2p+3q)$ and $(2p-3q)$ are sides of rectangle than its area is:'

(a) $2p^2+3q^2$ (b) $4p^2+3q^2$ (c) $4p^2-9q^2$ (d) $6p^2q^2$

Learning Outcomes

After compeletion of the chapter students are able to:

- *Identify Algebraic expression, its terms and their co-efficients.*
- *Identify variable and factors of a term.*
- *Define a Polynomial.*
- *Differentiate between an expression and polynomial.*
- *Define a monomial, Binomial and Trinomial.*
- *Differentiate between like and unlike terms.*
- *Apply operations of addition, subtraction and multiplication over polynomials and algebraic expressions.*
- *Apply multiplication of algebraic expression to find area of rectangle and volume of cuboid.*
- *Understand about identities and use identities in their daily life.*

Answers

Excercise 8.1

4.

	Term	Coefficient
(i)	$5xy$	5
	$-3zy$	-3
(ii)	2	2
	$2x$	2
	$-3x^2$	-3
(iii)	$4x^2y^2$	4
	$-4z^2$	-4
	$3xy$	3
(iv)	ab	1
	bc	1
	abc	1
	7	7
(v)	$\frac{x}{6}$	$\frac{1}{6}$
	$\frac{y}{6}$	$\frac{1}{6}$
	$2xz$	2
(vi)	$0.3a$	0.3
	$-0.5ab$	-0.5
(vii)	$\frac{xy}{2}$	$\frac{1}{2}$
	$7x$	7
	$\frac{3}{2}y$	$\frac{3}{2}$
(viii)	$0.4a$	0.4
	$-0.6ab$	-0.6
	$3b^2$	3
(ix)	$3xy^2$	3
	$5xyz$	5
	$-6y^2$	-6

5. (i) monomial (ii) monomial (iii) monomial (iv) binomial (v) binomial

(vi) trinomial (vii) trinomial (viii) trinomial (ix) trinomial (x) monomial

(xi) binomial (xii) trinomial

6. (i) $3ab - 6a^2b + abc + 3$

(ii) $-x + 4y + 7z - 2xyz - 8$

(iii) 0

(iv) $-2y + 2z$

(v) $-x^2y^2 + 4xy + 9$

(vi) $x^2 + y^2 + z^2$

7. (i) $8x - 4xy - 13y - 10$

(ii) $7\ell m + 4mn + 21n\ell$

(iii) $2ab - 3bc - ca - 5abc$

(iv) $2x - 3y - 4z - 10xyz$

(v) $0.4x + 0.6y - 11xyz$

(vi) $ab - 3bc + 3cd - 3abc$

8. (i) $abc + bc - cd$ (ii) $-x - y + 2z - 4xyz$ (iii) xy

(iv) $3xy + 5x + 4y - 5z$ (v) $0.5xy + 0.3zx$ (vi) $0.1xyz + 0.1xy^2$

9. $9x^2 + 10x - 2$

10. (i) a (ii) c (iii) a (iv) b (v) c (vi) c (vii) b

(viii) b (ix) c (x) a (xi) b

Excercise 8.2

1. (i) monomial (ii) monomial

2. (i) $24xy$ (ii) $8x$ (iii) $-12pq$ (iv) $-24p^2q$ (v) 0 (vi) $2p^3q$ (vii) $6p^2r$ (viii) $2pr$

3. xy ; $8\ell m$; $60mn$; $12mn^2$; $117\ a^3b^2c$; $6axpr$; $12mn^2p$; $2p^2qr$; $21x^4y^3$

4.

First monomial →	$2x$	$-5y$	$2x^2$	$-3xy$	$7x^2y$	$-9x^2y^2$
Second monomial ↓						
$-2y$	$-4xy$	$10y^2$	$-4x^2y$	$6xy^2$	$-14x^2y^2$	$18x^2y^3$
$3x$	$6x^2$	$-15xy$	$6x^3$	$-9x^2y$	$21x^3y$	$-27x^3y^2$
y^2	$2xy^2$	$-5y^3$	$2x^2y^2$	$-3xy^3$	$7x^2y^3$	$-9x^2y^4$
$-4xy$	$-8x^2y$	$20xy^2$	$-8x^3y$	$12x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$2x^2y^2$	$4x^3y^2$	$-10x^2y^3$	$4x^4y^2$	$-6x^3y^3$	$14x^4y^3$	$-18x^4y^4$

5. (i) $-84x^6$ (ii) $24xyz^2$ (iii) $\frac{abc}{24}$ (iv) $a^3b^3c^2d$ (v) $-24x^3y^3z^3$ (vi) $-12p^3qx^2$

6. (i) xyz (ii) $24xyz$ (iii) $14abc$ (iv) $120lmn$ (v) $a^3b^3c^3$ (vi) $\frac{abc}{24}$

7. (i) a (ii) b (iii) c (iv) c (v) b (vi) c

Excercise 8.3

1. (i) $4x^2 + 4xy$ (ii) $x^3 - 3x^2y$ (iii) $7x^2y + 7xy^2$ (iv) $4x^3 - 36x^2$ (v) 0 (vi) $a^2b^2 + ab^2c$

2. (i) $a^3b^3c^2 + a^2b^3c^3 + a^3b^2c^3$ (ii) $2x^2y + 2xy^2 + 2xyz$

(iii) $2p^2 + 2pq - 4pr$ (iv) $ab^2c + abc^2 - a^2bc$

3. (i) $a^4 - a^2b^2$ (ii) $-8x^2y - 12xy^2$ (iii) $a^3 - 2a^2b + ab^2$ (iv) $-4x^4 - 4x^2y^2 + 8x^3$

4. (i) $3x^2 + 2x - 7 ; -2 ; \frac{-21}{4}$ (iii) $x^3y^2 - x^2y^3 ; -4$

(ii) $2y^3 - 7y^2 + 8 ; 8 ; -1$ (iv) $a^2b + a^2b^2 + a^2b^2c ; 8$

5. (i) $x^2 - xy + y^2 - yz + z^2 - xz$

(ii) $2x^2 - 4xy - 2xz + 2yz - 2y^2$

6. (i) $10l^2 + 50ln + 5lm$

(ii) $4bc - 2ab + 2ac - 2a^2$

7. (i) $-5x^2 + 6x - 6$ **8.** $6xy^2 - 25xyz$

9. (i) b (ii) b (iii) c (iv) c

Excercise 8.4

1. (i) $x^2 + 9x + 20$ (vi) $x^2 - 2xy - 3y^2$

(ii) $2x^2 - 11x - 21$ (vii) $p^2 + 2pq - 3q^2$

(iii) $x^2 - 5x - 24$ (viii) $8p^2 - 18pq + 9q^2$

(iv) $2x^2 - 11x + 12$ (ix) $a^3 + a^2b^2 - ab - b^3$

(v) $2x^2 + 7xy + 6y^2$ (x) $\frac{7}{2}x^3 - xy + x^2y^2 - \frac{2}{7}y^3$

(xi) $0.6x^2y + 0.5xy^2 - 20y^3$ (xii) $p^4 - q^2$

2. (i) $y^2 + 19$ (ii) $a^2b^2 + 5a^2 - 3b^2 - 23$

(iii) $y^2x + y^3 - 7x + 6y$ (iv) $3x^2 - 5y^2$

(v) 0 (vi) $x^2 - y^2$

(vii) $2p^2 + 2pq - r^2$ (viii) $x^2 - y^2 - 2x^2y - 2xy^2$

(ix) $ln + mn - 2m^2$ (x) $2x^3 - 17x^2 + 37x$

Excercise 8.5

1. (i) $x^2 + 2xy + y^2$ (ii) $y^2 + 4xy + 4x^2$

(iii) $a^2 + 14ab + 49b^2$ (iv) $4a^2 - 4ab + b^2$

(v) $4x^2 - 12xy + 9y^2$ (vi) $x^2 - xy + \frac{1}{4}y^2$

(vii) $4x^2 + 12xy + 9y^2$ (viii) 9999

(ix) $x^2 - \frac{y^2}{100}$ (x) 2200

(xi) $\frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$ (xii) 2484

(xiii) $p^2 - q^2$

2. (i) $x^2 + 5x + 6$ (ii) $x^2 - 3x - 10$

(iii) $x^2 - 4x - 21$ (iv) $16x^2 + 24x + 5$

(v) $49p^2 + 21p - 18$ (vi) $25y^4 + 5y^2 - 2$

3. (i) $x^2y^2 + 6xyz + 9z^2$ (ii) $\frac{4}{9}x^2 - 2xy + \frac{9}{4}y^2$

(iii) $a^2 - 2ac + c^2$ (iv) $1.44p^2 - 3.6pq + 2.25q^2$

(v) $x^4 + 9y^4 + 6x^2y^2$ (vi) $x^2 + y^4 z^2 - 2xy^2z$

4. (i) $x^4 + x^4y^2 + 12x^2y + 9y^2 + 9$ (ii) $29m^2 + 29n^2 + 40mn$

(iii) $a^2b^2 + b^2c^2$ (iv) $-180pq$

6. (i) 9801 (ii) 10609

(iii) 26.01 (iv) 96.04

(v) 4899 (vi) 0.9996

7. (i) 1800 (ii) 2800

(iii) 0.20 (iv) 84

8. (i) 10710 (ii) 26.52

(iii) 2254 (iv) 9682

(v) 85.56 (vi) 100.94

9. (i) b (ii) a (iii) c (iv) b (v) c (vi) b

(vii) c (viii) c

CHAPTER

9

Mensuration

Learing Objectives

In this chapter, you will learn

- *To Differentiate between plane figures (triangles, quadrilaterals, pentagoes etc.) and solid figures.*
- *To find perimeter and area of plane figures (quadrilaterals).*
- *To find area of polygon.*
- *To find surface area and volume of some solids. (Cuboid, Cube, Cylinder) and use the concept in daily life.*

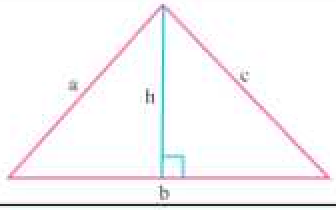
9.1 Introduction

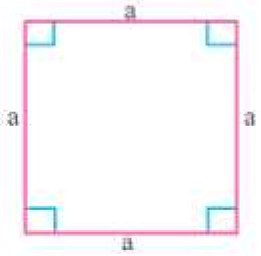
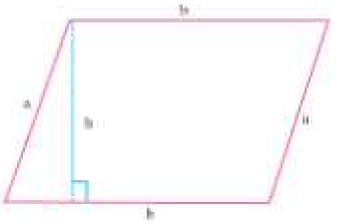
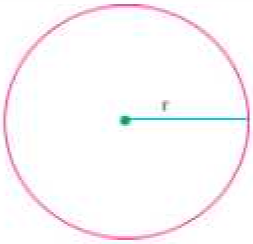
In earlier classes we have learnt about the perimeter and area of some closed plane figures. The perimeter of a figure is the total length of its boundary and the area of a figure is the region enclosed by it. We have already learnt about the perimeter and area of some plane figures such as triangles, rectangles, squares and circles etc. We have also learnt to find the area of pathways or borders of rectangular shapes.

In this chapter we will learn to find the area of simple quadrilaterals and some special types of quadrilaterals. We will also learn about surface area and volume of solids like cuboid, cube, cylinder etc.

9.2 Let us Recall

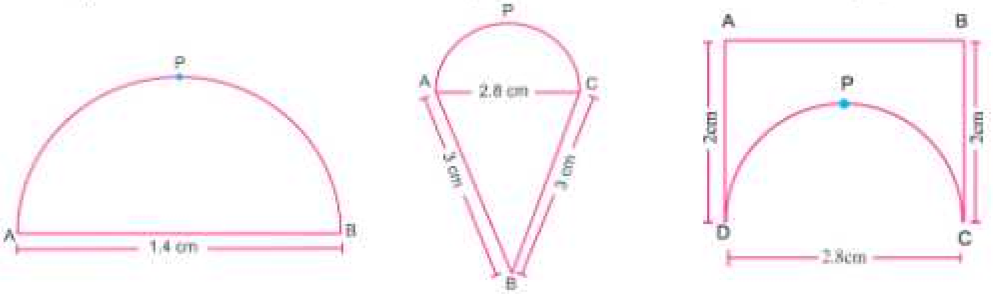
To review the previous knowledge let us recall the geometrical shapes and formulae (expressions) to find their perimeter and area.

Shape	Diagram	Perimeter	Area
Triangle		a + b + c	$\frac{1}{2} \times b \times h$
Rectangle		a + b + a + b = 2(a + b)	a × b

Square		a + a + a + a = 4a	a × a
Parallelogram		a + b + a + b = 2(a + b)	a × h
Circle		2πr	πr^2

For circle we should use word circumference in place of perimeter.

Example 9.1 **Find the perimeter of the following figures:**



(i) **(ii)** **(iii)**

Figure 9.1

Sol. (i) We have, diameter of the semi circle = 1.4cm

∴ Radius (r) of the semi circle $= \frac{1.4}{2} = 0.7\text{cm}$

So Perimeter (circumference) of the semi circle = πr + 2r

$$= \frac{22}{7} \times 0.7 + 2 \times 0.7 = 2.2 + 1.4 = 3.6 \text{ cm}$$

(ii) Perimeter of the figure = AB + BC + arc APC

$$= 3 + 3 + \pi r$$

$$= 6 + \frac{22}{7} \times 1.4 \left[\because \text{Diameter} = 2.8\text{cm}; \ r = \frac{2.8}{2} = 1.4\text{cm}\right]$$

$$= 3 + 4.4 = 7.4 \text{ cm}$$

(iii) Perimeter of the figure $= AB + BC + \text{arc } CPD + AD$

$= 2.8 + 2 + \pi r + 2$

$= 6.8 + \frac{22}{7} \times 1.4 = 6.8 + 4.4 = 11.2\text{cm}$

Example 9.2 The length and breadth of a rectangular field are in 3:2. If the area of the field is 294m², Find the cost of fencing the field at ₹8 per metre.

Sol. Let the length be 3x and breadth be 2x.

Given, Area of rectangle $= 294 \text{ m}^2$

i.e. length × breadth $= 294$

$\Rightarrow$ $3x \times 2x = 294$

$\Rightarrow$ $6x^2 = 294$

$\Rightarrow$ $x^2 = 49$

$x^2 = 7^2$

$\Rightarrow$ $x = 7$

$\therefore$ length $= 3x = 3 \times 7 = 21\text{m}$

And breadth $= 2x = 2 \times 7 = 14\text{m}$

Now, Perimeter of the rectangle $= 2 (\ell + b)$

$= 2 (21 + 14) = 2 \times 35 = 70\text{m}$

$\therefore$ The cost of fencing the field = ₹(70×8) = ₹560

Example 9.3 A park is rectangular in shape having length 30 m and breadth 20 m. There are four square flower beds of size 2m × 2m each in the park as shown in figure 9.2 and rest has grass on it. Find

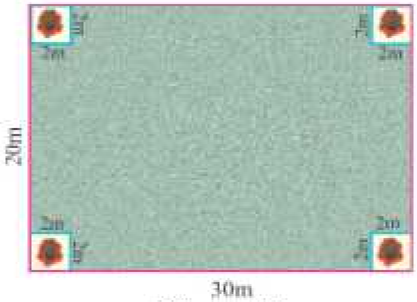


Figure 9.2

(i) Perimeter of park

(ii) Area of park

(iii) Total Area of all four flower beds

(iv) Area of park covered by grass

Sol. (i) Perimeter of park = 2 (length + breadth)

$= 2 (30 + 20) \text{ m} = 100 \text{ m}$

(ii) Area of park = length × breadth

$= (30 \times 20) \text{ m}^2 = 600 \text{ m}^2$

(iii) Area of all four flower beds = 4 × area of one flower bed $= 4 \times (2 \times 2) \text{ m}^2$

$= (4 \times 2 \times 2) \text{ m}^2$

$= 16\text{m}^2$

(iv) Area of park covered by grass

= Total area of park – Area of all four flower beds

$= (600 - 16) \text{ m}^2 = 584 \text{ m}^2$

Example 9.4 **The shape of a garden is rectangular in the middle and semi circular at the ends as shown in figure 9.3. Find the area and perimeter of the garden.**

Figure 9.3

Sol. The area of garden

= Area of rectangular part + 2 × Area of semi circle

$$= (\text{Length} \times \text{breadth}) + 2 \times \frac{1}{2} \times \pi r^2$$

$$= (30 \times 14)\ \text{m}^2 + \left(2 \times \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right)\ \text{m}^2 \qquad \left[\begin{array}{l} \because \text{Diameter} = 14\text{m} \\ \text{So, radius } (r) = \frac{14}{2} = 7\text{cm} \end{array}\right]$$

$$= 420\ \text{m}^2 + 154\ \text{m}^2 = 574\ \text{m}^2$$

Perimeter of garden = length of boundry

= 2 × length of rectangular part + 2 × circumference of semi circle

$$= (2 \times 30 + 2 \times \pi r)\ \text{m} = \left(60 + 2 \times \frac{22}{7} \times 7\right)\ \text{m}$$

$$= (60+44)\text{m} = 104\ \text{m}$$

Example 9.5. **The area of a floor is 1080 m². Flooring tiles are in the shape of parallelogram shape whose base is 24 cm and corresponding height 10 cm are available. How many such tiles are required to cover the floor. (If required we can split the tiles to fill up corner.)**

Sol. Area of floor = 1080 m² = (1080 × 100 × 100) cm² = 10800000 cm²

(Here note that area of floor is given in m² unit where as the size of tile is given in cm² unit. So to find the number of tiles, we have to convert them in same units.)

Area covered by one tile = b × h = (24 × 10) cm² = 240 cm²

$$\text{So Number of Tiles required} = \frac{\text{Area of floor}}{\text{Area covered by one tile}}$$

$$= \frac{10800000}{240} = 45000$$

Exercise 9.1

1. Find the perimeter and area of the following figures :

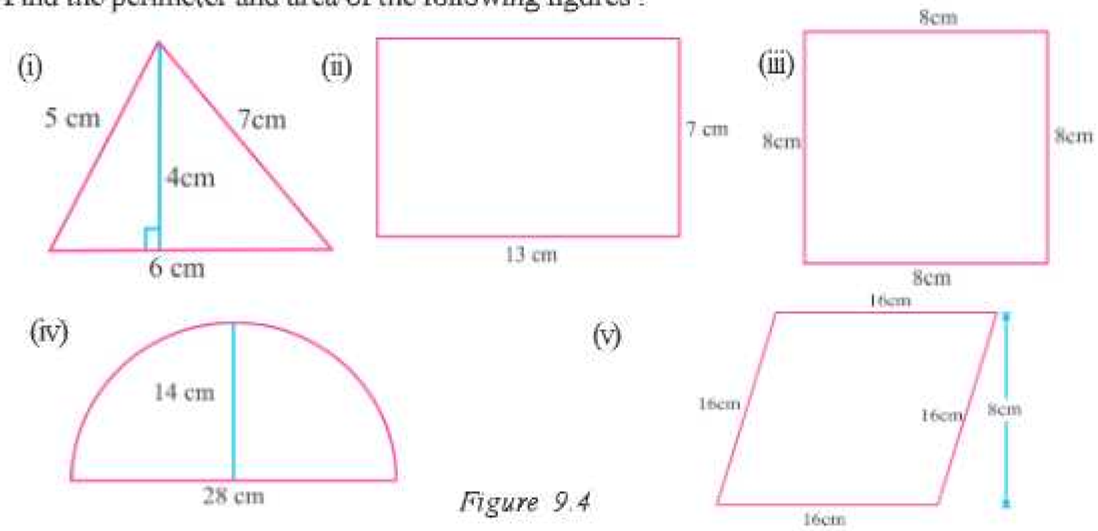


Figure 9.4

2. Find the area and perimeter of the following figures :

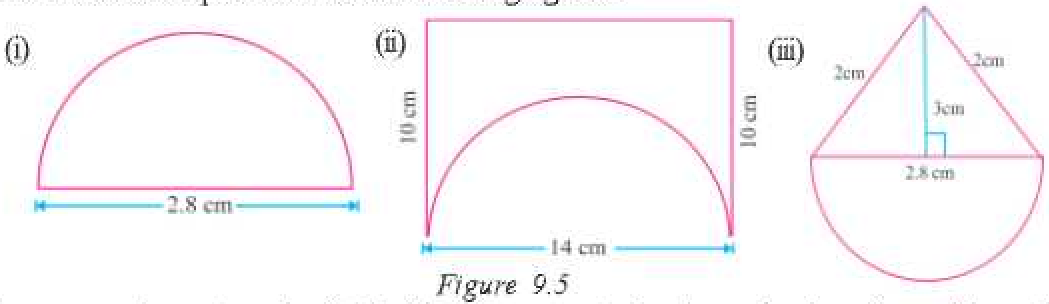


Figure 9.5

3. A square and a rectangular field with measurements (as shown in given figures) have the same perimeter. Which field has larger area and how much ?

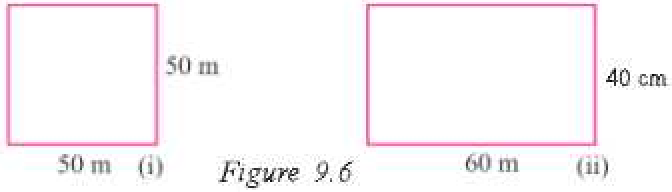


Figure 9.6

4. A park is of length 30 m and breadth 20 m. There is a path of one metre running inside along the perimeter of the park (fig. 9.7). The path has to be cemented. If 1 bag of cement is required to cement 4 m² area. How many bags of cement are required to construct the path.

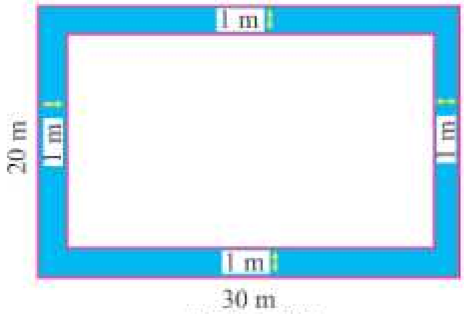


Figure 9.7

5. Mr. Sandeep has a square plot as shown in figure 9.8. and he wants to construct a house in the middle of plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of ₹ 60 per m^2.

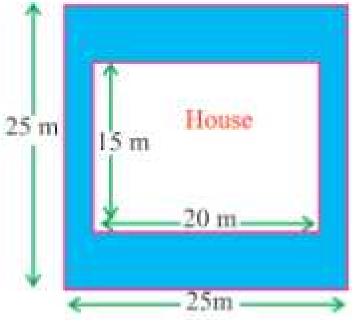


Figure 9.8

9.3 Area of a Quadrilateral

We know that a quadrilateral is a polygon having four sides. A quadrilateral can be split into two triangles by drawing one of its diagonal. It is known as triangulation. This "triangulation" helps us to find a formula to calculate the area of a quadrilateral. Study the figure 9.9.

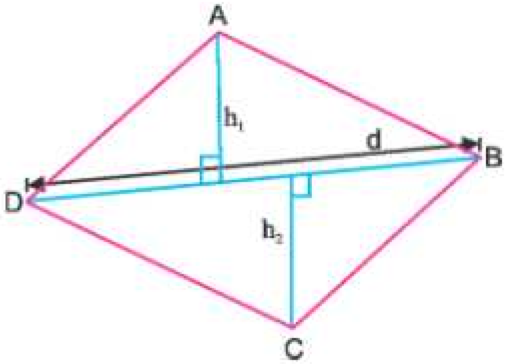


Figure 9.9

Area of quadrilateral ABCD

= Area of ΔABD + Area of ΔCDB

$$= \left(\frac{1}{2} \times DB \times h_1\right) + \left(\frac{1}{2} \times DB \times h_2\right)$$

$$= \frac{1}{2} DB \times (h_1 + h_2) = \frac{1}{2} d(h_1 + h_2)$$

where d denotes the length of diagonal DB.

Example 9.6 Find the area of quadrilateral ABCD shown in figure 9.10.

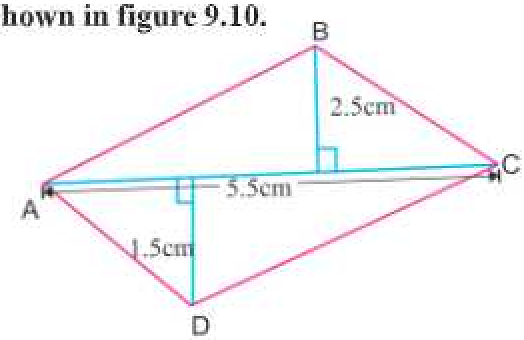


Figure 9.10

Sol. In this case,

$d = AC = 5.5$ cm, $h_1 = 2.5$ cm

and $h_2 = 1.5$ cm

Using Area of quaderilateral $= \frac{1}{2} d(h_1 + h_2)$

Area of quadrilateral ABCD $= \frac{1}{2} \times 5.5 \times (2.5 + 1.5)$ cm^2

$$= \frac{1}{2} \times 5.5 \times 4 \text{ cm}^2 = 11 \text{ cm}^2$$

9.4. Area of Special Quadrilaterals :

9.4.1 Area of Rhombus. We can use the same method of splitting a rhombus into two triangles (which is called triangulation) to find its area. We already know that rhombus is a parallelogram with all sides of equal length and the diagonals of rhombus are perpendicular bisectors of each other.

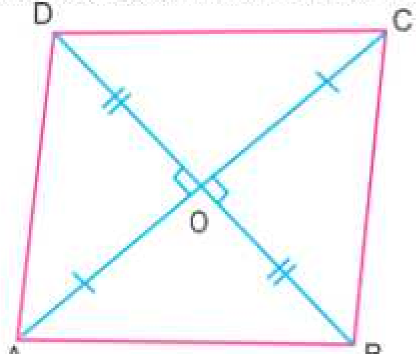


Figure 9.11

Now Area of Rhombus ABCD

= Area of triangle ACD + Area of triangle ABC

$$= \left(\frac{1}{2}\times AC\times OD\right)+\left(\frac{1}{2}\times AC\times OB\right)$$

$$= \frac{1}{2}\times AC\times(OD+OB)=\frac{1}{2}\times AC\times BD$$

$$= \frac{1}{2}\times d_1\times d_2 \qquad \text{(where } d_1 = AC \text{ and } d_2 = BD\text{)}$$

$$= \frac{1}{2}\times \text{(Product of diagonals)}$$

> **Note :** We know that every square is a rhombus. So we can also calculate the area of a square in the form of diagonals. As the diagonals of a square are equal.
>
> $\therefore$ Area of a square $= \frac{1}{2}\times$ (product of diagonals) $= \frac{1}{2}\, d\times d = \frac{1}{2}\, d^2$

Example 9.7 Find the area of a rhombus whose diagonals are of lengths 20 cm and 8.2 cm.

Sol. Area of rhombus $= \frac{1}{2}\times d_1\times d_2$ (where d_1 and d_2 are the diagonals of rhombus)

$$= \frac{1}{2}\times 20\times 8.2 = 10\times 8.2 = 82\ \text{cm}^2$$

Example 9.8 Find the area of a square whose diagonal is 12cm.

Sol. Given, diagonal (d) of a square = 12cm

$$\text{Area of a square} = \frac{1}{2}\times \text{(product of diagonals)}$$

$$= \frac{1}{2}\times d^2 = \frac{1}{2}\times 12\times 12 = 72\ \text{cm}^2$$

Example 9.9 Find the area of a rhombus whose base is 10 cm and height is 6.8 cm.

Sol. Given, base = 10cm and height = 6.8 cm

Area of a rhombus = base × height

$= 10\times 6.8 = 68\ \text{cm}^2$

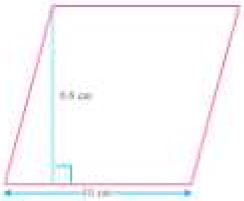


Figure 9.12

Example 9.10 The area of a rhombus is 120cm^2 and one of its diagonal is 16cm. Find the length of the other diagonal.

Sol. We know,

Area of a rhombus= $\frac{1}{2}$ × (product of diagonals)

$$\Rightarrow \quad 120 = \frac{1}{2} \times 16 \times d \quad \Rightarrow \quad d = \frac{120}{8} = 15\text{cm.}$$

Example 9.11 Find the area of a rhombus whose side is 6cm and altitude is 4cm. If length of one diagonal is 8cm then find the length of the other diagonal.

Sol. Given, side = 6cm and altitude = 4cm

∴ Area of Rhombus = Side × altitude

= 6 × 4 = 24 cm^2

Also Area of Rhombus = $\frac{1}{2}$ × (product of diagonals)

i.e. $24 = \frac{1}{2} \times 8 \times d$

$$\Rightarrow \quad d = \frac{24}{4} = 6\text{cm}$$

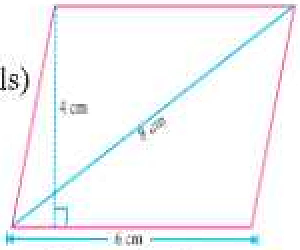


Figure 9.13

> **Note :** See that by drawing a diagonal in a parallelogram (or rhombus) it is divided into two congruent triangles. So we can also find the area of a parallelogram (or rhombus) by finding the area of triangle on one side of its diagonal and then by doubling it.

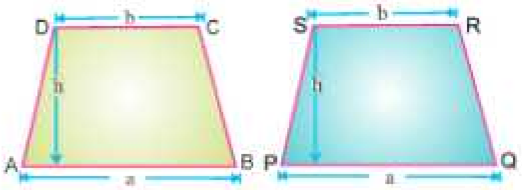
Now we shall find the area of a trapezium with the help of an activity.

Activity: Find area of trapezium with the help of an activity.

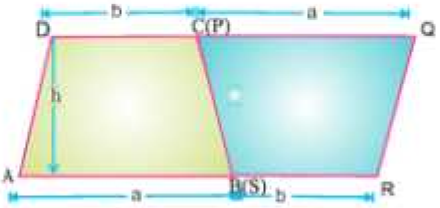
Material Required: Coloured paper, pair of scissors, scale, pencil, eraser, glue etc.

Previous Knowledge: Students know the area of parallelogram

Procedure:



(i)



(ii)

1. On two pieces of coloured paper of different colours, draw two congruent trapezium as in fig (i) viz ABCD and PQRS.
2. Flip one of the trapezium PQRS and place it along other trapezium as in fig (ii) to obtain parallelogram ARQD
3. Side of parallelogram ARQD is AR = a + b and height is h.

Now area of parallelogram ARQD = AR × h

2 × Area of trapezium ABCD = (a + b) × h

$$\text{Area of trapezium ABCD} = \frac{1}{2}(a + b) \times h$$

$$\therefore \quad \text{Area of trapezium ABCD} = \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height}$$

Example 9.12 Find the area of trapezium shown in figure 9.14 (i) and 9.14 (ii).

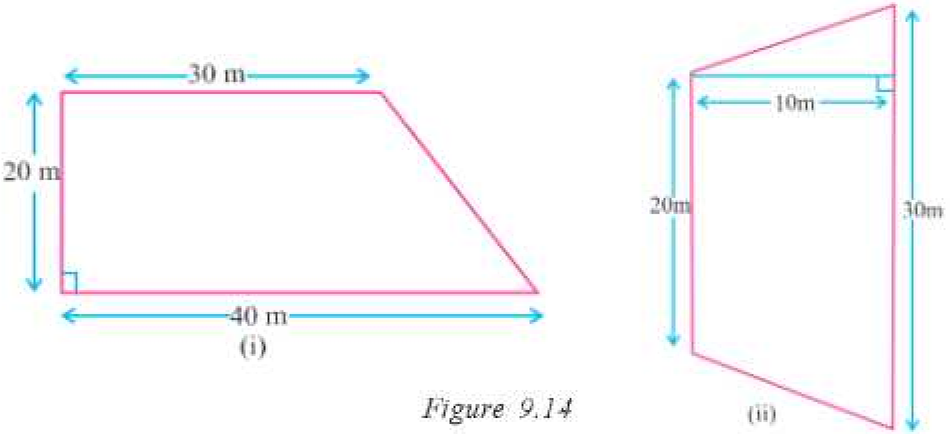


Figure 9.14

Sol. Figure 9.14 (i) and (ii) are both of trapezium shape.

(i) Given parallel sides a = 30 m, b = 40 m and distance between them (h) = 20 m

Area of trapezium = $\frac{1}{2}(a + b) \times h$

$\therefore$ Area = $\frac{1}{2}(30 + 40) \times 20 = \frac{1}{2} \times 70 \times 20 = 700 \text{ m}^2$

(ii) Given parallel sides a = 20 m, b = 30 m and distance (h) = 10 m

So Area of trapezium = $\frac{1}{2}(20 + 30) \times 10 = 250 \text{ m}^2$.

Example 9.13. The area of a trapezium shaped field is 480 m². The distance between two parallel sides is 15 m. If one of the parallel side is 20 m. Find the other parallel side.

Sol. Here one of the parallel side of trapezium = 20 m.

Let another parallel side be b. Distance between parallel sides (h) = 15 m and Area of trapezium = 480 m²

Now using, area of trapezium = $\frac{1}{2} \times$ (sum of parallel sides) $\times$ h

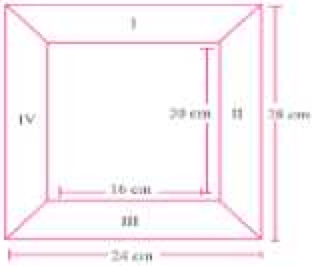
We have, $\quad 480 = \frac{1}{2} \times (20 + b) \times 15$

i.e. $\quad 20 + b = \frac{480 \times 2}{15} = 64$

So $\quad b = 64 - 20 = 44$ m

Hence other parallel side of trapezium = 44 m

Example 9.14. **In given fig, the picture frame has outer dimensions of size 24cm × 28cm and inner dimensions of size 16cm × 20cm. Find the area of each section of the frame, if the width of each section is same.**



Sol. Width of the frame along length $= \frac{1}{2}\ (28 - 20)$

$$= \frac{1}{2} \times 8 = 4\text{cm}$$

and width of the frame along breadth= $\frac{1}{2}(24-16) = \frac{1}{2} \times 8\ = 4\text{cm}$

∴ Each section of the frame in a trapezium with height 4cm.

Thus, Area of Section I $= \frac{1}{2} \times (16 + 24) \times 4$

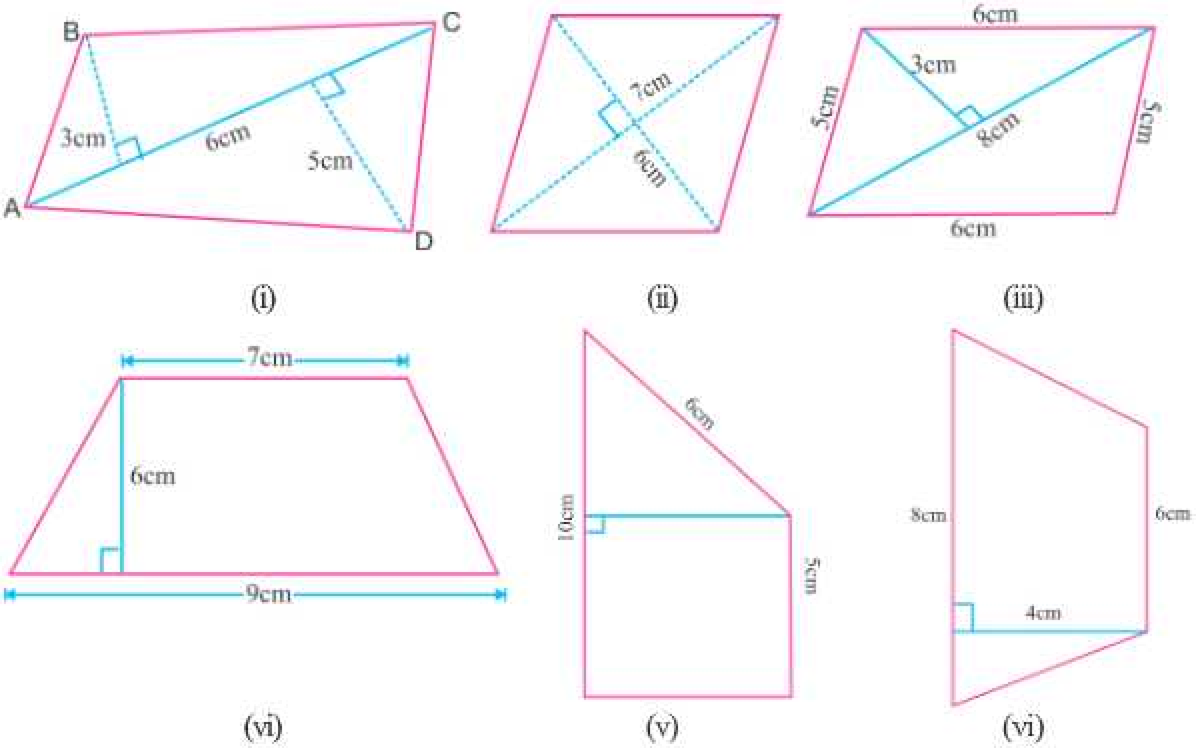
$$= \frac{1}{2} \times 40 \times 4 = 80\text{cm}^2 = \text{Area of section III}$$

Area of Section II $= \frac{1}{2} \times (20{+}28) \times 4 = \frac{1}{2} \times 48\ \times 4 = 96\text{cm}^2$

$=$ Area of Section IV

Exercise 9.2

1. Find the area of the quadrilaterals given below in figure 9.15.



(i) (ii) (iii)

(vi) (v) (vi)

Figure 9.15

2. The area of a rhombus is 320 cm^2. If length of its one diagonal is 16 cm. Find the length of the other diagonal.

3. One diagonal of a quadrilateral field is 24 m and the altitudes dropped on it from the opposite vertices are 8 m and 13 m. Find the area of the field (fig. 9.16).

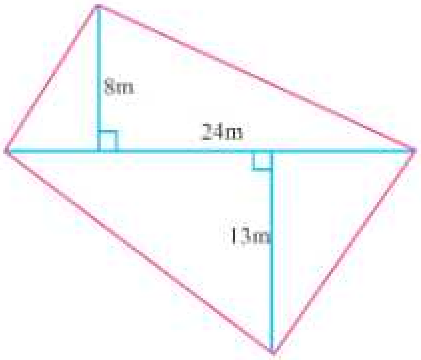


Figure 9.16

4. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

5. Find the area of a square whose length of diagonal is 10 cm.

6. Find the area of a rhombus with side 8 cm and altitude 4.8 cm.

7. Find the area of rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of its diagonal is 8 cm. Find the length of other diagonal.

8. Find the area of a trapezium shaped field if the parallel sides are of length 250 m and 160 m and the distance between them is 100 m.

9. Find the other parallel side of trapezium if its area is 300 m^2. One parallel side is 15 m and distance between parallel sides is 15 m.

10. Find the area of a trapezium whose parallel sides are 1m and 1.2m and perpendicular distance between them is 0.8 m.

11. The floor of a building consists of 2400 tiles which are rhombus shaped having diagonals 45cm and 32cm in length. Find the total cost of polishing the floor, if the cost per m^2 is ₹4.

12. **Multiple Choice Questions :**

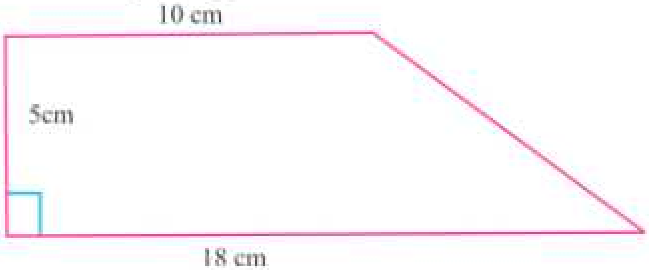
(i) Find the area of a rhombus whose diagonals are 4cm and 6cm.

(a) 24cm^2 (b) 12cm^2 (c) 10cm^2 (d) 18cm^2

(ii) Find the area of a square whose diagonal is d.

(a) d^2 (b) $\frac{1}{2}d$ (c) $2d^2$ (d) $\frac{1}{2}d^2$

(iii) Find the area of the given figure:



(a) 70cm^2 (b) 180cm^2 (c) 90cm^2 (d) 120cm^2

9.5 Area of a Polygon

We know that a simple closed figure made up of only line segments is called a polygon. So far we have learnt about the area of three sided polygons *i.e.* triangles and four sided polygon *i.e.* quadrilaterals. Now we will try to find the area of a five sided polygon namely pentagon, six Sided polygon namely hexagon and so on.

By Joining the diagonals, we have divided a quadrilateral into triangles and found its area. Similar methods can be used to find the area of a polygon. Observe the following pentagon.

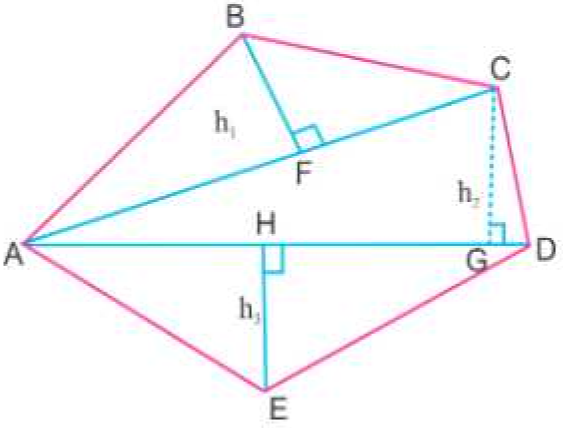


Figure 9.17

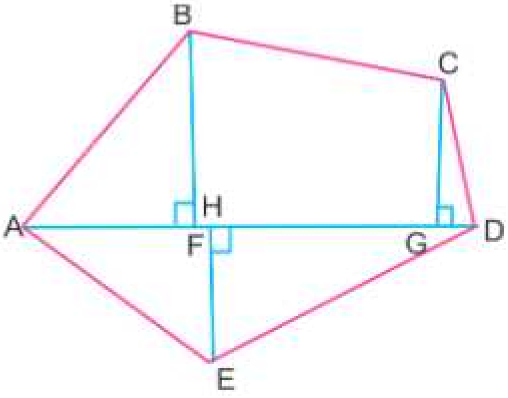


Figure 9.18

In fig. 9.17 by constructing two diagonals AC and AD the pentagon ABCDE is divided into three parts.

So Area of ABCDE = Area of ΔABC + Area of ΔACD + Area of ΔAED

We can find area of ABCDE in another way also.

In fig. 9.18, by constructing one diagonal AD and three perpendiculars BF, CG and EH on it, Pentagon ABCDE is divided into four parts. So area of ABCDE = area of right angled triangle AFB + Area of trapezium BFGC + Area of Right angled triangle CGD + Area of triangle AED.

Example 9.15. Find the area of the Pentagon shown in figure 9.19

Sol. Here pentagon ABCDE is divided into right triangle AFB, trapezium BFHC, right triangle CHD and triangle AED.

So Area of Pentagon ABCDE

= Area of right ΔAFB + Area of trapezium BFHC + Area of right ΔCHD +

Area of right Δ AED

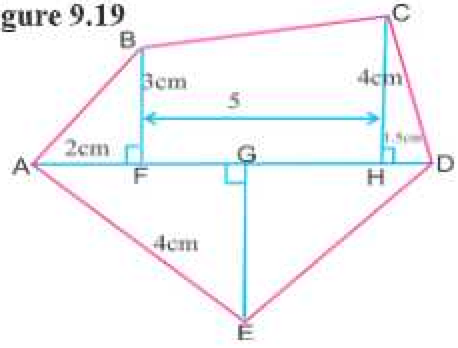


Figure 9.19

$$= \left(\frac{1}{2}\times AF\times BF\right) + \left(\frac{1}{2}(BF+CH)\times FH\right) + \left(\frac{1}{2}\times HD\times CH\right) + \left(\frac{1}{2}\times AD\times EG\right)$$

$$= \left(\frac{1}{2}\times 2\times 3\right) + \left[\frac{1}{2}(3+4)\times 5\right] + \left(\frac{1}{2}\times 1.5\times 4\right) + \left(\frac{1}{2}\times 8.5\times 4\right)$$

$= 3 \text{ cm}^2 + 17.5 \text{ cm}^2 + 3 \text{ cm}^2 + 17 \text{ cm}^2 = 40.5 \text{ cm}^2$

Example 9.16 Find the area of a Pentagon shown in figure 9.20.

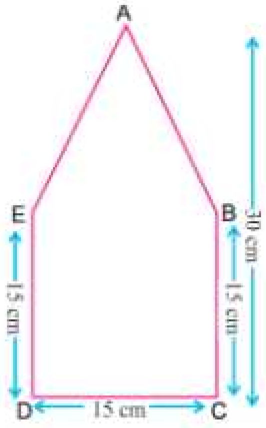


Figure 9.20

Sol. We can find the area of Pentagon ABCDE by two different methods.

Method 1 :

From A draw perpendicular AF on DC as shown in figure 9.21.

AF divides the pentagon into two congruent trapeziums. You can verify it by paper folding.

$$\text{Now Area of trapezium AFDE} = \frac{1}{2}(\text{AF} + \text{ED}) \times \text{DF}$$

$$= \frac{1}{2}(30 + 15) \times 7.5 \text{ cm}^2 \left[\text{DF} = \frac{1}{2}\text{DC}\right]$$

$$= 168.75 \text{ cm}^2$$

So Area of Pentagon ABCDE = 2 × Area of trapezium AFDE

$= 337.5 \text{ cm}^2$

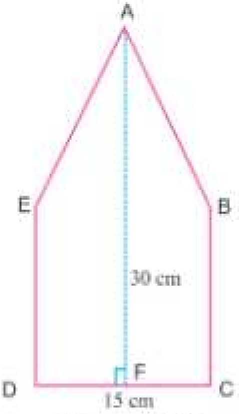


Figure 9.21

Method 2 :

We can also split this pentagon into two parts as shown in figure 9. 22

Now Area of Pentagon ABCDE = Area of ΔAEB + Area of square EBCD

$$= \left(\frac{1}{2} \times \text{EB} \times \text{AF}\right) + (\text{DC} \times \text{CB})$$

$$= \left(\frac{1}{2} \times 15 \times 15\right) \text{cm}^2 + (15 \times 15) \text{ cm}^2$$

$$= 112.5 \text{ cm}^2 + 225 \text{ cm}^2 = 337.5 \text{ cm}^2$$

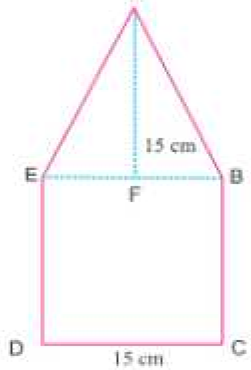


Figure 9.22

Example 9.17 Find the area of hexagon MNOPQR shown in figure 9.23 Where NM = NO = QP = QR

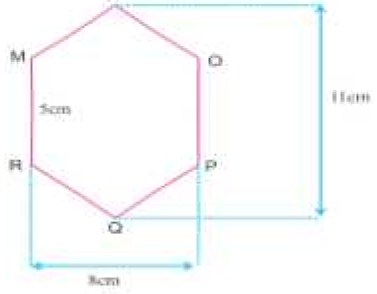


Figure 9.23

Sol. We can find its area by two different methods.

Method 1 :

NQ divides the hexagon into two congruent trapeziums as shown in figure 9.24 You can verify it by paper folding.

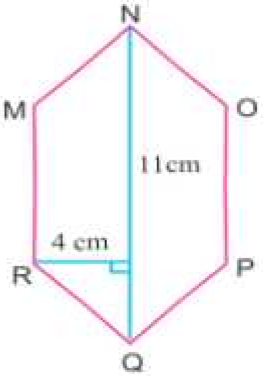


Figure 9.24

Now Area of trapezium MNQR

$$= \frac{1}{2}(5 + 11) \times 4 \text{ cm}^2 = 32 \text{ cm}^2$$

So Area of hexagon MNOPQR

$$= 2 \times 32 \text{ cm}^2 = 64 \text{ cm}^2$$

Method 2 :

We can also split the hexagon as shown in figure 9.25. Here ΔMNO and ΔRQP are congruent triangles with perpendicular 3 cm.

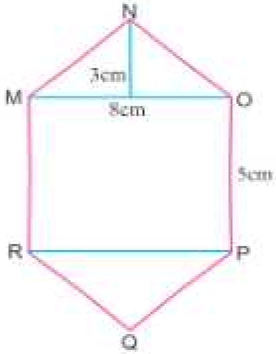


Figure 9.25

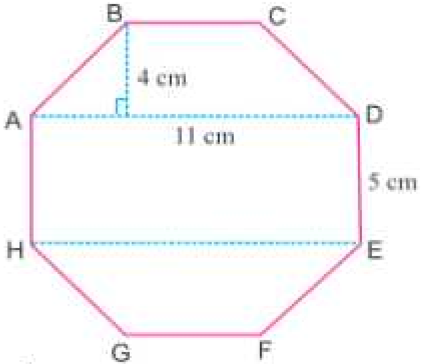
Area of hexagon MNOPQR

= 2 Area of ΔMNO + Area of Rectangle MOPR

$$= 2\times\left(\frac{1}{2}\times 8\times 3\right) \text{ cm}^2 + (8 \times 5) \text{ cm}^2 = 2 \times \frac{1}{2} \times 8 \times 3 \text{ cm}^2 + 40 \text{ cm}^2$$

$= 64 \text{ cm}^2$

Example 9.18 **Find the area of regular octagon as shown in figure 9.26**



Sol. Area of octagon ABCDEFGH=
Area of trapezium ABCD +
Area of rectangle ADEH+ Area of trapezium EFGH

$$= \left[\frac{1}{2}(11+5)\times 4\right] \text{ cm}^2 + (11 \times 5) \text{ cm}^2 + \left[\frac{1}{2}(11+5)\times 4\right] \text{ cm}^2$$

Figure 9.26

(Here trapezium ABCD and EFGH are congruent.)

$= 32 \text{ cm}^2 + 55 \text{ cm}^2 + 32 \text{ cm}^2 = 119 \text{ cm}^2.$

Exercise 9.3

1. **Find the area of pentagon ABCD shown in figure 9.27.**

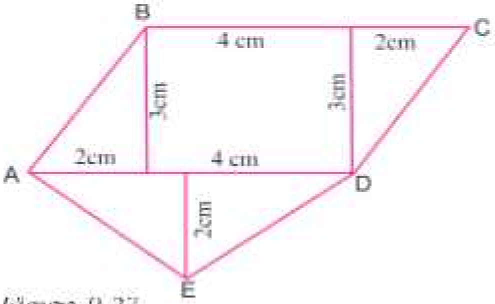


Figure 9.27

2. **There is a pentagonal shaped park as shown in figure 9.28. Jyoti and Kavita divided it in two different ways. Find the area of park using both ways.**

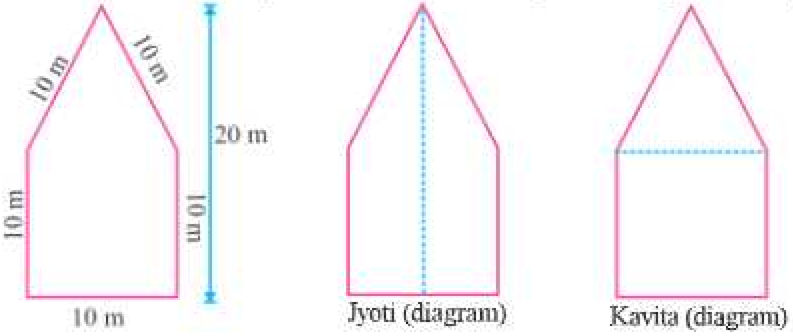


Figure 9.28

3. **Find the area of hexagon shown in figure 9.29 by two different ways as shown in figure 9.29 (a) and 9.29 (b). Where AB = BC = CD = DE = EF = FA**

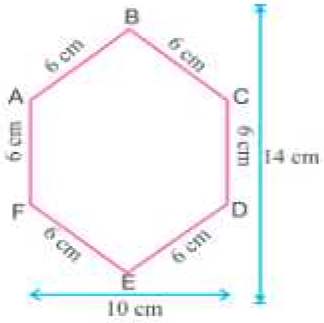


Figure 9.29

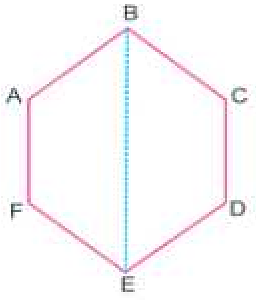


Figure 9.29 (a)

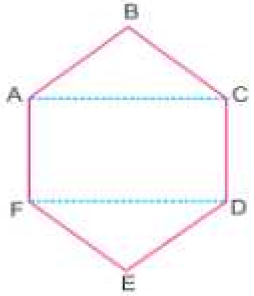


Figure 9.29 (b)

4. **Find the area of octagon as shown in figure 9.30.**

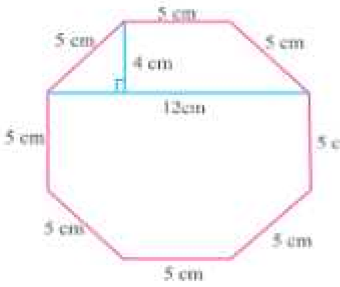


Figure 9.30

5. Find the area of Hexagon shown in the figure 9.31

where

MP = 9 cm MD = 7 cm

MC = 6 cm MB = 4 cm

MA = 2cm AN = 2.5 cm

OC = 3 cm QD = 2 cm

RB = 2.5 cm

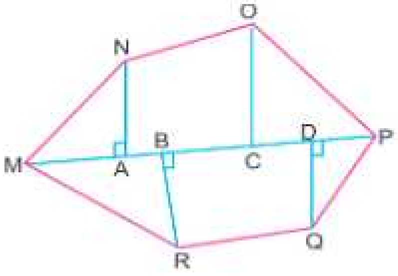


Figure 9.31

9.6 Solid Shapes

In your earlier classes you have studied that two dimensional figures can be identified as the faces of three dimensional shapes. Observe the solids which we have discussed so far in figure 9.32.

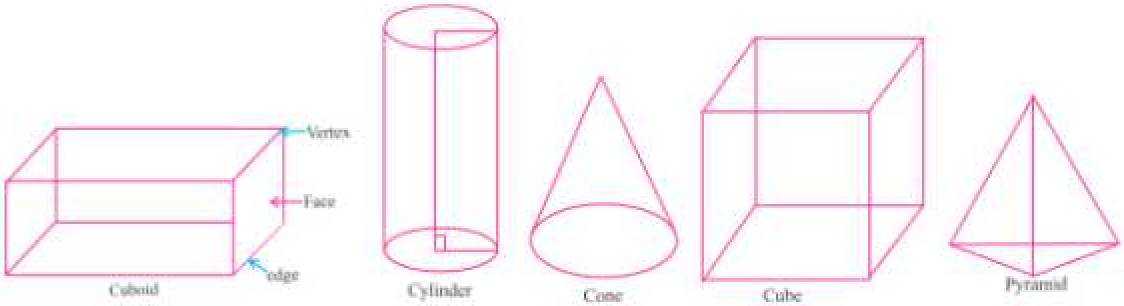


Figure 9.32

Observe that some shapes have two or more than two identical (congruent) faces. In case of **cuboid** all six faces are rectangular and opposite faces are identical. So there are three pairs of identical faces.

In case of a **cube,** all six faces are square and all are identical whereas **cylinder** has one curved surface and two circular faces which are identical and parallel to each other.

Observe in cylinder, the line segment joining the centre of circular faces is perpendicular to the base (fig. 9.32). Such cylinders are known as **right circular cylinders**. We are going to study only this type of cylinder though there are other types of cylinders as well.

9.7 Surface Area of Cuboid, Cube and Cylinder

To find the total surface area of cube, cuboid and cylinder, we will find the area of each face. The surface area of a solid is the sum of the area of its faces. To clarify further, we take each shape one by one.

9.7.1 Cuboid

We know the cuboid has three pairs of identical faces.

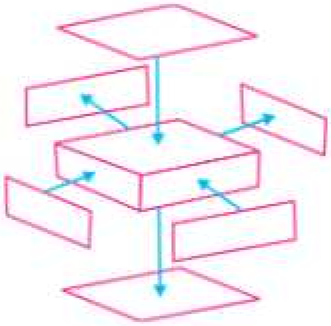


Figure 9.33

Now cut open a cuboidal box and lay it flat (figure 9.33), we can see a net as shown in (figure 9.34).

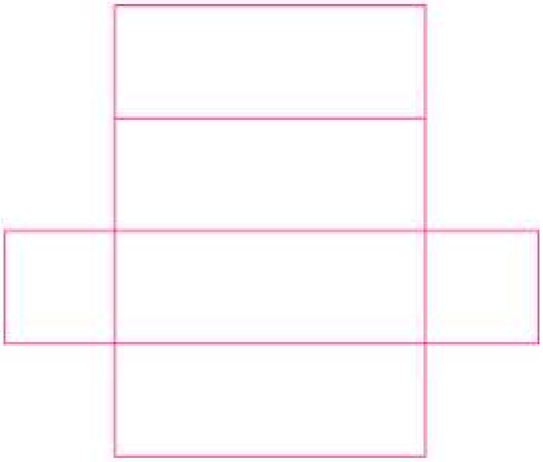


Figure 9.34

Activity

Find the formula of total surface area of the cuboid by activity.

Required Material : Chart Paper, Geometry Box, Coloured sketch or Pencils.

Procedure :

1. Take a chart paper and draw a net of cuboid as shown.
2. Cut the net from the chart paper.
3. Complete net is divided into 6 rectangles.
4. Here, Fig I and Fig V, Fig III and Fig IV; Fig II and Fig VI are congruent rectangles.
5. When we fold this net from partitions, we shall get a cuboid of dimensions.

   5cm × 4cm × 3cm i.e. $(l \times b \times h)$

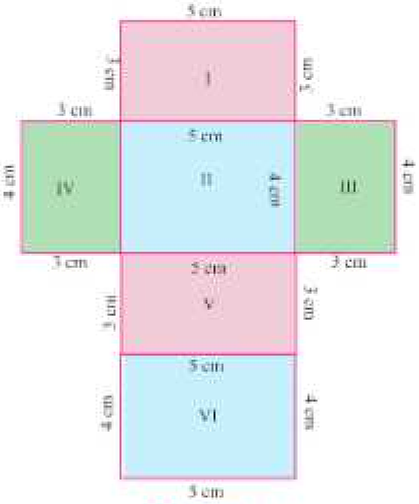


Figure 9.35

Observation:

Total Surface of Area of a cuboid = Area of all six rectangles

= Area of Rectangle (I+II+III+IV+V+VI)

= 2× Area of Rectangle (I + II + III)

$= 2 \times (5 \times 3 + 5 \times 4 + 3 \times 4)$

If $\ell = 5$cm, $b = 4$cm and $h = 3$cm

Then Total Surface Area of a cuboid $= 2 \times (l \times h + l \times b + h \times b)$

$= 2\,(lb + bh + hl)$

Note : The side walls (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of the room. If l, b and h are length, breadth and height of the room respectively, then lateral surface area of room

$$= (l \times h) + (b \times h) + (l \times h) + (b \times h)$$

$$= 2\,l\,h + 2bh$$

$$= 2h(l + b)$$

This is also known as Area of four walls.

VIVA VOCE

Q 1. What is the base of a cuboid?

Ans: Rectangle

Q.2. What is lateral surface area of a cuboid?

Ans: $2\,(l + b) \times h$

Q 3. What is total surface area of a cuboid?

Ans: $2\,(lb + bh + hl)$

Example 9.19 Find the total surface area and lateral surface area of a room having length 6 m, breadth 5 m and height 4m.

Sol. Given, $l = 6$m, b = 5m and h = 4m

Total surface area of room= 2(length × breadth + breadth × height + height × length) m^2

$$= 2(6 \times 5 + 5 \times 4 + 4 \times 5)\ m^2 = 140\ m^2$$

Lateral surface of room = 2 × (length + breadth) × height

$$= 2 \times (6 + 5) \times 4 = 2 \times 11 \times 4$$

$$= 88\ m^2$$

9.7.2 Cube

We know that if length, breadth and height of a cuboid are equal then it is known as a cube. All faces of a cube are square in shape. All faces are of equal area.

Lateral surface Area = 4 × Area of one face = $4l^2$

And Total surface area of cube = 6 × Area of one face

$$= 6\,l^2$$

where l is the length of each side of the cube.

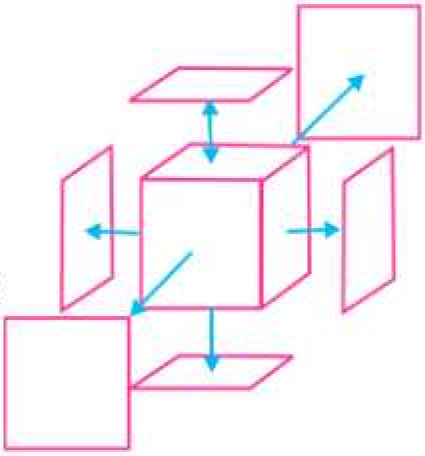


Figure 9.36

Example 9.20 Find the total surface area and lateral surface area of cube having side 10 cm.

Sol. We know that total surface area of cube = $6l^2$, where l is the length of side.

Given, side (l) of a cube = 10cm

So Total surface area = $6 \times (10 \times 10)$ cm² = 600 cm²

Now lateral surface area of cube= $4l^2 = 4 \times (10)^2$

$= 400$ cm²

Example 9.21 Find the side of a cube whose surface area is 600cm².

Sol. Give, Surface area of a cube = 600cm²

$$\Rightarrow \quad 6\ell^2 = 600 \Rightarrow \ell^2 = \frac{600}{6} = 100$$

$$\Rightarrow \quad \ell^2 = 10^2 \quad \Rightarrow \quad \ell = 10$$

Thus, Side of the cube = 10cm.

Example 9.22 Vasu is painting the walls and ceiling of a cuboidal hall with dimensions 15m×10m×7m. From each can of paint, 100m² of area can be painted. How many cans of paint will he need to paint the room?

Sol. Dimensions of the hall are 15m × 10m × 7m

Since, he is painting walls and ceiling

∴ Surface area of the hall = Area of four walls + Area of top

$= 2(\ell+b)h + \ell b$

$= 2(15+10) \times 7 + 15 \times 10$

$= 2 \times 25 \times 7 + 150 = 350 + 150 = 500\text{m}^2$

⇒ Surface Area of the hall = 500m²

Given,Area to be painted with 1 can = 100m²

$$\therefore \quad \text{Number of Cans} = \frac{500}{100} = 5$$

Thus, 5 cans of paint will be needed to paint the room.

Example 9.23 What will happen to the surface area of the cube if its edge is doubled?

Sol. Let the edge of the cube be l

then, Surface Area of the cube = $6l^2$

Now, If the edge is doubled, i.e. new edge = $2l$

∴ Surface area $6(2\ell)^2 = 4(6\ell^2)$ = 4 × (Original surface area)

Thus, If edge of a cube is doubled then the total surface area will be four times the original surface area.

Example 9.24 **Three cubes each of side 4cm are joined end to end. Find the surface area of the cuboid so formed.**

Sol. The cuboid formed by joining three cubes is shown, whose dimensions are

$l = 4 + 4 + 4 = 12$cm, b = 4cm,

h = 4cm

$\therefore$ Total Surface area of the cuboid

$= 2\,(lb + bh + hl)$

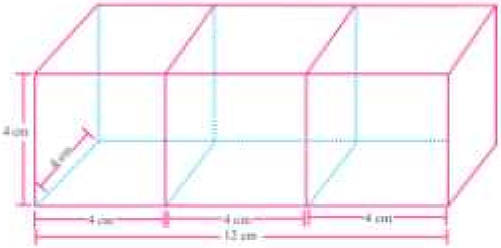
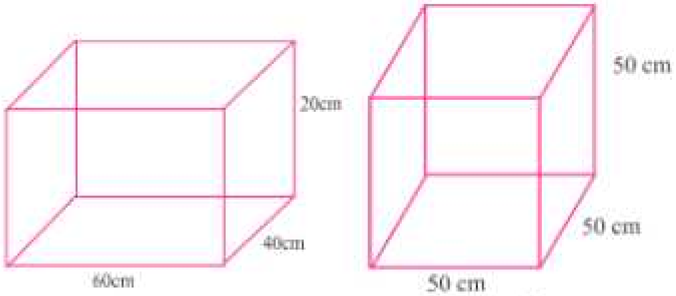


Figure 9.37

$= 2\,(12 \times 4 + 4 \times 4 + 4 \times 12)$

$= 2\,(48 + 16 + 48) = 224\text{cm}^2$

Example 9.25 **There are two cuboidal boxes as shown in the adjoining figure 9.38. Which box requires the lesser amount of material to make ?**



(a) *Figure 9.38* *(b)*

Sol. To find the amount of material to make the box (a) and (b), we have to find the total surface area of the boxes

Total surface area of box (a) $= 2\,(\ell b \times bh + he)$

$= 2\,(60 \times 40 + 40 \times 20 + 20 \times 60)\ \text{cm}^2$

$= 8800\ \text{cm}^2$

Total surface area of box (b) $= 6l^2 = 6 \times 50 \times 50 = 15000\ \text{cm}^2$

So box (a) requires lesser amount of material to make.

9.7.3 Right Circular Cylinder

The cylinder has two congruent circular faces that are parallel to each other. The line segment

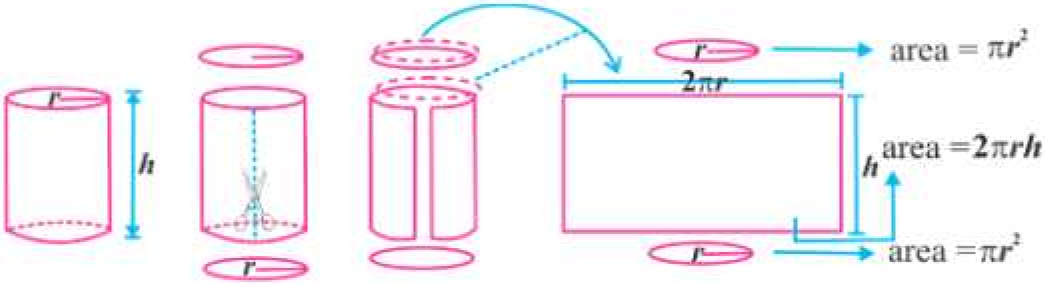


Figure 9.39

joining the centre of circular faces is perpendicular to the base in most of the cylinders, these are right circular cylinders. For example, water pipes, tube lights, round pillars etc.

To find the surface area of the cylinder, cut a cylinder as shown in figure 9.39.

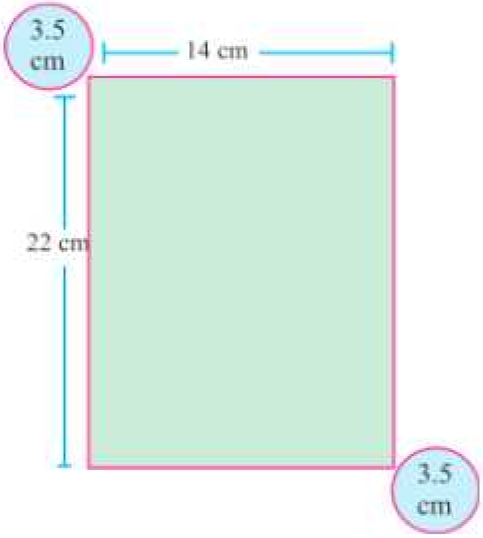
Curved surface area of a cylinder is the area of its curved part, part of cylinder is formed from a rectangle having length 2πr and width h. So curved surface area of a cylinder is 2πrh.

Activity

To find the total surface area of a cylinder by an activity.

Material Required : Chart paper, Geometry box, Coloured sketch or Pencils.

Procedure :



1. Take a chart paper and draw a net of closed cylinder as shown in which we have a rectangle of size 22cm × 14cm and two congruent circles of radius 3.5cm each.
2. Cut the net from the chart paper.
3. Now, when rectangle is rolled along side14cm,there is formation of an open cylinder and height of cylinder will be 14cm.
4. Close the open cylinder from top and bottom with two congruent circles of radius 3.5cm (whose circumference = $2\pi r = 2 \times \frac{22}{7} \times 3.5 = 22$cm)
5. Thus, A closed cylinder is formed by suitable transformation.

Observation:

Total Surface of Area of a cylinder = Area of the given net a closed cylinder

= Area of Rectangle + 2 × Area of circle

= 22cm × 14cm + 2 × πr^2

= $(2\pi r) \times h + 2\pi r^2$

= $2\pi r\ (h + r)$

By Step 4, $2\pi r$ = 22 and in step 3, h = 14

Thus,Total Surface area of a closed cylinder = $2\pi r\ (h + r)$

Note :- The word 'Lateral surface Area' is used for solids with plain surface like cube, cuboid etc. Where the word 'Curved surface Area' is used for solids with curved surfaces, e.g. cylinder, cone, sphere.

VIVA VOCE

Q 1. What is the base of a right circular cylinder?

Ans: Circle.

Q.2. What is the area of base of cylinder?

Ans: πr^2

Q 3. **What is the curved surface area of a cylinder?**
Ans: $2\pi rh$

Example 9.26 **Find the total and curved surface area of cylinder shown in figure 11.40.**

Sol. Radius (r) of base of cylinder is 2 m and height (h) is 5 m.

So total surface area of cylinder = $2\pi r\,(h + r)$

$2\pi\times 2\ (5+ 2)\ m^2 = 2\pi \times 2 \times 7\ m^2$

$= 2 \times \frac{22}{7} \times 2 \times 7\ m^2 = 88\ m^2$

Figure 9.40

and Curved surface area = $2\pi rh = 2\pi \times 2 \times 5 = 20\pi = 20 \times \frac{22}{7} m^2 = \frac{440}{7} m^2$

Example 9.27. **Find the curved area of a cylinder whose circumference of the base is 22cm and height is 7cm.**

Sol. Curved surface area of cylinder $= 2\pi rh$

$= (2\pi r) \times h =$ (Circumference of the base) × h

$= 22 \times 7 = 154 cm^2$

Example 9.28. **The curved surface area of a hollow cylinder is 4224cm². It is cut along its height and formed a rectangular sheet of width 33cm. Find the perimeter of the sheet.**

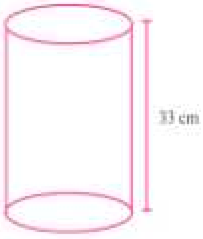


Figure 9.41

Sol. Curved surface area = $4224\ cm^2$

⇒ (Circumference of the base) × h = $4224\ cm^2$

⇒ (Circumference of the base) × 33 = $4224\ cm^2$

Circumference of the base = $\frac{4224}{33} = 128$ cm

Since, circumference of the base = length of the rectangle = 128cm

∴ Perimeter of the rectangular sheet = $2\ (l + b)$

$= 2\ (128 + 33) = 2 \times 161 = 322$cm

Example 9.29. **A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84cm and length 1m.**

Sol. Diameter of the road roller = 84cm

∴ radius (r) of the road roller = $\frac{84}{2}$ = 42cm

and length (h) = 1m = 100cm

Area of road leveled in 750 revolutions = 750 × curved surface area of roller

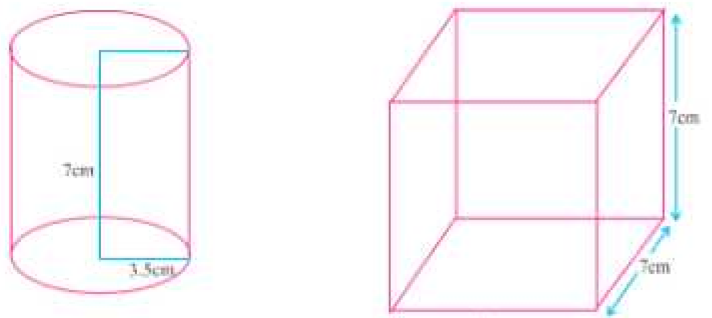
$= 750 \times 2\pi rh$

$= 750 \times 2 \times \frac{22}{7} \times 42 \times 100$

$= 19800000\ cm^2$

$$= \frac{19800000}{100 \times 100} \text{ m}^2 \quad = 1980 \text{ m}^2$$

Example 9.30 Find the curved surface Area (lateral surface area) of the figures 9.42 (a) and (b).



(a) *Figure 9.42* (b)

Sol. Figure 9.42 (a) is cylinderical in shape. So curved surface area of figure 9.42 (a)

$$= 2\pi rh \text{ cm}^2$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 7 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Figure 9.42 (b) is cubical in shape, so curved surface area of fig. 9.42 (b) is

$$= 4l^2 = 4 \times 7 \times 7 \text{ cm}^2$$

$$= 196 \text{ cm}^2$$

Example 9.31 A company packs its milk powder in cylindrical container whose base has a diameter 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in figure 9.43). If the label is placed 1 cm from top and bottom. What is the area of label ?

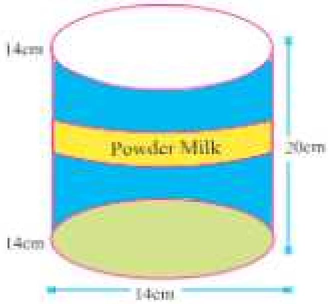


Figure 9.43

Sol. We have to find the area of the label. As label is placed around the cylinderical container having radius 7 cm and height 20 cm. Since the label is placed 1 cm from top and bottom. So height of label is 18 cm.

Hence area of label = (2 × π × radius × height) cm²

$$= \left(2 \times \frac{22}{7} \times 7 \times 18\right) \text{ cm}^2 = 792 \text{ cm}^2$$

Exercise 9.4

1. Find the lateral and total surface area of the cuboid having dimensions:

   (i) 6cm × 5cm × 4cm (ii)15m × 12m × 8m (iii) 8m × 10m × 8m

2. Find the lateral and total surface area of the cubes having edge:

   (i) 8cm (ii) 12m (iii) 15mm

3. Find the side of a cube whose surface area is 2400 cm^2.

4. Neetu painted the outside of a cabinet of measure 3m × 2m × 1.5 m. How much surface area she covered if she painted all cabinet except bottom.

5. Ashima painted her room of measure 15m × 12m × 7m. How much surface area did he cover if he painted all except the floor?

6. Manu wants her room to be painted. If the measures of her room is 20m × 12m × 15m then find the cost of painting the room except the floor at ₹ 6 per m^2?

7. A suitcase with measurement 80 cm × 48 cm × 24 cm is to be covered with a cloth. How many metres of cloth of width 96 cm is required to cover the suitcase ?

8. What will happen to the surface area of a cube if its edge is (i) tripled (ii) halved.

9. Three cubes each of side 5cm are joined end to end. Find the surface area of the cuboid so formed.

10. Find the curved and total surface area of a cylinder whose dimensions are

    (i) r = 7cm, h = 20cm (ii) r = 14cm, h = 15m (iii) diameter = 7cm, h = 12cm

11. Find the curved surface area of a cylinder whose circumference of the base is 77cm and height is 12cm

12. Find the radius of cylinder whose curved surface area is 1056cm^2 and height 12cm.

13. Find the height of cylinder whose radius is 7 cm and total surface area is 968 cm^2.

14. A cylindrical pipe open from both sides has radius 21 cm and height 50 cm. What is its surface area ?

15. A road roller takes 950 complete revolutions to move once over to level a road. Find the area of road leveled if the diameter of road roller is 84 cm and length is 1 m.

16. A closed cylindrical tank of radius 7 m and height 3m is made from a sheet of matel. What is cost of tank if rate of matel sheet is 20 Rs per m^2.

17. **Multiple Chocie Questions :**

    (i) Lateral surface area of cube is:

    (a) $6l^2$ (b) $5l^2$ (c) $4l^2$ (d) $2l^2$

    (ii) Curved surface area of cylinder is:

    (a) $2\pi rh$ (b) πrh (c) $2\pi r$ (d) πr^2h

    (iii) If the edge of a cube is doubled then what will happen to the surface area?

    (a) 2 times (b) 4 times (c) 3 times (d) Half

9.8 Volume of Cuboid, Cube and Cylinder

Amount of space occupied by a three dimensional object is called its volume. We can compare the volume of objects surrounding us. Volume of a room is greater than the volume of a almirah kept inside it. Similarly volume of almirah is greater than a box kept inside it.

Remember to find the area of a region we use square units. Here we will use cubic units to find the volume of a solid. For finding the area we divide the region into square units. Similarly to find the volume of a solid we need to divide it into cubical units.

Observe that the volume of each of the adjoining solid is 8 cubic units. (Figure 9.44)

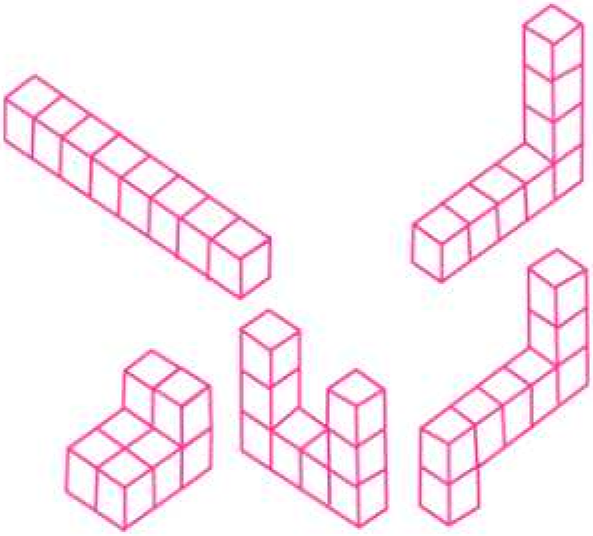


Figure 9.44

We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which are generally use to measure volume are

$$1 \text{ cubic cm} = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$$

$$1 \text{ cubic m} = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3$$

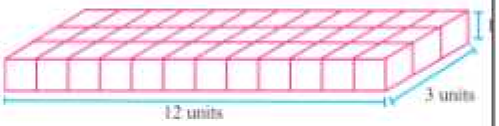
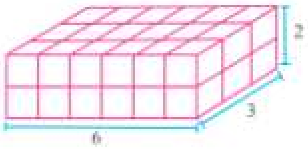
$$1 \text{ cubic mm} = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm} = 1 \text{mm}^3$$

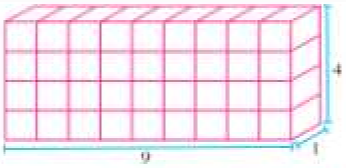
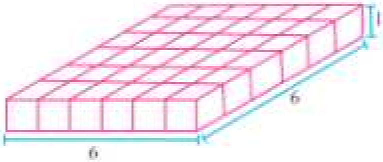
Now we find some expressions to find volume of a cuboid, cube and cylinder.

9.8.1 Cuboid

Take 36 cubes of equal size (*i.e.* length of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways as shown in the following table :

Table

	Cuboid	**length**	**breadth**	**height**	$l \times b \times h = V$
(*i*)		12	3	1	$12 \times 3 \times 1 = 36$
(*ii*)					

(*iii*)					
(*iv*)					

What do we observe ?

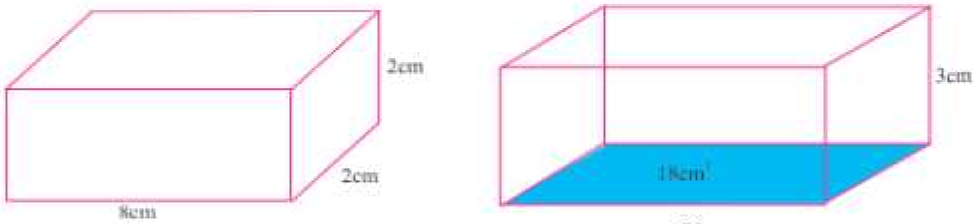
As we used 36 cubes to form these cuboids, so volume of each cuboid is 36 cubic units. We can see that volume of each cuboid is equal to product of length, breadth and height of the cuboid.

So Volume of Cuboid $= l \times b \times h$

As $l \times b$ is the area of base, so we can also say that

Volume of Cuboid = area of base × height

Example 9.32 **Find the volume of cuboid shown in figure 9.45. (a) and 9.45 (b)**



(a) (b)

Figure 9.45

Sol. In figure 9.45 (a) length, bredath and height of cuboid is 8 cm, 2 cm and 2 cm respectively.

So Volume of Cuboid $= l \times b \times h = (8 \times 2 \times 2)\ cm^3$

$= 32\ cm^3$

In figure 9.45 (b), Area of base of Cuboid is 18 cm^2 and height is 3 cm.

So Volume of Cuboid = Area of base × height $= (18 \times 3)\ cm^3 = 54\ cm^3$

9.8.2 Cube

As we know that cube is a special case of cuboid, where $l = b = h$.

Hence Volume of Cube $= l \times l \times l = l^3$ *i.e.* $(side)^3$

Example 9.33. **Find the volume of cube having side (i) 5 cm (ii) 2.5 cm.**

Sol. We know Volume of cube $= (side)^3$

(i) Side = 5cm

$\therefore$ Volume $= (5)^3 = 125\ cm^3$

(ii) Side = 2.5cm

$\therefore$ Volume = $(2.5)^3 = 15.625$ cm³

Example 9.34 Find the height of a cuboid whose base area is 180 cm² and volume is 900 cm³.

Sol. Given, Volume of Cuboid = 900 cm³

and its base area = 180 cm² ; Let h cm be the height of cuboid.

We know that

Volume of Cuboid = Base Area × height

So 900 = 180 × h

i.e. $h = \frac{900}{180} = 5$ cm

Example 9.35 A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes of side 6 cm can be placed in the given cuboid ?

Sol. Volume of cuboid = (60 × 54 × 30) cm³ = 97200 cm³

Volume of a small cube = (6 × 6 × 6) cm³ = 216 cm³

$\therefore$ Number of small cubes that can be placed in the cuboid $= \frac{\text{Volume of Cuboid}}{\text{Volume of Cube}} = \frac{97200}{216} = 450$

9.8.3 Cylinder

In case of cuboid we have seen that volume of cuboid = $l \times b \times h$

Also we have seen that volume of cuboid = Area of base × height

Can we find the volume of a cylinder in the same way ?

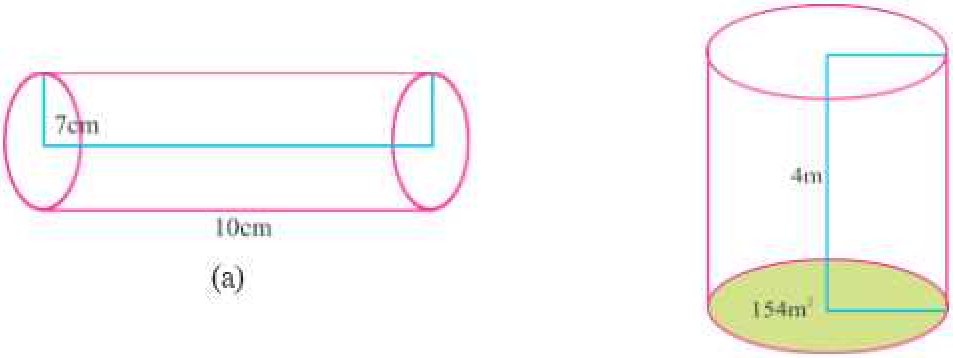
Like cuboid, top and bottom of a cylinder are congruent and parallel to each other. Its curved surface is also perpendicular to the base, just like cuboid.

So the Volume of Cylinder = Area of base × height

$= \pi r^2 \times h = \pi r^2 h$

where r is the radius of circular face and h is the height of cylinder.

Example 9.36 Find the volume of cylinder shown in figure 9.46 (a) and 9.46 (b)



(a)

(b)

Figure 9.46

Sol. In figure 9.46 (a), radius (r) of base = 7 cm and height (h) of cylinder = 10 cm

So Volume of cylinder $= \pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 10\right) \text{ cm}^3$

$= 1540 \text{ cm}^3$

In figure 9.46 (b), Area of base of cylinder = 154 m^2 and height of cylinder = 4 m,

So Volume of cylinder = Area of base × height

$= (154 \times 4) \text{ m}^3 = 616 \text{ m}^3$

9.9 Volume and Capacity

Volume refers to the amount of space occupied by an object and capacity refer to the quantity that a container holds. For example if a water bottle holds 1000 cm^3 of water, then the capacity of the water bottle is 1000 cm^3.

Capacity is also measured in terms of litres. The relation between litre and cm^3 is 1 $m\ell$ = 1 cm^3; 1L = 1000 cm^3.

Now 1 $m^3 = (100 \times 100 \times 100) \text{ cm}^3 = 1000000 \text{ cm}^3 = 1000L$

Example 9.37 A milk tank is in the form of a cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank ?

Sol. Volume of milk tank $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 1.5 \times 1.5 \times 7\right) \text{ m}^3$$

$$= 49.5 \text{ m}^3 = 49.5 \times 1000l \qquad [\therefore 1\text{m}^3 = 1000l]$$

$$= 49500\ l$$

Example 9.38 A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of cylinder.

Sol. Length of the paper becomes the circumfrence of the base of the cylinder and width becomes the height of cylinder.

Let radius of cylinder = r cm and height = 4 cm

Now perimeter of the base of cylinder = circumference of circle

$$\Rightarrow \quad 2\pi r = 11$$

$$2 \times \frac{22}{7} \times r = 11 \Rightarrow \quad r = \frac{7}{4} \text{ cm}$$

Volume of cylinder V $= \pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4\right) \text{ cm}^3 = 38.5 \text{ cm}^3$$

Exercise 9.5

1. Find the volume of a cuboid having dimension

   (i) 4m, 3m, 5m (ii)12cm, 8cm, 10cm (iii) 1.5cm , 2m, 3.4m

2. Find the volume of a cube having edge:

   (i) 6cm (ii) 12cm (iii) 1.5m

3. Find the volume of cuboid whose area of base is 24 cm^2 and height is 3 cm.

4. By doubling the side of cube, how many times (a) its surface area becomes (b) its volume becomes.

5. Find the height of a cuboid whose volume is 275 cm^3 and base area is 25 cm^2.

6. A godown is in the form of a cuboid of measure 60 m × 32 m × 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 8 m^3.

7. Find the volume of a cylinder whose:

   (i) $r = 7cm, h = 12cm$ (ii) $r = 3.5cm, h = 15cm$ (iii) $r = 14m$ $h = 10m$

8. Find the height of the cylinder whose volume is 1.54 m^3 and whose diameter of the base is 140 cm.

9. Find the volume of a cylinder having base area 1.54 m^2 and height 3.5 m.

10. A rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder.

11. Water is pouring into a cuboidal reservoir at the rate of 60 litres per minutes. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill up reservoir.

12. **Mulitple Choice Questions :**

    (i) Find the area of base of a cylinder.

    (a) $\pi r^2 h$ (b) πr^2 (c) $2\pi rh$ (d) $2\pi r$

    (ii) Find the volume of a cuboid having dimension 4m × 2.5m × 2m

    (a) $20m^3$ (b) $40m^3$ (c) $30m^3$ (d) $200m^3$

    (iii) If edge of cube is doubled then what will happen to its volume?

    (a) Double (b) 4 times (c) 8 times (d) 6 times

    (iv) 1*l* = ...cm^3

    (a) 1000 (b) 100 (c) 10 (d) *l*

    (v) The volume of a cube with edge 1.1 is

    (a) 13.31 (b) 1.331 (c) 133.1 (d) 1331

Learning Outcomes

After completion of the chapter, the students are now able to:

- *Differentiate between plane and solid figures.*
- *Find perimeter and area of the plane figures (quadrilateral)*
- *Find area of a polygon.*
- *Find surface area and volume of some solids (cuboid, cube, cylinder) and use the concept in daily life.*

Answers

Exercise 9.1

1. (i) P-18 cm, A-12 cm^2 (ii) P-40 cm, A-91 cm^2

(iii) P-32 cm, A-64 cm^2 (iv) P-72 cm, A-308 cm^2 (v) P- 64cm A- 128cm^2

2. (i) A-3.08 cm^2, P-7.2 cm   (ii) A-63 cm^2, 46 cm (iii) A-7.28 cm^2, P-8.4 cm

3. square, 100 m^2 **4.** 24   **5.** ₹ 19500

Exercise 9.2

1. (i) 24 cm^2 (ii) 21 cm^2 (iii) 24 cm^2 (iv) 48 cm^2 (v) 37.5 cm^2 (vi) 28 cm^2

2. 40 cm   **3.** 252 m^2 **4.** 45 cm^2 **5.** 50 cm^2 **6.** 38.4 m^2 **7.** 24 cm^2;6cm

8. 20500m^2 **9.** 25m   **10.** 0.88m^2 **11.** 691.20

12. (i) (b)   (ii) (d)   (iii) (a)

Exercise 9.3

1. 24 cm^2 **2.** 150 m^2 **3.** 100 cm^2 **4.** 128 cm^2 **5.** 31.75 cm^2

Exercise 9.4

1. (i) 88 cm^2, 148cm^2 (ii) 432cm^2, 792m^2 (iii) 288m^2; 448m^2

2. (i) 256 cm^2; 384cm^2 (ii) 576cm^2; 864m^2 (iii) 900mm^2; 1350mm^2

3. 20 cm   **4.** 21m^2 **5.** 558m^2 **6.** ₹7200   **7.** 144cm

8. (i) 9 times   (ii) One fourth   **9.** 350cm^2

10. (i) 880cm^2 ; 1188cm^2 (ii) 1320cm^2 ; 2552cm^2 (iii) 264cm^2 ; 341cm^2

11. 924cm **12.** 14cm **13.** 15cm **14.** 6600cm^2 **15.** 2508m^2

16. ₹ 8800 **17.** (i) (c), (ii) (a), (iii) (b)

Exercise 9.5

1. (i) 60 m^3 (ii) 960cm^3 (iii) 10.4m^3

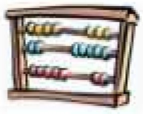
2. (i) 216cm^3 (ii) 1728cm^3 (iii) 3.375m^3

3. 72 cm^3 **4.** (a) 4 times (b) 8 times

5. 11 cm **6.** 7200 **7.** (i) 1848 cm^3 (ii) 577.5cm^3 (iii) 6160m^3

8. 1m **9.** 5.39m^3 **10.** 17600cm^3 **11.** 30h

12. (i) (b) (ii) (a) (iii) (c) (iv) (a) (v) (b)



CHAPTER

10

Exponents and Powers

Learning Objectives

In this chapter, you will learn:

- *About powers with negative integers as exponents.*
- *To understand laws of exponents.*
- *To compare very large and very small numbers.*
- *To convert very small/large numbers in standard form using exponents.*

10.1 Introduction

In earlier classes, we have already learnt about powers with positive integers as exponents, expressing the natural numbers as a product of prime factors.

For example $2^6 = 2\times2\times2\times2\times2\times2$

We say : 2 raised to power 6.

Here, 2 is called base and 6 is called exponent.

We also know that how to write numbers like 2475 in expanded form using exponents as

$2 \times 10^3 + 4 \times 10^2 + 7 \times 10^1 + 5 \times 10^0$

Here we say 2 raised to the power 6.

Exponent

i.e. $(2)^6$

Base

10.2 Power with Negative Exponents

We know that $3^4 = 3 \times 3 \times 3 \times 3$, $5^3 = 5 \times 5 \times 5$ and $2^n = 2 \times 2 \times 2 \times$ .... n times (n is a natural number)

Now try to find out value of 2^{-3}

as we know

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 = 8 = 8 \div 1 \\ 2^2 &= 2 \times 2 = 4 = 8 \div 2 \\ 2^1 &= 2 = 2 = 4 \div 2 \\ 2^0 &= 1 = 2 \div 2 \end{aligned}$$

In every step the upcoming result is divided by the base 2 from the previous result

So continuing the above pattern, we get

$$2^0 = 1$$

$$2^{-1} = 1 \div 2 = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2} \div 2 = \frac{1}{2\times2} = \frac{1}{2^2}$$

$$2^{-3} = \frac{1}{2^2} \div 2 = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$$

Now consider the following

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$3^3 = 3 \times 3 \times 3 = 27 = \frac{81}{3}$$

$$3^2 = 3 \times 3 = 9 = \frac{27}{3}$$

[In every step, the upcoming result is divided by the base 3 from the previous result.]

$$3^1 = 3 = \frac{9}{3}$$

$$3^0 = 1 = \frac{3}{3}$$

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

From above discussion we have

$$2^{-2} = \frac{1}{2^2} \text{ or } 2^2 = \frac{1}{2^{-2}}$$

$$2^{-3} = \frac{1}{2^3} \text{ or } 2^3 = \frac{1}{2^{-3}}$$

$$3^{-2} = \frac{1}{3^2} \text{ or } 3^2 = \frac{1}{3^{-2}} \text{ etc.}$$

In general we can say

For any non zero integer 'a'

$a^{-m} = \frac{1}{a^m}$, where m is a positive integer or a^{-m} is the multiplicative inverse of a^m i.e. $a^{-m} \times a^m = 1 = a^m \times a^{-m}$

Note : a^m and a^{-m} multiplicative inverse of each other

Also if $\frac{p}{q}$ is a non zero rational number and m is positive integer. Then

$$\left(\frac{p}{q}\right)^{-m} = \frac{1}{\left(\frac{p}{q}\right)^m} = \left(\frac{q}{p}\right)^m$$

Example 10.1 **Evaluate :**

(i) 4^{-3} **(ii)** $(-5)^{-2}$ **(iii)** $\left(\frac{3}{2}\right)^{-3}$

Solution : (i) $4^{-3} = \frac{1}{4^3} = \frac{1}{4\times4\times4} = \frac{1}{64}$

(ii) $(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{(-5)\times(-5)} = \frac{1}{25}$

(iii) $\left(\frac{3}{2}\right)^{3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2\times2\times2}{3\times3\times3} = \frac{8}{27}$

Example 10.2 **Find the multiplicative inverse of**

(i) 3^{-4} **(ii)** 8^{-2} **(iii)** $\left(\frac{3}{4}\right)^3$

Solution : As we know multiplicative inverse of $a^{-m} = a^m$

(i) Multiplicative inverse of $3^{-4} = 3^4 = 3\times3\times3\times3 = 81$

(ii) Multiplicative inverse of $8^{-2} = 8^2 = 8 \times 8 = 64$

(iii) Multiplicative inverse of $\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{4}{3}\times\frac{4}{3}\times\frac{4}{3} = \frac{64}{27}$

Example 10.3 **Write the following numbers in expanded form using exponents**

(i) 1024.54 **(ii) 1286.256**

Solution : We have

(i) $1024.54 = 1 \times 1000 + 0 \times 100 + 2 \times 10 + 4 \times 1 + \frac{5}{10} + \frac{4}{100}$

$= 1\times 10^3 + 0 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1} + 4 \times 10^{-2}$

(ii) $1286.256 = 1 \times 1000 + 2 \times 100 + 8 \times 10 + 6 \times 1 + \frac{2}{10} + \frac{5}{100} + \frac{6}{1000}$

$= 1 \times 10^3 + 2 \times 10^2 + 8 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2} + 6 \times 10^{-3}$

10.3 Laws of Exponents

In class VII, we have learnt that for any non-zero integer a, $a^m \times a^n = a^{m+n}$; when m and n are natural numbers.

It is also true for any non zero rational number $\frac{p}{q} = x$, where p and q are integers and q is not equal to zero.

Now $\left(\frac{p}{q}\right)^m \times \left(\frac{p}{q}\right)^n = \frac{p^m}{q^m} \times \frac{p^n}{q^n} = \frac{p^{m\ n}}{q^{m\ n}} = \left(\frac{p}{q}\right)^{m\ n}$

i.e. $\boxed{x^m \times x^n = x^{m+n}}$

Does this law also hold for negative exponents?

Observe the following :

(i) We know that $2^{-2} = \frac{1}{2^2}$ and $2^{-3} = \frac{1}{2^3}$

$\left(\because \quad a^{-m} = \frac{1}{a^m}\right.$ for any non zero rational number a)

Therefore $2^{-2} \times 2^{-3} = \frac{1}{2^2} \times \frac{1}{2^3} = \frac{1}{2^2 \times 2^3} = \frac{1}{2^{2\ 3}} = \frac{1}{2^5} = 2^{-5}$

i.e. $2^{-2} \times 2^{-3} = 2^{-5}$

(ii) $(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$

$$= \frac{1}{(-3)^4(-3)^3} = \frac{1}{(-3)^{4\ 3}} = \frac{1}{(-3)^7} = (-3)^{-7}$$

That is $(-3)^{-4} \times (-3)^{-3} = (-3)^{-7}$

(iii) $5^{-3} \times 5^4 = \frac{1}{5^3} \times 5^4 = \frac{5^4}{5^3} = 5^{4-3} = 5^1 \quad \left[\because \quad \frac{a^m}{a^n} = a^{m-n}\right]$

(When m, n are natural numbers (m > n) and $a \neq 0$)

In general we can say that if a is any non-zero rational number and m, n are integers then

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^m = a^m \times b^m$

(v) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

(vi) $a^0 = 1$

Example 10.4. **Simplify and write in exponential form.**

(i) $(-2)^{-3} \times (-2)^{-4}$ **(ii)** $3^{5} + 3^{-6}$ **(iii)** $p^{3} \times p^{-10}$

(iv) $5^{2} \times 5^{-3} \times 5^{6}$

Solution : (i) $(-2)^{-3} \times (-2)^{-4} = (-2)^{-3+(-4)} = (-2)^{-3-4}$ (Since $a^{m} \times a^{n} = a^{m+n}$)

$= (-2)^{-7} = \frac{1}{(-2)^{7}}$ (Since $a^{-m} = \frac{1}{a^{m}}$)

(ii) $3^{5} \div 3^{-6} = 3^{5-(-6)} = 3^{5+6} = 3^{11}$ (Since $\frac{a^{m}}{a^{n}} = a^{m-n}$)

(iii) $p^{3} \times p^{-10} = p^{3+(-10)} = p^{3-10} = p^{-7} = \frac{1}{p^{7}}$ (Since $a^{m} \times a^{n} = a^{m+n}$)

(iv) $5^{2} \times 5^{-3} \times 5^{6}$

$= 5^{2+(-3)} \times 5^{6} = 5^{2-3} \times 5^{6} = 5^{-1} \times 5^{6}$

$= 5^{-1+6} = 5^{5}$

Example 10.5. **Express 4^{-5} as a power with base 2.**

Solution : We have $4 = 2 \times 2 = 2^{2}$

Therefore $4^{-5} = (2^{2})^{-5} = 2^{2\cdot(-5)}$ [Since $(a^{m})^{n} = a^{mn}$]

$= 2^{-10}$

$= \frac{1}{2^{10}}$ (Since $a^{-m} = \frac{1}{a^{m}}$)

Example 10.6. **Simplify :**

(i) $(-4)^{-3} \times 5^{-3} \times (-5)^{-3}$ **(ii)** $(-3)^{4} \times \left(\frac{5}{3}\right)^{4}$ **(iii)** $\frac{1}{8} \times 5^{-3}$

Solution : (i) We have $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

$= (-4 \times 5 \times -5)^{-3}$ (Using the law $a^{m} \times b^{m} = (ab)^{m}$)

$= (100)^{-3}$

$= \frac{1}{100^{3}}$ [Using $a^{-m} = \frac{1}{a^{m}}$]

(ii) We have

$(-3)^{4} \times \left(\frac{5}{3}\right)^{4} = (-1 \times 3)^{4} \times \frac{5^{4}}{3^{4}} = (-1)^{4} \times 3^{4} \times \frac{5^{4}}{3^{4}}$

$= (-1)^{4} \times 5^{4} = 5^{4}$ [Since $(-1)^{4} = 1$]

(iii) $\frac{1}{8} \times 5^{-3}$

$$= \frac{1}{2^3}\times 5^{-3} = 2^{-3}\times 5^{-3} = (2\times 5)^{-3} = 10^{-3} = \frac{1}{10^3}$$

Example 10.7. Find x so that

$$(-5)^{x+1}\times(-5)^3 = (-5)^7$$

Solution : We have

$$(-5)^{x+1}\times(-5)^3 = (-5)^7$$

$$\Rightarrow\quad (-5)^{x+1+3} = (-5)^7$$

$$\Rightarrow\quad (-5)^{x+4} = (-5)^7$$

On both sides, powers have the same base different from 1 & –1. (see note below) So their exponents must be equal.

Equating the exponents on both sides

We have $x + 4 = 7 \Rightarrow x = 7 - 4$

$\Rightarrow\quad x = 3$

Note : $x^n=1$ only if $n=0$. This will work for any x except $x = 1$ or $x = -1$.

For $x = 1$, $1^1 = 1^2 = 1^3 .... = 1^{-2}$ or $1^n = 1$ for infinitely many n.

For $x = -1$, $(-1)^0 = (-1)^2 = (-1)^{-2} = ... = 1$ or $(-1)^m = 1$ for any even integer m.

Example 10.8. Find the value of $\left(\frac{2}{3}\right)^{-4}$

Solution : $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4}$

$$= \frac{3^4}{2^4} = \frac{3\times3\times3\times3}{2\times2\times2\times2} \qquad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right]$$

$$= \frac{81}{16}$$

Example 10.9. Evaluate : (i) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$ (ii) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{1}$

Solution : (i) $\left(\frac{5}{8}\right)^{7}\times\left(\frac{8}{5}\right)^{-4} = \left(\frac{5}{8}\right)^{7}\times\left(\frac{5}{8}\right)^{4} = \left(\frac{5}{8}\right)^{7-4} = \left(\frac{5}{8}\right)^{3}$

$$= \left(\frac{8}{5}\right)^{3} = \frac{8^3}{5^3} = \frac{8\times8\times8}{5\times5\times5} = \frac{512}{125}$$

(ii) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}^{-1}$

$=(3-4)^{-1}=(-1)^{-1}$

$=\frac{1}{(-1)^{1}}=\frac{1}{-1}=-1$

Exercise 10.1

1. Evaluate (i) 5^{-2} (ii) $(-3)^4$ (iii) $\left(\frac{1}{3}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-3)^5 \div (-3)^7$ (ii) $\left(\frac{1}{2^3}\right)^2$ (iii) $(-5)^4 \times \left(\frac{3}{5}\right)^4$

(iv) $3^{-2} \times (-5)^{-2}$ (v) $(5^{-1} \times 6^{-1}) \times 2^{-1}$ (vi) $(2^{-7} \div 2^{-10}) \times 2^{-5}$

3. Find the value of

(i) $(2^0+4^{-1}) \times 3^2$ (ii) $(8^{-2} \times 2^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}+\left(\frac{1}{5}\right)^{-2}$ (iv) $(2^{-1}+3^{-1}+4^{-1})^0$

(v) $\left\{\left(\frac{-3}{4}\right)^{-2}\right\}^2$ (vi) $\frac{8^{-1} \times 5^2}{2^{-3}}$

4. Find the value of p for which $5^p \div 5^{-3} = 5^5$

5. Find m, $\left(\frac{-2}{3}\right)^{13} \times \left(\frac{3}{-2}\right)^8 = \left(\frac{-2}{3}\right)^{2m-1}$

6. Evaluate :

(i) $\left(\frac{5}{6}\right)^7 \times \left(\frac{6}{5}\right)^3$ (ii) $\left[\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-2}$

7. Simplify :

(i) $\frac{3^{-5} \times 10^{-5} \times 25}{5^{-7} \times 6^{-5}}$ (ii) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$

8. (i) Express 8^{-3} as a power with base 2.

(ii) Express 125^{-4} as a power with base 5.

9. **Multiple Choice Questions :**

(i) $a^m \times a^n =$...............

(a) a^{m-n} (b) a^{mn} (c) a^{m+n} (d) a^{m^n}

(ii) $3^7 \div 3^8 =$ ....

(a) 3 (b) $\frac{1}{3}$ (c) 3^{15} (d) 3^{56}

(iii) $(3^2 + 4^2)^0 =$ ....

(a) 2 (b) 25 (c) 1 (d) 7

(iv) Value of $(5^3)^4$ is = ...

(a) 5^7 (b) 5^{12} (c) 5^{-1} (d) 5^8

(v) Find x if $5^{2x-1} = 5^5$

(a) 2 (b) 3 (c) 4 (d) 5

10.4.1 Uses of Exponents to express numbers in standard form

First we shall discuss that, how to express very large number in standard form. For example,

(i) Mass of earth is 5970, 000, 000, 000, 000, 000, 000, 000

$= 5.97 \times 10^{24}$

(Here decimal is moved 24 places to left)

(ii) Distance of earth to the sun is 149, 600, 000km

$= 1.496 \times 10^8$km

(Here decimal is moved 8 places to left)

It is clear from above examples that any number can be expressed as a decimal number between 1 and 10 (including 1 but excluding 10) multiplied by a power of 10. such a form of number is called standard form. In the same manner, we can express very small numbers in standard form.

For Example

(i) Take a small number 0.00000005

We have

$0.00000005 = \frac{5}{100000000} = \frac{5}{10^8} = 5\times10^{-8}$ (Here decimal is moved 8 places to the right)

(ii) The size of a plant cell is 0.00001275m

$= \frac{1275}{100000000} = \frac{1275}{10^3 \times 10^5} = \frac{1.275}{10^5}$

$= 1.275 \times 10^{-5}$m (Here decimal is moved 5 places to right)

From the above discussion we find that very small numbers can be expressed in standard form by shifting the decimal to the right.

Example 10.10. Write the following numbers in standard form.

(i) 0.00000021 (ii) 15240000 (iii) 6020000000000000

Solution : (i) $0.00000021 = \frac{21}{100000000} = \frac{2.1}{10^7} = 2.1 \times 10^{-7}$

(ii) $15240000 = 1.524 \times 10^7$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

10.4.2 Conversion from standard form number to usual decimal form

In last section, we have learnt about the standard form of numbers. In this section, we shall discuss the conversion of standard form in usual form. Let us consider some examples:-

Example 10.11 Express the following numbers in usual form

(i) 4.63×10^6 (ii) 7.89×10^{-4} (iii) 5×10^{-8}

Solution (i) $4.63 \times 10^6 = \frac{463}{100}\times10^6 = 463\times10^4 = 4630000$

(ii) $7.89 \times 10^{-4} = \frac{7.89}{10^4} = \frac{789}{100\times10^4} = 0.000789$

(iii) $5\times10^{-8} = \frac{5}{10^8} = \frac{5}{100000000} = 0.00000005$

10.4.3 Comparing very large very small numbers :

Example 10.12. Compare the following:

(i) Weight of A is 2.34×10^9 kg and weight of B is 1.17×10^8 kg

(ii) Number 8.02×10^{-5} and 0.802×10^{-6}

Sol. (i) $\frac{\text{Weight of A}}{\text{Weight of B}} = \frac{2.34\times10^9}{1.17\times10^8} = \frac{2.34\times10^9}{1.17\times10^8} = 2 \times 10 = 20$

∴ Weight of A is 20 times the weight of B.

(ii) Ratio $= \frac{8.02\times10^{-5}}{0.802\times10^{-6}} = \frac{8.02\times10^{-5}}{8.02\times10^{-7}} = 10^2 = 100$

∴ First number is 100 times the second number.

Example 10.13. Find the total mass if mass of hydrogen element is 1.674×10^{-27}kg and mass of sliver element is 1.79×10^{-25}kg ?

Sol. Total mass = Mass of Hydrogen element + Mass of silver element

$= 1.674 \times 10^{-27} + 1.79 \times 10^{-25} = 1.674 \times 10^{-2} \times 10^{-25} + 1.79 \times 10^{-25}$

$= 0.01674 \times 10^{-25} + 1.79 \times 10^{-25}$

$= (0.01674 + 1.79) \times 10^{-25} = 1.80674 \times 10^{-25}$

Example 10.14. During solar eclipse, distance between sun and earth is 1.496×10^{11}km and the distance between earth and moon is 3.84×10^{8}km then find the distance between the Sun and the Moon.

Sol. During solar eclipse, moon comes between earth and sun.

∴ Distance between sun and moon

= Distance between sun and earth – Distance between earth and moon

$= 1.496 \times 10^{11} - 3.84 \times 10^{8} = 1496 \times 10^{8} - 3.84 \times 10^{8}$

$= 10^{8} \times (1496 - 3.84) = 10^{8} \times 1492.16 = 1.492 \times 10^{11}$km

Exercise 10.2

1. Express the following numbers in standard form.

(i) 0.000085 (ii) 0.00000000837 (iii) 4050000

(iv) 37860000000 (v) 0.00000000000000942

2. Express the following numbers in usual form:

(i) 2.5×10^{4} (ii) 5×10^{-8} (iii) 7.59×10^{-4}

(iv) 1.01001×10^{9} (v) 6.8×10^{12} (vi) 8.61492×10^{-6}

3. Express the number appearing in the following statements in standard form:

(i) Thickness of a thick paper is 0.07mm

(ii) 1 micron is equal to $\frac{1}{1000000}$m.

(iii) Charge on an electron is 0.000,000,000,000,000,000 16 coulomb.

(iv) The speed of light is 300,000,000m/sec.

(v) Mass of the Earth is 5,970,000,000,000,000,000,000,000kg

(vi) Diameter of a wire on a computer chip is 0.00000 3m

(vii) Thickness of class VIII Mathematics book is 20mm.

4. Compare the following:

If size of plant cell = 0.000012m

Diameter of a wire on a computer chip = 0.000003m

& Thickness of a piece of paper = 0.000016m

(i) Size of plant cell to thickness of piece of paper.

(ii) Size of plant cell to diameter of a wire on a computer chip.

(iii) Thickness of a piece of paper to diameter of a wire on a computer chip.

5. In a stack, there are 5 books each of thickness 26mm and 5 paper sheets each of thickness 0.014mm. What is the total thickness of the stack.

6. **Multiple choice Questions :**

(i) Standard form of 1040352 is.

(a) 1.040352×10^6 (b) 1.040352×10^7

(c) 10.40352×10^6 (d) 10.40352×10^7

(ii) Usual form of number 1.6×10^4 is:

(a) 16000 (b) 1600

(c) 160000 (d) 1.60000

(iii) Which of following is standard form of 0.00001225?

(a) 1.225×10^{-5} (b) 1.225×10^5

(c) 122.5×10^{-7} (d) 1.22×10^{-5}

(iv) Usual form of 3.2805×10^{-4} is:

(a) 302805 (b) 32805

(c) 0.32 805 (d) 0.00032805

(v) 3.03×10^6 is equal to:

(a) 303000 (b) 30300000

(c) 3030000 (d) 300000

Learning Outcomes

After completion of the chapter, students are able to

- *Use powers with negative integers as exponents.*
- *Understand laws of exponents.*
- *Compare very large and small numbers.*
- *Convert very large/small numbers in standard form.*

Answers

Exercise 10.1

1. (i) $\frac{1}{25}$ (ii) 81 (iii) 243

2. (i) $\frac{1}{9}$ (ii) $\frac{1}{64}$ (iii) 81 (iv) $\frac{1}{225}$ (v) $\frac{1}{60}$ (vi) $\frac{1}{4}$

3. (i) $\frac{45}{4}$ (ii) $\frac{1}{32}$ (iii) 50 (iv) 1 (v) $\frac{256}{81}$ (vi) 25

4. (i) p=2 (ii) 9

5. m = 11

6. (i) $\frac{625}{1296}$ (ii) $\frac{1}{16}$

7. (i) 625 (ii) $\frac{625 \times t^4}{2}$

8. (i) 2^{-9} (ii) 5^{-12}

9. (i) c (ii) b (iii) c (iv) b (v) b

Exercise 10.2

1. (i) 8.5×10^{-5} (ii) 8.37×10^{-9}

(iii) 4.05×10^{6} (iv) 3.786×10^{10}

(v) 9.42×10^{-15}

2. (i) 25000 (ii) 0.00000005 (iii) 0.000759

(iv) 1010010000 (v) 6800000000000 (vi) 0.00000861492

3. (i) 7.0×10^{-2}mm (ii) 1.0×1.0^{-6}m (iii) 1.6×10^{-19}coulomb

(iv) 3.0×10^{8}m/sec (v) 5.97×10^{24}kg (vi) 3.0×10^{-6}m

(vii) 2.0×10mm

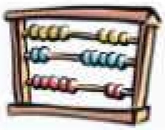
4. (i) Size of a plant cell is approximately $3/4$ of thickness of piece of paper.

(ii) Size of plant cell is approximately 4 times the diameter of a wire on a computer chip.

(iii) Thickness of piece of paper is approximately $\frac{16}{3}$ times the diameter of a wire on a computer chip.

5. 130.07mm

6. (i) a (ii) a (iii) a (iv) d (v) c



CHAPTER

11

Direct and Inverse Proportions

Learning Objectives

In this chapter you will learn:

- *About direct and inverse proportions.*
- *About various terms like demand and supply.*
- *About Growth in population and land available and other daily life problems.*

11.1 Introduction:-

In our day to day life, we come across many such situations where variation in one quantity brings variation in other quantity. For example,

1. If the number of articles (of same kind) purchased increases, the total cost also increases.
2. More the money deposited in a bank, more will be the interest.
3. With increase in speed of a vehicle, the time taken to cover the same distance will decrease.
4. More the number of workers, less time will be taken to complete the same work.

Observe that change in one quantity leads to change in other quantity. Consider an example, Aman prepares tea for herself. She uses 200 m*l* of water, 1 spoon of sugar, half spoon of tea leaves and 30m*l* of milk. How much quantity of each item will she need to make tea for five persons?

To answer such type of questions,we now study concept of variation in which we shall discuss two types of variation or proportion i.e. direct and inverse.

11.2 Direct Proportion

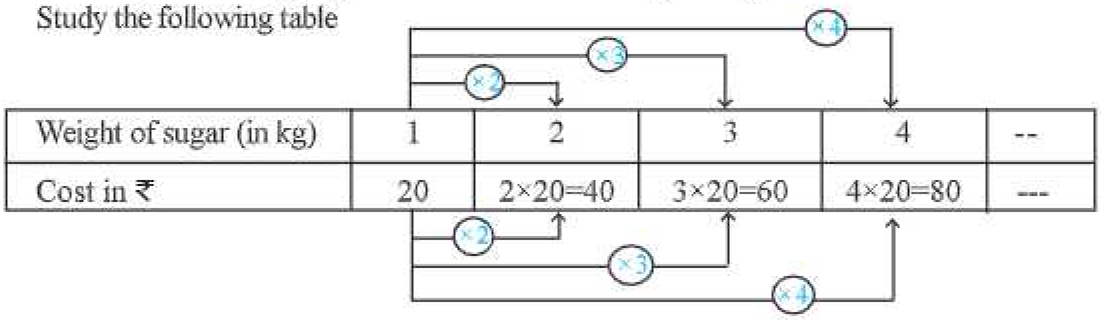
Two quantities are said to be in direct proportion when increase in one quantity (variable) leads to increase in other quantity (variable) in same ratio or when decrease in one quantity leads to decrease in other quantity in same ratio. Consider if cost of 1kg of sugar is ₹20 then what would be the cost of 4kg sugar?

We see that cost of 4 kg sugar is 4 times cost of 1kg of sugar. The cost will be 4 × 20= ₹80.

∴ Increase in quantity of sugar (i.e. 1 kg to 4kg) leads to increase in the cost of sugar (i.e. ₹20 to ₹80). So these quantities (variables) are in direct proportion.

Similarly cost of 6kg sugar will be ₹120 and 10 kg of sugar wil be ₹200.

Study the following table



Weight of sugar (in kg)	1	2	3	4	--
Cost in ₹	20	2×20=40	3×20=60	4×20=80	---

Observe that as weight of sugar increases its cost also increases in such a manner that their ratio remains same.

In above example, Ratio = $\frac{\text{weight of sugar}}{\text{cost}} = \frac{1}{20}$ (in all cases)

Consider one more example. Let the mileage of a car is 21km/litre. How far will it travel in 10 litres? The answer is 210km. How did we calculate it? Since in one litre it travel 21kms. So in 10 litres, it will travel (21 × 10) = 210 kms. Similarly in 20 litres it will travel (21 × 20) kms = 420 kms. Let the consumption of petrol be x litres and corresponding distance travelled is y km then complete the following table.

Petrol in litres (x)	1	10	20	25	30	45	50
Distance in km (y)	21	210	420	--	--	--	--

As the value of x increases, the value of y also increases. In such a way that the ratio $\frac{x}{y}$ does not change; it remains constant (say k). In this case, it is $\frac{1}{21}$.

We say x and y are in direct proportion if $\frac{x}{y} = k$ i.e $x = ky$

In this example, $\frac{1}{21} = \frac{10}{210} = \frac{20}{420} =$ ------

Where in numerator 1, 10, 20.... is the petrol consumed in litres (x) and in denominator 21, 210, 420, ... is the corresponding distance travelled in kms (y). So when x and y are **in direct proportion** we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where y_1 and y_2 are the values of y corresponding to the values x_1 and x_2 of x respectively.

The consumption of petrol and the distance travelled by a car is a case of **direct proportion**. Similarly the total amount spent and the number of articles purchased of same kind is also an example of **direct proportion.**

Now observe the following example. Let the present age of a boy, his father and mother are 15years, 45 years and 40 years respectively. Make the following table

	Present Age	**Age after Five years**	**Age after ten years**
Boy's age (B)	15	20	25
Father's age (F)	45	50	55
$\frac{B}{F}$	$\frac{15}{45} = \frac{1}{3}$	$\frac{20}{50} = \frac{2}{5}$	$\frac{25}{55} = \frac{5}{11}$

Similarly you can make a table to find the ratio of his age to the corresponding age of his mother. Now what do you observe? Do B and F increases (or decreases) together? The answer will be yes.

Now Is $\frac{B}{F}$ (ratio) same everytime? The answer is **No.** So they are not in direct proportion. You can repeat this activity with other friends and write down the observations.

Note : Variables increasing (or decreasing) together need not always be in direct proportion.

Let us consider some examples where we would use the concept of direct proportion.

Example 11.1 Which of the following quantities x and y are in the direct proportion?

(i)

x	**8**	**15**
y	**40**	**75**

(ii)

x	**15**	**35**
y	**25**	**45**

(iii)

x	**8**	**9**
y	**6**	**12**

Sol. As we know that two quantities x and y are in direct proportion if $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ or $\frac{x}{y} = k$ constant.

(i) Here $x_1 = 8$, $x_2 = 15$, $y_1 = 40$, $y_2 = 75$

Now, $\frac{x_1}{y_1} = \frac{8}{40} = \frac{1}{5}$ and $\frac{x_2}{y_2} = \frac{15}{75} = \frac{1}{5}$

$$\Rightarrow \quad \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{1}{5}$$

Thus, x and y are in direct proportion.

(ii) Here $x_1 = 15$, $x_2 = 35$, $y_1 = 25$, $y_2 = 45$

Now, $\frac{x_1}{y_1} = \frac{15}{25} = \frac{3}{5}$ and $\frac{x_2}{y_2} = \frac{35}{45} = \frac{7}{9}$

$$\Rightarrow \quad \frac{x_1}{y_1} \neq \frac{x_2}{y_2}$$

Thus, x and y are not in direct proportion.

(iii) Here $x_1 = 8$, $x_2 = 9$, $y_1 = 6$, $y_2 = 12$

Now, $\frac{x_1}{y_1} = \frac{8}{6} = \frac{4}{3}$ and $\frac{x_2}{y_2} = \frac{9}{12} = \frac{3}{4}$

$$\Rightarrow \quad \frac{x_1}{y_1} \neq \frac{x_2}{y_2}$$

Thus, x and y are not in direct proportion.

Example 11.2 **Find value of a in the following parts, if x and y are in direct proportion:**

(i)

x	8	13
y	48	a

(ii)

x	a	12
y	45	60

Sol. (i) Given x and y are in direct proportion.

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2} \qquad \Rightarrow \qquad \frac{8}{48} = \frac{13}{a}$$

$$\Rightarrow \quad 8 \times a = 13 \times 48 \;\Rightarrow\; a = \frac{13 \times \cancel{48}^{6}}{\cancel{8}_{1}} = 78$$

(ii) Given x and y are in direct proportion.

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2} \qquad \Rightarrow \qquad \frac{a}{45} = \frac{12}{60}$$

$$\Rightarrow \quad a \times 60 = 12 \times 45$$

$$\Rightarrow \quad a = \frac{\cancel{12}^{1} \times \cancel{45}^{9}}{\cancel{60}_{\cancel{5}_{1}}} = 9$$

Example 11.3. **The cost of 3 metres cloth is ₹105. Tabulate the cost of 5 metres, 7 metres, 10 metres and 13 metres of cloth of the same type.**

Sol. Suppose the length of cloth is x metres and its cost is ₹ y.

As the length of cloth increases, cost of the cloth also increases in the same ratio. It is a case of direct proportion.

x	3	5	7	10	13
y	105	y_2	y_3	y_4	y_5

So we will use the relationship $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

(i) Here $x_1 = 3$, $y_1 = 105$, $x_2 = 5$ is y_2=?

So using $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

We get $\frac{3}{105} = \frac{5}{y_2}$ or $3y_2 = 5 \times 105$ or $y_2 = \frac{5 \times \cancel{105}^{35}}{\cancel{3}_{1}} = 175$

(ii) Here $x_3 = 7$ so $\frac{3}{105} = \frac{7}{y_3}$ or $3y_3 = 7 \times 105$ or $y_3 = \frac{7 \times \cancel{105}^{35}}{\cancel{3}_{1}} = 245$

(iii) Here $x_4 = 10$ so $\frac{3}{105} = \frac{10}{y_4}$ or $3y_4 = 10 \times 105$ or $y_4 = \frac{10 \times \cancel{105}^{35}}{\cancel{3}_1} = 350$

(iv) Here $x_5 = 13$ so $\frac{3}{105} = \frac{13}{y_5}$ or $3y_5 = 13 \times 105$ or $y_5 = \frac{13 \times \cancel{105}^{35}}{\cancel{3}_1} = 455$

> Here, Note that after finding y_2 we can also use $\frac{x_2}{y_2} = \frac{x_3}{y_3}$
>
> i.e $\frac{5}{175} = \frac{7}{y_3}$ in place of $\frac{x_1}{y_1} = \frac{x_3}{y_3}$ to find the value of y_3 and so on, as ratio will remain constant.

Example 11.4 **Following are the car parking charges near a bus stop**

Upto 4 hours	**₹60**
Upto 8 hours	**₹100**
Upto 12 hours	**₹140**
Upto 24 hours	**₹180**

Check if the parking charges are in direct proportion.

Sol. Let us make the table from the given data, taking time upto hours as x and corresponding parking charges as ₹ y

Time in hours (x)	4	8	12	24
Charges in ₹ (y)	60	100	140	180

Now $\frac{x_1}{y_1} = \frac{1}{15}; \frac{x_2}{y_2} = \frac{8}{100} = \frac{2}{25}; \frac{x_3}{y_3} = \frac{12}{140} = \frac{3}{35}; \frac{x_4}{y_4} = \frac{24}{180} = \frac{2}{15}$

Here, we observe that although charges of parking increases with hours of parking, but their ratio is not same. So parking charges are not in direct proportion.

Example 11.5 **Complete the table if x and y are in direct proportion.**

x	2	4	x_3	24	x_5	x_6	50
y	7	y_2	28	y_4	98	112	y_7

Sol. As x and y are in direct proportion, So $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} =$...........

(i) $\frac{2}{7} = \frac{4}{y_2}$ so $2y_2 = 4 \times 7$ or $y_2 = \frac{4 \times 7}{2} = 14$

(ii) $\frac{2}{7} = \frac{x_3}{28}$ or $7x_3 = 28 \times 2$ or $x_3 = \frac{28 \times 2}{7} = 8$

(iii) $\frac{2}{7}=\frac{24}{y_4}$ or $2y_4 = 24 \times 7$ or $y_4 = \frac{24\times7}{2} = 84$

(iv) $\frac{2}{7}=\frac{x_5}{98}$ or $7x_5 = 2 \times 98$ or $x_5 = \frac{2\times98}{7} = 28$

(v) $\frac{2}{7}=\frac{x_6}{112}$ or $7x_6 = 2 \times 112$ or $x_6 = \frac{2\times112}{7} = 32$

(vi) $\frac{2}{7}=\frac{50}{y_7}$ or $2y_7 = 50 \times 7$ or $y_7 = \frac{50\times7}{2} = 175$

Example 11.6 A machine in a soft drink factory fills 840 bottles in 6 hours. How many bottles will it fill in 5 hours?

Sol. Let the number of bottles that will be filled in 5 hours be x.

Now we put the given information in the form of a table as shown below:

Number of Bottles	840	x
Time Taken (hours)	6	5

More the number of bottles, more time will be taken. So both terms are in direct proportion.

$$\therefore \quad \frac{840}{6}=\frac{x}{5} \quad \Rightarrow \quad x \times 6 = 840 \times 5$$

$$\Rightarrow \quad x = \frac{840\times5}{6} = 700$$

Thus, 700 bottles will be filled in 5 hours.

Example 11.7 Parveen has a road map with a scale of 1cm representing 22km. He drives on a road for 88km. What would be corresponding distance shown on the map?

Sol. Let the required distance be x cm.

Scale (in cm)	1	x
Distance (in km)	22	88

More the scale on the map, more the distance will be there. So both terms are in direct proportion.

$$\therefore \frac{1}{22} - \frac{x}{88} \Rightarrow x - \frac{88}{22} - 4$$

So, the distance shown on map will be 4 cm.

Example 11.8 If the weight of 12 sheets of a paper is 36 grams, how many sheets of the same paper will weight 300 grams?

Sol. Let the number of sheets with weight 300 gram be x. We put above information in the form of table as shown below.

Number of sheets	12	x
Weight of sheets (in grams)	36	300

As more the number of sheets of same type, more will be weight. So number of sheets and their weights are directly proportional to each other

So $\frac{12}{36} = \frac{x}{300}$ or $x = \frac{12 \times 300}{36} = 100$

Thus, the number of sheets of papers weighing 300 grams be 100.

Example 11.9 **A truck is moving with a uniform speed of 45 km/hour.**

(i) How far will it travel in one and half hour?

(ii) How much time truck will take to cover a distance of 495 km?

Sol. Let the distance travelled (in km) in one and half hour be x and time taken to travel 495 km is y mintues.

Distance travelled (in km)	45	x	495
Time taken(in minutes)	60	90	y

$\because$ 1 hour = 60 min

$\therefore 1\frac{1}{2}$ hours = 90 min

As speed of truck is uniform, so the distance covered would be directly proportional to time

(i) So we have $\frac{45}{60} = \frac{x}{90}$ or $x = \frac{45 \times 90}{60} = 67.5$

So truck will cover 67.5 km in one and half hour

(ii) Also $\frac{45}{60} = \frac{495}{y}$ or $y = \frac{495 \times 60}{45} = 660$

So time taken to cover 495 km is 660 minute = 11hours

Example 11.10 **A 5m 60cm high vertical pole casts a shadow 3m 20cm long. Find at the same time**

(i) The length of the shadow by another pole of height 10m 50cm.

(ii) The height of the pole whose shadow is 5m long.

Sol. Let the height of pole be x metres and length of shadow is y metres, from the given question we can form the table shown below. We know that 1m = 100cm

Height of pole (in metres)	5.6	10.5	x
Length of shadow (in metres)	3.2	y	5.0

$\because$ 5m 60cm = 5.6m

3m 20cm = 3.2m

10m 50cm = 10.5m

The case is of direct proportion, so $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

(i) Here, $\frac{5.6}{3.2} = \frac{10.5}{y} \Rightarrow 5.6 \times y = 3.2 \times 10.5$ so $y = \frac{3.2 \times 10.5}{5.6} = 6$

So length of shadow of pole having height 10.5m is 6m

(ii) $\frac{5.6}{3.2}=\frac{x}{5} \Rightarrow x \times 3.2 = 5.6 \times 5$ so $x = \frac{5.6\times 5}{3.2} = 8.75$

So Height of pole having shadow 5 m is 8.75 m.

Note: This Question can be solved by converting units in centimetres also.

Exercise 11.1

1. Which of the following quantities x and y are in direct variation?

(i)

x	9	12
y	54	72

(ii)

x	18	24
y	27	36

(iii)

x	12	14
y	20	24

(iv)

x	15	9
y	18	15

(v)

x	6	13
y	9	19.5

2. Find the value of missing quantity if x and y are in direct variation.

(i)

x	12	–
y	48	88

(ii)

x	13	7
y	–	56

(iii)

x	–	17
y	84	102

3. Complete the table if x and y are in direct proportion.

x	2	a	8	c	15	e
y	8	20	b	52	d	80

4. A machine in a factory fills 680 bottles in 5 hours. How many bottles will it fill in 3 hours?

5. Picture of Bacteria enlarged 60000 times attains a length of 3cm. What is the length of bacteria if it is enlarged 10000 times only.

6. A bus travels 40km in 30 minutes. If the speed of the bus remain same, how far can it travel in 3 hours?

7. If the weight of 25 precious stones is 50 grams. How many precious stones of the same type would weigh 4500 grams?

8. A 15 metres high pole casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

9. If the weight of 12 sheets of a thick paper is 40gm, how many sheets of the same paper would weigh $2\frac{1}{2}$ kg?

10. In a library, 126 copies of a certain book requires a shelf-length of 3.4 metres. How many copies of the same book would occupy a shelf length of 5.1 metres?

11. A mixture of paint is prepared by mixing 1 part of blue pigment with 5 parts of base. In the following table, find the parts of base that need to be added.

Parts of blue pigment	1	4	9	12
Parts of base	5	–	–	–

12. The cost of one litre of milk is ₹55. Tabulate the cost of 2, 4 and 10 litres of milk.

13. A train is running at the uniform speed of 75km/h.

    (i) How much distance will be covered in 20 minutes?

    (ii) How much time it will take to cover 250 km?

14. The cost of 12 chocolates is ₹180.

    (i) What is the cost of 18 such chocolates?

    (ii) How many such chocolates will be there in ₹330?

15. **Multiple Choice Questions :**

    (i) Find 'a' if the given quantities are in direct variation.

x	12	18
y	a	30

(a) 15 (b) 20 (c) 18 (d) 16

(ii) If x and y are in direct variation then which of the following is true?

(a) $xy = k$ (b) $x + y = k$ (c) $x - y = k$ (d) $\frac{x}{y} = k$

(iii) If the cost of 5 pencils is ₹15. Find the cost of 12 such pencils.

(a) ₹15 (b) ₹18 (c) ₹36 (d) ₹24

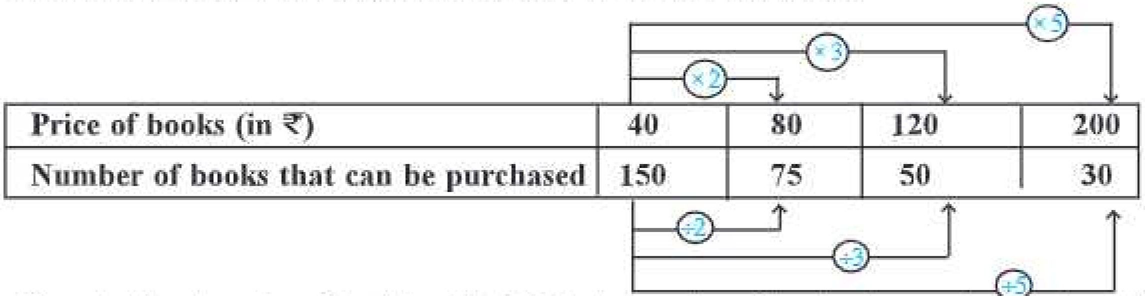
(iv) A car is moving at a uniform speed of 75km/h. How for it will travel in 3 hours?

(a) 300 km (b) 225 km (c) 275 km (d) 150 km

11.3 Inverse proportion:-

Two quantities (variables) are said to be in inverse proportion if increase in one quantity leads to decrease in other quantity in same ratio or vice-versa. For example, (i) If the number of workers increases, time taken to finish the same work decreases. (ii) if the speed of a vehicle increases, the time taken to cover the same distance decreases.

Let us consider an example. A school wants to spend ₹6000 on mathematics books. If the rate of one book is ₹40, then how many books could be bought? Clearly 150 books can be bought. If the price of book is more than ₹40, then the number of books which could be purchased with the same amount of money will be less than 150. Observe the following table.



Price of books (in ₹)	**40**	**80**	**120**	**200**
Number of books that can be purchased	**150**	**75**	**50**	**30**

If we double the price of book i.e. 40×2=80, the number of books that can be purchased by same amount will be $150 \times \frac{1}{2} = 75$ (*i.e* halved). if we increase the price by three times i.e. 40×3=120, the number of books can be purchased with same amount will be $150 \times \frac{1}{3} = 50$ (*i.e* one third). We observe that price of book increases and number of books purchased by same amount decreases.

Note: The product of the corresponding values of two quantities is fixed constant i.e.
$40 \times 150 = 80 \times 75 = 120 \times 50 = 200 \times 30 = 6000$

If we represent the price of one book as x and number of books purchased as y, then as x increases, y decreases in same ratio and vice versa. It is important to note that the product xy remains fixed constant.

> **Note:** So two quantities x and y are said to vary inversely, If there exist a relation of the type $xy = k$, between them, where k is a constant.

If for x_1 , x_2 values of x there are corresponding y_1 and y_2 values of y then $x_1y_1 = x_2 y_2 = k$ (say) or $\frac{x_1}{x_2} - \frac{y_1}{y_2}$ we say that x and y are in inverse proportion.

> When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$ or $x = ky$ (where k is a constant of proportionality) and
> when two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$ or $x = \frac{k}{y}$ (where k is a constant of proportionality)

Let us consider some examples where we use the concept of inverse proportion.

Example 11.11. Which of the following quantities are in inverse proportion?

(i)

x	**12**	**36**
y	**15**	**5**

(ii)

x	**18**	**54**
y	**27**	**12**

(iii)

x	**24**	**8**
y	**12**	**36**

Sol. We know, if x and y are in inverse proportion then $x_1y_1 = x_2y_2$

(i) Here $x_1 = 12, x_2 = 36, y_1 = 15, y_2 = 5$

$\therefore$ $x_1 y_1 = 12 \times 15 = 180$ and $x_2 y_2 = 36 \times 5 = 180$

$\Rightarrow$ $x_1 y_1 = x_2 y_2 = 180$

Thus x and y are in inverse proportion.

(ii) Here $x_1 = 18, x_2 = 54, y_1 = 27, y_2 = 12$

$\therefore$ $x_1 y_1 = 18 \times 27 = 486$ and $x_2 y_2 = 54 \times 12 = 648$

$\Rightarrow$ $x_1 y_1 \neq x_2 y_2$

Thus x and y are not in inverse proportion.

(iii) Here $x_1 = 24, x_2 = 8, y_1 = 12, y_2 = 36$

$\therefore$ $x_1 y_1 = 24 \times 12 = 288$ and $x_2 y_2 = 8 \times 36 = 288$

$\Rightarrow$ $x_1 y_1 = x_2 y_2 = 288$

Thus x and y are in inverse proportion.

Example 11.12. **Find the value of 'a' if x varies inversely to y.**

(i)

x	9	36
y	a	6

(ii)

x	15	a
y	24	18

Sol. (i) Given x and y are in inverse proportion.

$\therefore \quad x_1 y_1 = x_2 y_2$

$\Rightarrow \quad 9 \times a = 36 \times 6 \quad \Rightarrow \quad a = \frac{36\times6}{9} = 24$

(i) Given x and y are in inverse proportion.

$\therefore \quad x_1 y_1 = x_2 y_2$

$\Rightarrow \quad 15 \times 24 = a \times 18 \quad \Rightarrow \quad a = \frac{15\times24}{18} = 20$

Example 11.13. **If 15 men can build a wall in 24 hours, how many men will be required to do the same work in 30 hours?**

Sol. Let the number of men required to build the wall in 30 hours be 'a'.

We have the following table:

Number of men	15	a
Number of hours	24	30

Obviously, more the number of men, less the number of hours required to build the wall. So, both quantities are in inverse proportion.

$\therefore \quad 15 \times 24 = a \times 30$

$\Rightarrow \quad a = \frac{15\times24}{30} = 12$

Thus, 12 men will be required to finish the work in 30 hours.

Example 11.14. **A School has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?**

Sol. Let timing of each period be x minutes if 9 periods are there.

We have the following table:

Number of Periods	8	9
Timing (in min)	45	x

Obviously, more the number of periods less the timing of periods. So, both quantities are in inverse proportion.

$\Rightarrow \quad 8 \times 45 = 9 \times x$

$$\Rightarrow \quad x = \frac{8 \times 45}{9} = 40$$

Thus, each period will be of 40 minutes, if 9 periods are there.

Example 11.15. 8 pipes are required to fill a tank in 2 hours. How long will it take if (i) 6 pipes (ii) 12 pipes of same type are used to fill the tank.

Sol. Let the required time to fill the tank be x_1 and x_2 minutes corresponding to 6 pipes and 12 pipes.

Thus we have the following table

Number of pipes	8	6	12
Time taken to fill tank (in minute)	120	x_1	x_2

1 hour = 60 min
2 hours = 60 × 2
= 120 min

As number of pipes decreases, time taken will increases and as the number of pipes increases, the time taken will decreases, so this is case of inverse proportion.

(i) $\therefore \quad 8 \times 120 = 6 \times x_1$ (As in inverse porportion $x_1y_1 = x_2 y_2$)

$$\Rightarrow \quad x_1 = \frac{8 \times 120}{6} = 160$$

Thus, time taken by 6 pipes to fill the tank will be 160 minutes i.e. 2 hours 40 minutes.

(ii) $8 \times 120 = 12 \times x_2$

$$\Rightarrow \quad x_2 = \frac{8 \times 120}{12} = 80$$

Thus, time taken by 12 pipes to fill the tank is 80 minutes i.e. 1 hour and 20 minutes.

Example 11.16. There are 100 students in a hostel. Food provision for them is for 21 days. Out of 100, 25 students have gone to their home for one month. How long will the provision of food last for the remaining students?

Sol. Suppose the provision last for y days when the number of students remains 100 – 25 = 75. we have the following table.

Number of students	100	75
Number of days	21	y

Notice that it is a case of inverse proportion

So $100 \times 21 = 75 \times y$

$$\text{hence } y = \frac{100 \times 21}{75} = 28$$

Thus, the provision will last for 28 days.

Exercise 11.2

1. Which of the following are in inverse proportion?

(i)

x	8	6
y	9	12

(ii)

x	15	5
y	18	56

(iii)

x	24	8
y	20	60

(iv)

x	12	18
y	24	20

(v)

x	25	10
y	20	50

2. Find the value of 'a' if x and y are in inverse proportion.

(i)

x	16	8
y	9	a

(ii)

x	12	27
y	a	4

(iii)

x	25	a
y	8	20

3. If a box of pens is given to 25 children, they will get 3 pens each. How many pens would each child get, if the number of children is reduced by 10?

4. A batch of tablets were packed in 10 boxes with 6 tablets in each box. If the same batch is packed using 12 tablets in each box. How many boxes would be needed?

5. A company requires 36 machines to make a product in 54 days. How many machines would be required to make the same product in 81 days?

6. 6 pipes are required to fill a tank is 1h 20 min. How long will it take if only 5 pipes of the same type are used?

7. A train takes 2 hours to reach a destination at speed of 60km/h. How long will it take to reach the destination at 80km/h?

8. A car can finish a certain journey in 10 hours at the speed of 32km/h. By how much should its speed be increased so that it may take only 8 hours to cover the same distance?

9. Two persons could fit the AC unit in a house in 2 hours. One person fell ill before the work started, how long would the job take now?

10. Arrangement of tables & chairs in an exam hall is done by 10 workers in 2 hours. How many workers will be required to do the same work in 4 hours?

11. A factory requires 42 machines to produce a given number of articles in 63 days? How many more machines would be required to produce the same number of articles in 54 days?

12. There are 200 students in a hostel. Food provision for them lasts for 10 days. How long will these provision last, if 50 more students join the hostel?

13. If a box of sweets is divided among 24 children, they will get 4 sweets each. How many would each get, if the number of children is reduced by 8.

14. In a television game show, the prize money of ₹1,00,000 is to be divided equally among winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportion to the numbers of winners.

No. of winners	1	2	4	5	8	10
Prize money of each winner	100000	50000	-	-		-

15. **Multiple Choice Questions :**

(i) If x and y are in inverse proportion then which of the following is true?

(a) $xy = k$ (b) $\frac{x}{y}=k$ (c) $x + y = k$ (d) $x - y = k$

(ii) Find a if x and y are in inverse proportion:

x	30	24
y	12	a

(a) 18 (b) 20 (c) 15 (d) 16

(iii) 10 men complete a work in 20 days. In how many days 25 men will complete the work?

(a) 4 (b) 16 (c) 12 (d) 8

(iv) A farmer has enough food to feed 20 animals in his cattle for 6 days? How long would the food last if there were 10 more animals in his cattle?

(a) 3 (b) 8 (c) 4 (d) 10

Learning Outcome

After completion of the chapter, the students are now able to:

- *Understand direct and inverse porportion.*
- *Understand various terms like demand and supply.*
- *Understand about growth in population and land available and their use in daily life.*

Answers

Exercise 11.1

1. (i), (ii) 2. (i) 22 (ii) 104 (iii) 14 3. a = 5, b = 32, c = 13, d = 60, e = 20

4. 408 bottles 5. 0.5cm 6. 240km 7. 2250 stones

8. 22.5m 9. 750 sheets 10. 189 copies

11. 20, 45, 60 12. ₹110, ₹220, ₹550 13. (i) 25km (ii) 3hr 20 min

14. (i) ₹270 (ii) 22 15. (i) b (ii) d (iii) c (iv) b

Exercise 11.2

1 (i), (iii), (v) 2. (i) 18 (ii) 9 (iii) 10 3. 5pen 4. 5 boxes 5.24 machines

6. 1 hour 48 min 7. 1 hour 30 min 8. 8km/h 9. 4 hours 10. 5 workers

11. 49 machines 12. 8 days 13. 6 sweets

14. 25,000; 20,000; 12,500; 10,000 15. (i) a (ii) c (iii) d (iv) c

CHAPTER

12

Factorisation

Learning Objectives

In this chapter, you will learn:

- *About factors of algebraic expressions.*
- *How to make factors.*
- *About different methods of factorisation.*

12.1 Introduction

In earlier classes, we have learnt about factors of composite numbers, let us consider a natural number, (say)

$42 = 1\times42$ Here 1 and 42 are factors of 42.

Or $42 = 1\times2\times21$ Here 1, 2 and 21 are factors of 42.

Or $42 = 1\times2\times3\times7$ Here 1, 2, 3 & 7 are factors of 42.

As we know that 1 is neither composite nor prime number.

Therefore 2, 3, 7 are prime factors of 42. Similarly the prime factors of 30 are 2, 3, & 5 and the prime factors of 70 are 2, 5, & 7.

Similarly algebraic expressions can also be expressed as the product of their factors.

12.2 Factors of Algebraic Expression:

In Class VII, we have learnt that terms of algebraic expressions are formed as product of factors. For example, in algebraic expressions 3xy, the term 3xy is formed by factors 3, x and y i.e. $3xy = 3\times x\times y$ here 3, x and y are factors of 3xy.

We observe that these factors 3x & y of 3xy which further cannot be expressed as a product of factors, called '**Irreducible factors**'.

Note that $3\times(xy)$ is not irreducible form of 3xy. Since the factor xy can be further expressed as a product of x and y i.e. $xy = x\times y$

Since 3xy can also be written as $1\times3\times x\times y$. Note 1 is a factor of 3xy. Infact 1 is a factor of every term. But we do not show 1 as a separate factor of any term, unless it is specially required.

Let us consider the expression 5x (x+3).

It can be written as a product of factors 5, x and (x+3) i.e. $5x (x+3) = 5 \times x \times (x + 3)$

Similarly Irreducible factors of 12x (y+3) (z+5) are 2, 2, 3, x, (y+3) and (z+5)

Also, the product of 2x+3 and $2x-3 = (2x + 3) (2x-3) = 4x^2 - 9$

We can say that 2x+3 and 2x–3 are factors of $4x^2-9$. we can write it as $4x^2-9= (2x+3) (2x-3)$

12.3 What is Factorisation

As we have learnt, when an algebraic expression can be written as product of two or more expressions then each of these expression is called a factor of the given expression. To find factors of a given expression means to obtain two or more expressions whose product is the given expression.

The process of writing an algebraic expression as the product of two or more algebraic expressions is called factorisation. Thus, factorisation is the reverse process of multiplication.

For Example

Product of 5xy and (3xy–7) is $5xy\,(3xy - 7) = 15x^2y^2 - 35xy$ and factors of $15x^2y^2 - 35xy$ are 5xy and (3xy–7)

Similarly product of (2a+3b) and (2a–3b) is $(2a + 3b)\,(2a - 3b) = 4a^2 - 9b^2$ and factors of $4a^2 - 9b^2$ are 2a + 3b and 2a – 3b.

Now we shall learn the factorisation of expressions by different methods.

12.3.1 Method of finding Common Factors

In this section, we shall learn about how to take out the common factor (s) and use the Distributive property

$$\begin{aligned} ab \pm ac &= a \times b \pm a \times c \\ &= a \times (b \pm c) \\ &= a\,(b \pm c) \end{aligned}$$

Example 12.1. Factorise : 2x+6

Solution- We shall write each term as a product of irreducible factors.

Here $2x = 2 \times x$ and $6 = 2 \times 3$

Hence $2x + 6 = 2 \times x + 2 \times 3$

Observe that both terms have '2' as common factor.

$\therefore\ 2x + 6 = 2 \times (x + 3) = 2\,(x + 3)$

Example 12.2. Factorise : $7a^2 + 14a$

Solution:- Here $7a^2 = 7 \times a \times a$ and $14a = 2 \times 7 \times a$

Observe that both terms have '7' and 'a' as common factors

$$\begin{aligned} \text{Hence } 7a^2 + 14a &= 7 \times a \times a + 2 \times 7 \times a \\ &= 7 \times a \times (a + 2) \\ &= 7a\,(a + 2) \end{aligned}$$

Example 12.3. Factorise : $5x^2y - 15xy^2$

Solution : We know $5x^2y = 5 \times x \times x \times y$ and $15xy^2 = 3 \times 5 \times x \times y \times y$

Both terms have 5, x and y as common factors.

$$\begin{aligned} \text{Hence } 5x^2y - 15xy^2 &= 5 \times x \times x \times y - 3 \times 5 \times x \times y \times y \\ &= 5 \times x \times y\,(x - 3y) \\ &= 5xy\,(x - 3y) \end{aligned}$$

Example 12.4 **Factorise : $14x^2y^2 + 10x^2y + 8xy^2$**

Solution : We know $14x^2y^2 = 2 \times 7 \times x \times x \times y \times y$

$10x^2y = 2 \times 5 \times x \times x \times y$

$8xy^2 = 2 \times 2 \times 2 \times x \times y \times y$

Therefore, $14x^2y^2 + 10x^2y + 8xy^2 = 2 \times 7 \times x \times x \times y \times y + 2 \times 5 \times x \times x \times y + 2 \times 2 \times 2 \times x \times y \times y$

$= 2 \times x \times y \times (7 \times x \times y + 5 \times x + 2 \times 2 \times y)$

Here three terms have 2, x and y as common irreducible factors.

$= 2xy\,(7xy + 5x + 4y)$

Example 12.5 **Factorise : $4x^2 + 9x + 18$**

Solution : Here $4x^2 = 2 \times 2 \times x \times x$, $9x = 3 \times 3 \times x$ and $18 = 2 \times 3 \times 3$

We observe that there is no common term in the given three terms:

In such cases, 1 is the common factor and the given expression will be written as it is.

Hence $4x^2 + 9x + 18 = 2 \times 2 \times x \times x + 3 \times 3 \times x + 2 \times 3 \times 3$

$= 1\,(2 \times 2 \times x \times x + 3 \times 3 \times x + 2 \times 3 \times 3)$

$= 4x^2 + 9x + 18$

Note: We cannot factorise such algebraic expression if there is no common factor among the terms.

Example 12.6 **Factorise the following:**

(i) $3a\,(x + y) - 5b\,(x + y)$ **(ii) $2(x - y)^2 + 5\,(x - y)$**

(iii) $6x\,(2a - 3b) - 5y\,(3b - 2a)$

Solution : (i) We have, $3a\,(x + y) - 5b\,(x + y)$

Observe that, we have two terms $3a\,(x + y)$ and $5b\,(x + y)$

Clearly, $(x + y)$ is a common factor among them

Hence, $3a\,(x + y) - 5b\,(x + y) = (x + y)\,(3a - 5b)$

(ii) We have, $2(x - y)^2 + 5\,(x - y)$

Here, we have two terms $2(x - y)^2$ and $5\,(x - y)$.

Clearly $(x-y)$ is a common factor among them

Hence, $2\,(x - y)^2 + 5\,(x - y) = (x - y)\,[2\,(x - y) + 5] = (x - y)\,(2x - 2y + 5)$

(iii) We have, $6x\,(2a-3b) - 5y\,(3b - 2a)$

Rewrite second term $-5y\,(3b - 2a) = -5y\,[-(2a - 3b)]$

$= 5y\,(2a - 3b)$

Hence, $6x\,(2a - 3b) - 5y\,(3b - 2a) = 6x\,(2a - 3b) + 5y\,(2a - 3b)$

$= (2a - 3b)\,(6x+5y)$. (Taking common $(2a - 3b)$)

12.3.2 Factorisation by Regrouping Terms

In last section, we have discussed the method of finding common factors in which given algebraic expressions have common factors. But sometimes, we have such algebraic expressions which cannot be factorised directly that can be factorised by regrouping terms. We shall discuss such factorisation as follows:

Consider the expression $3x + 3 + 4xy + 4y$

Here you can notice that no factor is common to all the terms but first two terms have common factor 3 and the last two terms have common factors 4 and y. So in this type of sums, we regroup the tems on follows:

In this case, we write $(3x + 3)$ and $(4xy + 4y)$

$$\begin{aligned} 3x + 3 &= 3 \times x + 3 \times 1 \\ &= 3 \times (x + 1) \\ \text{and } 4xy + 4y &= 4 \times x \times y + 4 \times y \\ &= 4 \times x \times y + 4 \times y \times 1 \\ &= 4y\,(x + 1) \end{aligned}$$

Hence $3x + 3 + 4xy + 4y = (3x + 3) + (4xy + 4y) = 3\,(x+1) + 4y\,(x+1)$

Now here we have two terms

Observe, we have common factor $(x+1)$ in both terms. Combining these two terms.

$3x + 3 + 4xy + 4y = 3\,(x + 1) + 4y\,(x + 1) = (x + 1)\,(3 + 4y)$

Thus, The algebraic expression $3x + 3 + 4xy + 4y$ is now in the form of a product of two irreducible factors.

Hence $(x+1)$ and $(3+4y)$ are the factors of $3x + 3 + 4xy + 4y$.

Suppose that the above expression was given as $3x + 4y + 4xy + 3$; Then it may not be easy to factorise directly. For that we have to rearrange the terms after observing.

Rearranging the expression as $3x + 3 + 4xy +4y$ allows us to form groups $(3x+3)$ and $(4xy +4y)$ leading to factorisation. This process is called regrouping. Regrouping may be possible in more than one way. Suppose, we can regroup the expression as $3x + 4xy + 3 + 4y$.

$$\begin{aligned} \text{Now } 3x + 4xy + 3 + 4y &= (3x + 4xy) + (3+4y) \\ &= x\,(3 + 4y) + 1\times(3 + 4y) \\ &= (3 + 4y)\,(x + 1) \end{aligned}$$

The factors are the same, although they appear in different order.

Example 12.7. Factorise the following expressions.

(i) $5xy + 7y - 5x^2 - 7x$ **(ii) $ax - ay + bx - by$**

(iii) $5p^2 - 8pq - 10p + 16q$

Solution : (i) Step I. Check if there is a common factor among all terms. Here, we have no common factor.

Step II. Observe that the first two terms have a common factor y;

i.e. $5xy + 7y = y(5x+7)$ ....(1)

Now, observe that in last two terms, there is '$-x$' as a common factor.

$$-5x^2 - 7x = -x\,(5x+7) \;\ldots\ldots\ldots(2)$$

putting (1) & (2) together, we have

$$\begin{aligned} 5xy + 7y - 5x^2 - 7x &= y(5x + 7) - x\,(5x + 7) \\ &= (5x + 7)\,(y - x) \quad \text{[Taking } (5x + 7) \text{ common]} \end{aligned}$$

(ii) Given that

$$\begin{aligned} & ax - ay + bx - by \\ =\; & a\,(x - y) + b\,(x - y) \\ =\; & (x - y)\,(a + b) \qquad \text{[Taking } (x - y) \text{ common]} \end{aligned}$$

Here taking common a from first two terms and b from next two terms

(iii) Given that

$$\begin{aligned} & 5p^2 - 8pq - 10p + 16q \\ =\; & p(5p - 8q) - 2\,(5p - 8q) \\ =\; & (5p - 8q)\,(p - 2) \qquad \text{[Taking common } (5p - 8q)\text{)} \end{aligned}$$

Taking common 'p' from first two terms and '-2' from next two terms

Exercise 12.1

1. Find the common factors of the given terms.

(i) $15x, 25$

(ii) $3y, 33xy$

(iii) $7pq,\ 28p^2q^2$

(iv) $2x, 3x^2, 5$

(v) $4abc,\ 24ab^2,\ 12a^2b$

(vi) $12x^3, -6x^2, 36x$

(vii) $4xy^3,\ 10x^3y^2,\ 8x^2y^2z$

(viii) $3x^2, 5x, 9$

2. Factorise the following expressions:

(i) $6x-48$

(ii) $7p-14q$

(iii) $-24z+30z^2$

(iv) $18\ell^2m+27a\,\ell m$

(v) $25x^2y^2z - 15x^2yz^2$

(vi) $a^2bc + ab^2c + abc^2$

(vii) $px^2y + qxy^2 + rxyz$

(viii) $10pq-15qr+20rp$

3. Factorise:

(i) $3a(2p-3q) -5b(2p - 3q)$

(ii) $15a\,(x^2+y^2) -10b\,(x^2+y^2)$

(iii) $4\,(x + y)^2 + 2\,(x + y)$

(iv) $(2a - 5b)^2 + 10b - 4a$

(v) $(5\ell + 3m)^2 -5l - 3m$

4. Factorise:

(i) $x^2 + xy + 6x + 6y$

(ii) $y^2-yz - 3y+ 3z$

(iii) $12xy - 8x + 3y-2$

(iv) $a^2b -ab^2 +4a - 4b$

(v) $x^3 - 6x^2 +x - 6$

(vi) $a^2 + ab\,(1+b) +b^3$ (Hint: First multiply middle term)

(vii) $3px - 6py - 8qy+ 4qx$

(viii) $r-7 + 7pq - pqr$

5. **Multiple choice Questions:**

(i) Common factor of 10xy and 12y is

(a) 10x (b) 2xy (c) 2y (d) 2x

(ii) Common factor of $5a^2b$ and $9xy^2$ is

(a) 1 (b) 0 (c) ab xy (d) ax

(iii) $8p^2 - 20pq + 28\, p^2q$

(a) $4p\,(2p + 5q - 7pq)$ (b) $4p\,(2p - 5q + 7p^2q)$

(c) $4q\,(2p - 5q + 7q)$ (d) $4p\,(2p - 5q + 7pq)$

(iv) $3(2l - m)^2 + (2l - m) =$

(a) $(2l - m)\,(6l - 3m + 1)$ (b) $(2l - m)\,(6l - 2m)$

(c) $3\,(2l - m)\,(2l - m + 1)$ (d) $(2l - m)\,(3 + 2l - m)$

(v) $p^2 - pq + pr - qr =$

(a) $(p - r)\,(p + q)$ (b) $(p + r)\,(q - p)$

(c) $(p + r)\,(p - q)$ (d) $(p - q)\,(r - p)$

12.3.3 Factorisation using Algebraic Identities

In last section, we have learnt about the factorisation of algebraic expressions using common factors method and regrouping method. In this section, we shall discuss the factorisation using algebraic identities.

We know that

(i) $(a + b)^2 = a^2 + 2ab + b^2$............(I) (ii) $(a - b)^2 = a^2 - 2ab + b^2$............(II)

(iii) $(a + b)\,(a - b) = a^2 - b^2$ ....(III)

When the given expression is in the form of $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ or $a^2 - b^2$ then it can be converted in the form of $(a + b)^2$, $(a - b)^2$ or $(a - b)\,(a + b)$ respectively. then it can be factorised by using above identity. The following examples illustrate it.

Example 12.8. Factorise :

(i) $x^2 + 10x + 25$ (ii) $y^2 - 6y + 9$ (iii) $25m^2 + 30m + 9$

(iv) $9p^2 - 24p + 16$ (v) $p^4 + 2p^2q^2 + q^4$

Solution : Given expression has three terms. therefore it does not fit identity (iii). Here two terms i.e. first and third terms are perfect squares with positive sign so it is of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

(i) We have , $x^2 + 10x + 25 = x^2 + 10x + 5^2$

First and third terms are perfect squares in form of a^2 and b^2 (where $a = x$, $b = 5$)

and middle term is in the form of $2ab = 2\,(x)\,(5)$

$\therefore\ x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$

Thus $x^2 + 10x + 5 = (x + 5)^2$ $\quad [\because a^2 + 2ab + b^2 = (a+b)^2]$

(ii) We have, $y^2 - 6y + 9 = y^2 - 6y + 3^2$

Here, First and third terms are perfect squares in form of a^2 and b^2 where $a = y$, $b = 3$ and middle term is in form of $2ab = 2(y)(3)$

$y^2 - 6y + 3^2 = y^2 - 2(y)(3) + 3^2$

Thus $y^2 - 6y + 9 = (y - 3)^2$ $\quad [\because a^2 - 2ab + b^2 = (a - b)^2]$

(iii) $25m^2 + 30m + 9 = 5^2m^2 + 30m + 3^2$

$= (5m)^2 + 30m + (3)^2$ [Here $a = 5m$, $b = 3$]

$= (5m)^2 + 2(5m)(3) + (3)^2 = (5m+3)^2$

Thus, $25m^2 + 30m + 9 = (5m + 3)^2$

(iv) $9p^2 - 24p + 16 = 3^2p^2 - 24p + 4^2$

$= (3p)^2 - 24p + 4^2$ [Here $a = 3p$, $b = 4$]

$= (3p)^2 - 2(3p)(4) + 4^2 = (3p - 4)^2$

Thus, $9p^2 - 24p + 16 = (3p - 4)^2$

(v)We have, $p^4 + 2p^2q^2 + q^4 = (p^2)^2 + 2p^2q^2 + (q^2)^2$

$= (p^2)^2 + 2(p)^2(q^2) + (q^2)^2$

$= (p^2 + q^2)$

Thus, $p^4 + 2p^2q^2 + q^4 = (p^2 + q^2)^2$

Example 12.9. Factorise the following :

(i) a^2-25 **(ii) $4x^2-9$** **(iii) $49x^2-36y^2$** **(iv) $\frac{9}{16}x^2y^2 - \frac{16}{25}z^2$**

(v) $16x^5 - 144x^3$

Solution : Given expressions has two terms, both are perfect squares with second term is negative. So these are of form $a^2 - b^2 = (a - b)(a + b)$

(i) $a^2-25 = a^2-5^2 = (a - 5)(a + 5)$

(ii) $4x^2 - 9 = 2^2x^2 - 3^2 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)$

(iii) $49x^2 - 36y^2 = 7^2x^2 - 6^2y^2 = (7x)^2 - (6y)^2 = (7x - 6y)(7x + 6y)$

(iv) $\frac{9}{16}x^2y^2 - \frac{16}{25}z^2 = \frac{3^2}{4^2}x^2y^2 - \frac{4^2}{5^2}z^2 = \left(\frac{3}{4}xy\right)^2 - \left(\frac{4}{5}z\right)^2$

$$= \left(\frac{3}{4}xy - \frac{4}{5}z\right)\left(\frac{3}{4}xy + \frac{4}{5}z\right)$$

(v) $16x^5 - 144x^3 = 16x^3(x^2-9)$ [Taking $16x^3$ common]

$= 16x^3(x^2-3^2) = 16x^3(x-3)(x+3)$

Example 12.10. Factorise the following :

(i) $a^4 - b^4$ (ii) $p^4 - 81$ (iii) $16x^4 - 1$

Solution : (i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2-b^2)\ (a^2+b^2)$ [Applying identity $a^2-b^2 = (a-b)\ (a+b)$]

$= (a-b)\ (a+b)\ (a^2+b^2)$ [Applying identity $a^2-b^2 = (a-b)\ (a+b)$]

(ii) $p^4 - 81 = (p^2)^2 - 9^2$

$= (p^2 - 9)\ (p^2 + 9)$

$= (p^2 - 3^2)\ (p^2 + 9)$ [Applying Identity $a^2-b^2 = (a-b)\ (a+b)$]

$= (p-3)\ (p+3)\ (p^2+9)$ [Applying Identity $a^2-b^2 = (a-b)\ (a+b)$]

(iii)$16x^4 - 1 = 4^2(x^2)^2 - 1^2$

$= (4x^2)^2 - (1)^2 = (4x^2 - 1)\ (4x^2+1)$

$= (2x^2 - 1^2)\ (4x^2 + 1)$

$= [(2x)^2 - 1^2]\ (4x^2 + 1)$

$= (2x - 1)\ (2x + 1)\ (4x^2 + 1)$

Example 12.11. Factorise the following :

(i) $x^2-2xy+y^2-z^2$ (ii) $25a^2 - 4b^2 + 28bc - 49c^2$

(iii) $x^4 - (x - 2)^4$

Solution : (i) We have, $x^2-2xy + y^2 - z^2$

$= (x^2 - 2xy + y^2) - z^2$

$= (x-y)^2 - z^2$ [$\because a^2-2ab + b^2 = (a-b)^2$]

$= (x - y - z)\ (x - y + z)$ [$\because a^2-b^2 = (a-b)\ (a+b)$]

(ii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 25a^2 - [(2b)^2 - 2 \times (2b) \times (7c) + (7c)^2]$

$= 25a^2 - (2b-7c)^2 = (5a)^2 - (2b - 7c)^2$ [$\because a^2-2ab + b^2 = (a-b)^2$]

$= [5a - (2b - 7c)]\ [5a + (2b - 7c)]$ [$\because a^2-b^2 = (a-b)\ (a+b)$]

$= [5a - 2b + 7c)]\ (5a + 2b - 7c)$

(iii) $x^4 - (x - 2)^4$

$= (x^2)^2 - [(x - 2)^2]^2$

$= [x^2 - (x - 2)^2]\ [x^2 + (x-2)^2]$

$= [x - (x - 2)\ (x + x - 2)]\ [(x^2 + (x - 2)^2]$

$= (x - x + 2)\ (x + x - 2)\ [x^2 + (x - 2)^2]$

$= 2\ (2x - 2)\ [x^2 + (x-2)^2]$

12.3.4 FACTORS OF THE FORM (x+a) (x+b)

In last section, we have learnt the factorization of algebraic expressions using algebraic identities. In this section, we shall discuss such type of algebraic expression which are in the form of $x^2 + \ell x + m$ i.e. which does not contain two perfect square terms. Let us discuss how we factorise such expressions.

> For factorising an algebraic expression of the type $x^2 + \ell x + m$, we use identity $x^2 + (a + b)x + ab = (x + a)(x + b)$. For this we find two factors a and b of m (i.e. the constant term) such that
>
> $ab = m$ and $a + b = \ell$
>
> i.e. Sum of both factors = Coefficient of x and Product of both factors = Constant term

Example 12.12. Factorise : $x^2 + 14x + 33$

Solution : Step I. Find the two numbers whose product is constant term (i.e. 33) and sum is the coefficient of x (i.e. 14)

Step II. Since the product is positive. Therefore both factors of 33 will be either positive or negative.

Step III. But the sum is positive so both factors of 33 will be positive.

Product = 33

1 × 33	**(–1) × (–33)**
3 × 11	**(–3) × (–11)**

Step IV. Factors of 33 are 1×33, 3×11 one pair from above factors is taken whose sum is 14 i.e. 3 & 11.

Therefore the required numbers are 3 & 11

$\therefore \quad x^2 + 14x + 33 = x^2 + (3 + 11)x + 33$

$= (x + 3)(x + 11)$ $\left[\begin{array}{l}\text{By using identity} \\ x^2 + (a+b)x + ab = (x + a)(x + b)\end{array}\right]$

Example 12.13. Factorise : $x^2 - 5x + 6$

Solution : We want to find two numbers (integers) whose sum is –5 and product is 6. Here sum is negative, therefore, both factors of 6 will be negative. i.e. $6 = (-1) \times (-6)$ or $(-2) \times (-3)$ required factor of 6 are (–2) and (–3).

Therefore $x^2 - 5x + 6 = x^2 + \{(-2) + (-3)\}x + (-2)(-3)$

$= (x - 2)(x - 3)$

Product = 6

1 × 6	**(–1) × (–6)**
2 × 3	**(–2) × (–3)**

Example 12.14 Factorise : $p^2 + 4p - 12$

Solution : We find two numbers whose product is (–12) and sum is 4.

Since product is negative, one number will be positive and the other number will be negative.

and the sum is positive, so numerically greater of two numbers will be positive. So the required factors are 6 and (–2).

Therefore $\quad p^2 + 4p - 12 = p^2 + \{6 + (-2)\}p - 12$

$= p^2 + 6p - 2p - 12$

$= p(p + 6) - 2(p + 6)$

$= (p + 6)(p - 2)$

Product = –12

(–1) × 12	**1× (–12)**
(–2) × 6	**2 × (–6)**
(–3) × 4	**3×(–4)**

Example 12.15 Factorise : $y^2 - 4y - 45$

Solution : We want to find two numbers whose product is (–45) and sum is (–4)

Since the product is negative therefore one of numbers is positive and the other number is negative.

Since sum is negative, therefore. numerically greater of two numbers is negative.

So, the required number are (–9) and 5.

$$\therefore \quad \begin{aligned} y^2 - 4y - 45 &= y^2 + (-9 + 5)\,y - 45 \\ &= y^2 - 9y + 5y - 45 \\ &= y\,(y - 9) + 5\,(y - 9) \\ &= (y - 9)\,(y + 5) \end{aligned}$$

Product = –45

$(-1) \times 45$	**$1 \times (-45)$**
$(-3) \times 15$	**$3 \times (-15)$**
$(-5) \times 9$	**$5 \times (-9)$**

Exercise 12.2

1. Factorise the following expressions:
   (i) $x^2 + 10x + 25$ (ii) $y^2 - 8y + 16$ (iii) $25p^2 + 30p + 9$
   (iv) $49a^2 + 84ab + 36b^2$ (v) $100x^2 - 80xy + 16y^2$
   (vi) $(p+q)^2 - 4pq$ (Hint expand $(p+q)^2$ first)
   (vii) $\ell^4 + 2\ell^2 m^2 + m^4$ (viii) $4x^2 - 8x + 4$ (Hint :First take common 4 from each term)
2. Factorise the following expressions:
   (i) $25a^2 - 64b^2$ (ii) $49x^2 - 36$ (iii) $28x^2 - 63y^2$
   (iv) $\frac{4}{25}x^2 - \frac{9}{49}y^2$ (v) $8x^5 - 72x^3$ (Hint : taking x common first)
   (vi) $(p+q)^2 - (p-q)^2$ (vii) $16a^2b^2 - 25$
   (viii) $(x^2-2xy+y^2) - z^2$ (Hint : First use identity $a^2 - 2ab + b^2 = (a-b)^2$ then another)
3. Factorise :
   (i) $x^4 - y^4$ (ii) $a^4 - 81$ (iii) $m^4 - 256$
   (iv) $p^4 - (q+r)^4$ (v) $a^4 - 2a^2b^2 + b^4$
4. Factorise the following:
   (i) $a^2 + 2ab + b^2 - c^2$ (ii) $1 - 9l^2 + 24lm - 16m^2$ (iii) $25p^2 - 40pq + 16q^2 - 49r^2$
5. Factorise the following expressions.
   (i) $x^2 + 7x + 12$ (ii) $y^2 - 10y + 21$ (iii) $a^2 + 3a - 18$
   (iv) $3p^2 + 18p - 48$ (Hint: taking common 3 from each term)
   (v) $q^2 - q - 6$ (vi) $x^2 - 11x - 42$ (vii) $5x^2 + 25x + 30$
   (viii) $3y^2 - 21y + 36$
6. **Multiple choice Questions :**
   (i) $4p^2 - 20pq + 25q^2$
   (a) $(4p-5q)^2$ (b) $(2p-5q)^2$ (c) $(2q-5p)^2$ (d) $(4q-25p)^2$
   (ii) $4x^3 - 9x =$
   (a) $x^2\,(4x-9)\,(4x+9)$ (b) $x\,(2x-3)\,(2x+3)$
   (c) $x^3\,(2x-3)\,(2x+3)$ (d) $x^2\,(2x-3)\,(2x+3)$

(iii) $(a+b)^2 - (a-b)^2$

(a) $-4ab$ (b) $2a + 2b$ (c) $2a-2b$ (d) $4ab$

(iv) $m^2 - 14m - 32 =$

(a) $(m + 16)(m - 2)$ (b) $(m-16)(m-2)$

(c) $(m-16)(m+2)$ (d) $(m+16)(m+2)$

(v) $p^3 - p$

(a) $p(p^2+1)$ (b) $(p^2-1)(p+1)$ (c) $p^2(p-1)$ (d) $p(p-1)(p+1)$

12.4 Division of Algebraic Expressions

We have learnt about addition, subtraction and multiplication of algebraic expressions. Now, we will learn how to divide one algebraic expression by another. We know that division is the inverse operation of multiplication. Thus $7 \times 5 = 35$,

gives $35 \div 5 = 7$ or $35 \div 7 = 5$

Similarly (i) $4x \times 3x^2 = 12x^3$

Therefore $12x^3 \div 4x = 3x^2$

Or $12x^3 \div 3x^2 = 4x$

(ii) $3x(x + 2) = 3x^2 + 6x$

Therefore $(3x^2 + 6x) \div 3x = x + 2$

Or $(3x^2 + 6x) \div (x + 2) = 3x$

Now we shall learn how the division of one expression can be done by another expression.

12.4.1 Division of a monomial by another Monomial :

In this section, we shall learn the division of a monomial by another monomial.

Consider $12x^3 \div 3x$

We may write $3x$ and $12x^3$ in irreducible factors.

$$3x = 3 \times x$$

and $$12x^3 = 2 \times 2 \times 3 \times x \times x \times x$$

Or $$12x^3 = 3 \times x \times 2 \times 2 \times x \times x = (3x) \times (4x^2)$$

(Here seperate 3x from factors of $12x^3$)

Therefore, $12x^3 \div 3x = 4x^2$

This process is tedious and time consuming, so there is a shorter way by cancellation of common factors like we do in division of numbers.

i.e. $$35 \div 5 = \frac{35}{5} = \frac{7 \times \cancel{5}}{\cancel{5}} = 7$$

Similarly $$12x^3 \div 3x = \frac{12x^3}{3x} = \frac{2 \times 2 \times \cancel{3} \times \cancel{x} \times x \times x}{\cancel{3} \times \cancel{x}} = 2 \times 2 \times x \times x = 4x^2$$

ALTERNATIVELY

Division of a monomial by a monomial

We have,

Quotient of two monomials = (Quotient of their coefficient) × (Quotient of their variables in two monomials)

For Example: $12x^3 \div 3x = \frac{12x^3}{3x} = \left(\frac{12}{3}\right)\left(\frac{x^3}{x}\right) = 4x^{3-1} = 4x^2$ (Using $a^m \div a^n = a^{m-n}$)

Example 12.16. Divide : (i) $12x^5$ by $-4x^3$ (ii) $(-18y^3)$ by $3y^2$ (iii) $-4x^2y^3$ by $12x^3y$

Solution : (i)

$$12x^5 = 2 \times 2 \times 3 \times x \times x \times x \times x \times x$$

and

$$-4x^3 = -2 \times 2 \times x \times x \times x$$

$$\text{Therefore } 12x^5 \div (-4x^3) = \frac{\cancel{2}\times\cancel{2}\times 3\times\cancel{x}\times\cancel{x}\times\cancel{x}\times x\times x}{-\cancel{2}\times\cancel{2}\times\cancel{x}\times\cancel{x}\times\cancel{x}} = -3 \times x \times x = -3x^2$$

ALITER

$$12x^5 \div (-4x^3) = \frac{12x^5}{-4x^3} = \left(\frac{12}{-4}\right)\left(\frac{x^5}{x^3}\right) = -3 \times x^{5-3} = -3x^2 \text{ (Using } a^m \div a^n = a^{m-n}\text{)}$$

(ii)

$$-18y^3 = -2 \times 3 \times 3 \times y \times y \times y$$

and

$$3y^2 = 3 \times y \times y$$

$$\text{Therefore } -18y^3 \div 3y^2 = \frac{-2\times\cancel{3}\times 3\times\cancel{y}\times\cancel{y}\times y}{\cancel{3}\times\cancel{y}\times\cancel{y}} = -2 \times 3 \times y = -6y$$

ALITER

$$-18y^3 \div 3y^2 = \frac{-18y^3}{3y^2} = \left(\frac{-18}{3}\right)\left(\frac{y^3}{y^2}\right)$$

$$= -6 \times y^{3-2} \text{ (Using } a^m \div a^n = a^{m-n}\text{)}$$

$$= -6y^1 = -6y$$

(iii)

$$(-4x^2y^3) \div 12x^3\,y = \frac{-4x^2y^3}{12x^3y} = \left(\frac{-4}{12}\right)\left(\frac{x^2}{x^3}\right)\left(\frac{y^3}{y}\right)$$

$$= \left(\frac{-1}{3}\right)\left(\frac{1}{x}\right)\left(y^2\right) = \frac{-y^2}{3x}$$

12.4.2 Division of a polynomial by a monomial

In last section, we have learnt the division of a monomial by another monomial. In this section we shall discuss the division of polynomial by a monomial.

Consider the division of a polynomial $6a^3 + 8a^2 + 4a$ by a monomial $2a$. We shall factorise first

i.e. $6a^3 + 8a^2 + 4a = 2a\,(3a^2 + 4a + 2)$

$$(6a^3 + 8a^2 + 4a) \div 2a = \frac{2a\left(3a^2 + 4a + 2\right)}{2a}$$

$$= 3a^2 + 4a + 2$$

We can divide it without making factors i.e. directly as follows:

$$(6a^3 + 8a^2 + 4a) \div 2a = \frac{6a^3 + 8a^2 + 4a}{2a}$$

(Divide each term by the given monomial)

$$= \frac{6a^3}{2a} + \frac{8a^2}{2a} + \frac{4a}{2a}$$

$$= 3a^2 + 4a + 2$$

Let us explain the above division by taking more examples.

Example 12.17. Divide: (i) $6x^4 + 24x^3 - 5x^2$ by $3x^2$ (ii) $(5y^8 - 10y^5 + 3y^2) \div (-5y^2)$

Solution : (i) We have $(6x^4 + 24x^3 - 5x^2) \div 3x^2 = \dfrac{6x^4 + 24x^3 - 5x^3}{3x^2}$

$$= \frac{6x^4}{3x^2} + \frac{24x^3}{3x^2} - \frac{5x^2}{3x^2}$$ **(Dividing each term by the given monomial)**

$$= 2x^2 + 8x - \frac{5}{3}$$

(ii) We have $(5y^8 - 10y^5 + 3y^2) \div (-5y^2) = \dfrac{5y^8 - 10y^5 + 3y^2}{-5y^2}$

$$= \frac{5y^8}{-5y^2} - \left(\frac{10y^5}{-5y^2}\right) + \left(\frac{3y^2}{-5y^2}\right) = -y^6 + 2y^3 - \frac{3}{5}$$

12.4.3 Division of a Polynomial by another Polynomial (Binomial)

In last section, we have learnt the division of a monomial or polynomial by a monomial. In this section we shall discuss the division of a polynomial by a binomial.

Here, we discuss the cases with zero remainder and methods for division of polynomial by binomial with the help of examples.

Example 12.18. Divide as directed :

(i) $5(2x+1)(3x+5) \div (2x+1)$ (ii) $20(y+4)(y^2+5y+3) \div 5(y+4)$

Solution : (i) We have $5(2x+1)(3x+5) \div (2x+1) = \dfrac{5\cancel{(2x+1)}(3x+5)}{\cancel{(2x+1)}}$

$$= 5(3x+5)$$

(ii) We have $20(y+4)(y^2+5y+3) \div 5(y+4) = \dfrac{\cancel{20}^{4}\cancel{(y+4)}(y^2+5y+3)}{\cancel{5}_{1}\cancel{(y+4)}}$

$$= 4(y^2+5y+3)$$

Example 12.19. Divide: $y^2 + 7y + 10$ by $y + 5$

Solution : First, factorise $(y^2 + 7y + 10)$

Therefore $y^2 + 7y + 10 = y^2 + (5 + 2)y + 5 \times 2$

$= (y + 5)(y + 2)$ (Using identity ...(iv))

Now $(y^2+7y+10) \div (y+5)$

$$= \frac{y^2+7y+10}{y+5} = \frac{\cancel{(y+5)}(y+2)}{\cancel{y+5}}$$

[Cancelling the common factor (y + 5) from numerator & denominator]

$$= y + 2$$

Example 12.20. **Divide as directed :**

(i) $(5p^2 - 25p + 20) \div (p-1)$ (ii) $12xy\,(9x^2 - 16y^2) \div 4xy\,(3x + 4y)$

Solution : (i) First factorise $(5p^2 - 25p + 20)$, We get

$$5p^2 - 25p + 20 = 5(p^2 - 5p + 4) = 5\,(p^2 - 4p - p + 4)$$
$$= 5\,[p\,(p-4) - 1\,(p-4)] = 5\,(p-4)\,(p-1)$$

Now, $$(5p^2 - 25 + 20) \div (p-1) = \frac{5p^2 - 25p + 20}{p-1} = \frac{5(p-4)\cancel{(p-1)}}{\cancel{p-1}}$$
$$= 5\,(p-4)$$

(ii) First Factorise $(9x^2 - 16y^2)$, We get

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)\,(3x + 4y)$$

$$\text{Now, } 12xy\,(9x^2 - 16y^2) \div 4xy\,(3x + 4y) = \frac{12xy\left(9x^2 - 16y^2\right)}{4xy\,(3x + 4y)}$$

$$= \frac{\overset{3}{\cancel{12xy}}\,(3x - 4y)\cancel{(3x + 4y)}}{\underset{1}{\cancel{4xy}}\cancel{(3x + 4y)}} = 3\,(3x - 4y)$$

Let us learn another method of division of polynoments.

Example 12.21. **Divide $(3 - 11x + 6x^2)$ by $(-1 + 3x)$**

Solution : Write the dividend & divisor in decreasing order of powers of variable.

Step 1. Dividend: $6x^2 - 11x + 3$

Divisor : $3x - 1$

Step 2. Divide the first term of dividend i.e. $6x^2$ by the first term of divisor $3x$, we get $\frac{6x^2}{3x} = 2x$

$$\begin{array}{r|l} & \;2x \\ \hline 3x-1) & 6x^2 - 11x + 3\,(\end{array}$$

i.e. We get first term of quotient as $2x$.

Step 3. Multiply the divisor $(3x - 1)$ by $2x$ **(Resulting expression of step 2)**

We get $2x\,(3x-1) = 6x^2 - 2x$

Subtract this from dividend $(6x^2 - 11x + 3)$

to get remainder $(6x^2 - 11x + 3) - (6x^2 - 2x) = -9x + 3$

$$\begin{array}{r|l} & \;2x \\ \hline 3x-1) & 6x^2 - 11x + 3\,(\\ & \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline & \qquad -9x + 3 \end{array}$$

Step 4. Now consider this remainder $-9x + 3$ as new dividend. Divide the first term of new dividend $(-9x)$ by first term of divisor $3x$.

We get $\frac{-9x}{3x} = -3$ (As in step 2)

i.e. 2nd term of quotient.

$$\begin{array}{r|l} & \;2x - 3 \\ \hline 3x-1) & 6x^2 - 11x + 3\,(\\ & \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline & \qquad -9x + 3 \end{array}$$

Step 5. Multiply the divisor $(3x-1)$ by -3 **(Resulting expression of step 4)**

We get $-3\,(3x-1) = -9x + 3$.

Subtract this from new dividend to get remainder

$[(-9x+3) - (-9x + 3)]$

$= -9x + 3 + 9x - 3$

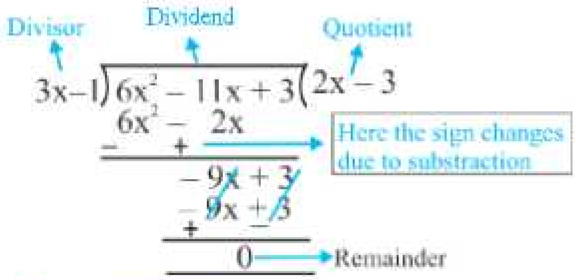
$= 0$

$$\begin{array}{r|l} & \;2x - 3 \\ \hline 3x-1) & 6x^2 - 11x + 3\,(\\ & \underset{-}{6x^2} \underset{+}{-} 2x \\ \hline & \qquad -9x + 3 \\ & \qquad \underset{+}{-}9x \pm 3 \\ \hline & \qquad\qquad 0 \end{array}$$

The remainder is zero and quotient is 2x–3

Hence, $(6x^2 - 11x + 3) \div (3x-1) = 2x - 3$

The above procedure is called long division method. It is shown below.



Exercise 12.3

1. **Carry out the following divisions.**

(i) $20x^4 \div 10x^2$ (ii) $(-35y^4)$ by $(-7y^3)$ (iii) $16a^4$ by $-6a^2$

(iv) $7x^2y^2z^2 \div 21xyz$ (v) $24p^8q^8 \div (-8\ p^6q^4)$ (vi) $(-15x^2y^3z^2) \div 10x^2y\ z^2$

(vii) $8l^2m^3 \div (-16l^4m^2)$ (viii) $(-12x^2y) \div 20xy^2z$

2. **Divide the given polynomial by given monomial**

(i) $(3x^2 - 4x) \div 7x$ (ii) $(-12a + 22a^2 - 16a^3 + 4) \div 2a$

(iii) $(-8y^3 + 16y^2 + 14y + 1) \div 4y$

(iv) $(ax^8 - bx^6 + cx^4) \div x^4$ (v) $(15x^2y^3 - 10x^3y^2 + 2xy) \div (-5xy^2)$

3. **Divide as directed:**

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

(ii) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

(iii) $9a^2b^2(3c - 24) \div 27ab(c-8)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $(x^3y^6 - x^6y^3) \div x^3y^3$

(vi) $48xyz(3x-12)(5y-30) \div 72(x - 4)(y - 6)$

4. **Using factor method, divide the following polynomial by a binomial.**

(i) $(x^2 + 6x + 8)$ by $(x + 2)$ (ii) $(x^2 - x - 42)$ by $(x + 6\qquad)$

(iii) $(p^2 - 6p - 27)$ by $(p - 9)$ (iv) $(7x^2 + 14x)$ by $(x + 2)$

(v) $(a^2 - 7a + 12)$ by $(a - 3)$ (vi) $(x^4 + 3x^2 - 10)$ by $(x^2 + 5)$ (Hint put $x^2 = y$)

5. **Divide the following polynomial by a binomial using long division method.**

(i) $(p^2 + 12p + 35)$ by $(p + 7)$ (ii) $(9y^2 - 6y - 8)$ by $(3y - 4)$

6. Divide :

(i) $z(5z^2 - 80)$ by $5z(z+4)$ (ii) $10pq(p^2-q^2)$ by $2p(p+q)$

(iii) $15ab(16a^2 - 25)$ by $10ab(4a+5)$ (iv) $44(x^4 - 5x^3 - 24x^2)$ by $11(x^2 - 8x)$

(v) $39x^3(50x^2 - 98)$ by $26x^2(5x + 7)$

7. Multiple Choice Questions :

(i) $(4x^2-8x) \div (-4x^2) =$

(a) $-1 + 2x$ (b) $\frac{2}{x}$ (c) $-1+\frac{2}{x}$ (d) $2x$

(ii) $(x^2yz + xy^2z + xyz^2) \div xyz =$

(a) xyz (b) $x+y+z$ (c) $x^2 + y^2 + z^2$ (d) $\frac{xy}{z}$

(iii) $2x^2(x+1)(x+3) \div 4x(x+3) =$

(a) $2x(x+1)$ (b) $2x^2(x+1)$ (c) $\frac{x^2(x+1)}{2}$ (d) $\frac{x(x+1)}{2}$

(iv) $(72x^2 - 50) \div (6x - 5) =$

(a) $2(6x + 5)$ (b) $12x + 5$ (c) $12x^2 + 5$ (d) $2(12x + 5)$

(v) $(x^2-8x-20) \div (x-10) =$

(a) $(x - 2)$ (b) $(x + 2)$ (c) $x - 3$ (d) $x + 4$

Learning Outcomes

After completion of this chapter, students are now able to:

- *Apply different methods of finding the factors of an algebraic expressions.*
- *Do division of algebraic expression and division of polynmials by a polynomial.*
- *Apply identities $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$ or $(a-b)(a+b) = a^2-b^2$ etc.*

Answers

Exercise 12.1

1. (i) 5 (ii) $3y$ (iii) $7pq$ (iv) 1 (v) $4ab$ (vi) $6x$ (vii) $2xy^2$
(viii) 1

2. (i) $6(x-8)$ (ii) $7(p-2q)$ (iii) $-6z(4-5z)$ or $6z(5z-4)$
(iv) $9\ell m(2\ell+3a)$ (v) $5x^2yz(5y-3z)$ (vi) $abc(a+b+c)$
(vii) $xy(px+qy+rz)$ (viii) $5(2pq-3qr+4rp)$

3. (i) $(2p-3q)(3a-5b)$ (ii) $5(x^2+y^2)(3a-2b)$ (iii) $2(x+y)(2x+2y+1)$
(iv) $(2a-5b)(2a-5b-2)$ (v) $(5\ell+3m)(5\ell+3m-1)$

4. (i) $(x+y)(x+6)$ (ii) $(y-z)(y-3)$ (iii) $(3y-2)(4x+1)$
(iv) $(a-b)(ab+4)$ (v) $(x-6)(x^2+1)$ (vi) $(a+b)(a+b^2)$
(vii) $(x-2y)(3p+4q)$ (viii) $(r-7)(1-pq)$

5. (i) c (ii) a (iii) d (iv) a (v) c

Exercise 12.2

1. (i) $(x+5)^2$ (ii) $(y-4)^2$ (iii) $(5p+3)^2$ (iv) $(7a+6b)^2$
(v) $4(5x-2y)^2$ (vi) $(p-q)^2$ (vii) $(\ell^2+m^2)^2$ (viii) $4(x-1)^2$

2. (i) $(5a+8b)(5a-8b)$ (ii) $(7x+6)(7x-6)$
(iii) $7(2x-3y)(2x+3y)$ (iv) $\left(\frac{2}{5}x+\frac{3}{7}y\right)\left(\frac{2}{5}x-\frac{3}{7}y\right)$
(v) $8x^3(x+3)(x-3)$ (vi) $4pq$
(vii) $(4ab+5)(4ab-5)$ (viii) $(x-y+z)(x-y-z)$

3. (i) $(x+y)(x-y)(x^2+y^2)$ (ii) $(a+3)(a-3)(a^2+9)$
(iii) $(m-4)(m+4)(m^2+16)$ (iv) $(p+q+r)(p-q-r)[p^2+(q+r)^2]$
(v) $(a+b)^2(a-b)^2$

4. (i) $(a+b-c)(a+b+c)$ (ii) $(1-3\ell+4m)(1+3\ell-4m)$
(iii) $(5p-4q-7r)(5p-4q+7r)$

5. (i) $(x + 3)(x + 4)$ (ii) $(y - 3)(y - 7)$ (iii) $(a - 3)(a + 6)$

(iv) $3(p + 8)(p - 2)$ (v) $(q - 3)(q + 2)$ (vi) $(x - 14)(x + 3)$

(vii) $5(x + 2)(x + 3)$ (viii) $3(y-3)(y-4)$

6. (i) b (ii) b (iii) d (iv) c (v) d

Exercise 12.3

1. (i) $2x^2$ (ii) $5y$ (iii) $\frac{-8}{3}a^2$ (iv) $\frac{1}{3}xyz$ (v) $-3p^2q^4$ (vi) $\frac{-3}{2}y^2$

(vii) $\frac{-m}{2\ell^2}$ (viii) $\frac{-3x}{5yz}$

2. (i) $\frac{3x}{7} - \frac{4}{7}$ (ii) $-6+11a - 8a^2 + \frac{2}{a}$ (iii) $-2y^2 + 4y + \frac{7}{2} + \frac{1}{4y}$

(iv) $ax^4 - bx^2 + c$ (v) $-3xy + 2x^2 - \frac{2}{5y}$

3. (i) $5(3x + 5)$ (ii) $(x + 2)(x + 3)$ (iii) ab (iv) $2z(z - 2)$ (v) $y^2 - x^3$

(vi) $10xyz$

4. (i) $x + 4$ (ii) $x - 7$ (iii) $p + 3$ (iv) $7x$ (v) $a - 4$

(vi) $x^2 - 2$

5. (i) $p+5$ (ii) $3y + 2$

6. (i) $(z - 4)$ (ii) $5q(p - q)$ (iii) $\frac{3}{2}(4a - 5)$ (iv) $4x(x + 3)$

(v) $3x(5x - 7)$

7. (i) c (ii) b (iii) d (iv) a (v) b

CHAPTER
13
Introduction to Graphs

Learning Objectives

In this chapter, you will learn:

- *To read and interpret data from different types of graphs, used in daily life.*
- *To understand the trends and compare the information given in graphs.*
- *To understand the concept of x-axis, Y-axis, origin, etc. in cartesian coordinate system.*
- *To plot points on the plane and read coordinates of a point from a graph.*

13.1 Introduction:-

The graphs are visual representation of collected data. We can represent a data in tabular form but graphical represtation of the data is easier to understand. We have seen graphs in newspapers, on television, magazines, books etc. The purpose of the graph is to show numerical facts in visual form for understanding quickly, easily and clearly.We have already discussed many graphs like Bar Graph, Histogram and Pie Chart in previous chapter 'DATA HANDLING'.

In this chapter, we shall discuss 'line graph' in detail.

13.2 Linear Graph

A line graph consists of bits of line segments joined consecutively. Some times the graph may be a whole unbroken line. Such a graph is called a **linear graph**.

To draw such a line we need to locate some points on the graph sheet. In this section We will learn to locate points on a graph sheet and plot the points on graph sheet.

13.2.1 Location of a Point : To locate a point on a plane, two observations are required. First, How far it is from the left edge (Vertical line) of the plane and second, how far it is from the horizontal line in the plane.

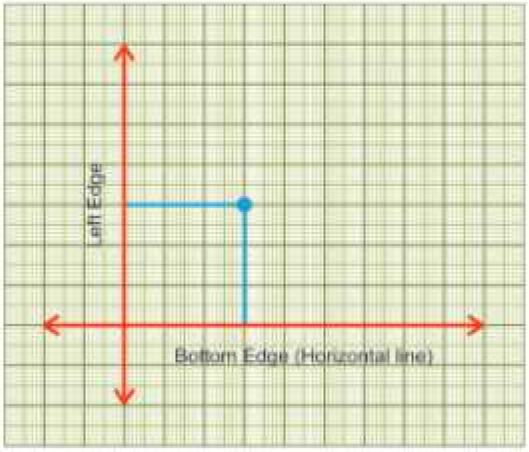


Figure 15.1

*The 17th century mathematician **Rene Descartes** noticed the movements of an insect near a corner of the ceiling and began to think of determining the position of a given point in a plane. His system of fixing a point with the help of two measurement horizontal and vertical came to known as cartesian system in his honour.*

13.2.2 Co-ordinates : Suppose you go to a Cinema Hall and search for your reserved seat. You need two numbers, the row numbers and the seat. This is the basic method for fixing a point in the plane.

To find the position of a point on a graph sheet we draw two axes (like left edge and bottom edge) The horizontal line is known as **x-axis** and vertical line is known as **y-axis**. The point of intersection of x-axis and y-axis is called **origin** having coordinates (0,0). The graph sheet itself is a square grid. Now observe in the figure how the point (3, 4) which is 3 units from y-axis and 4 units from x-axis is plotted on the graph.

The x-coordinate 3 is called **abscissa** and the y-co-ordinate 4 is called **ordinate** of the point.

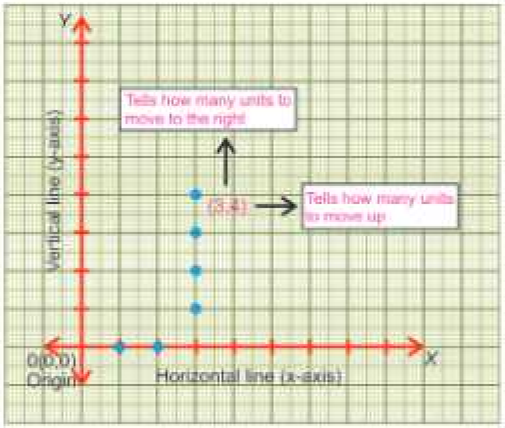


Figure 13.2

We say that the co-ordinates of the point are (3, 4)

Example 13.1 From the figure choose the letter that indicates the location of the points given below.

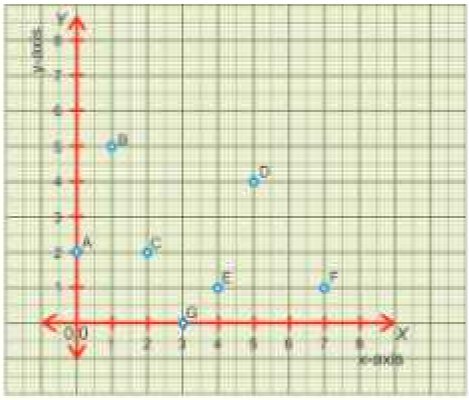


Figure 13.3

(i) **(7, 1)** **(ii)** **(5, 4)** **(iii)** **(0, 2)**
(iv) **(1, 5)** **(v)** **(3, 0)** **(vi)** **(2, 2)**
(vii) **(4, 1)**

Sol. (i) (7, 1) is point F (ii) (5, 4) is point D
(iii) (0, 2) is point A (iv) (1, 5) is point B
(v) (3, 0) is point G (vi) (2, 2) is point C
(vii) (4, 1) is point E

Example 13.2 **Plot the following points on the graph paper.**

(i) **A(2, 3)** **(ii)** **B(5,1)** **(iii)** **C(1, 2)**
(iv) **D(2, 5)** **(v)** **E(0, 4)** **(vi)** **F(4, 5)**
(vii) **G(3, 0)** **(viii)** **H(5, 5)**

Sol.

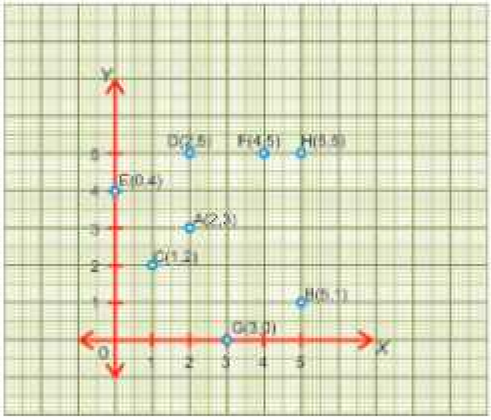


Figure 13.4

Example 13.3 **Plot the point (3, 5) on a graph sheet is it same as (5, 3)?**

Sol. As in (3, 5), x-coordinate is 3 and y coordinate 5. On graph sheet draw the x axis and y axis. Start from O (0, 0) move 3 units to the right and then 5 units up. you will reach the point (3, 5). Similarly locate the point (5, 3), you can see that (3,5) and (5, 3) are different points

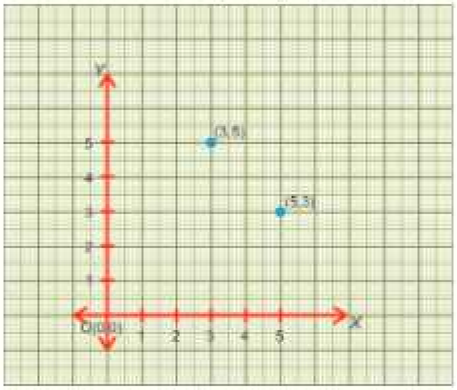


Figure 13.5

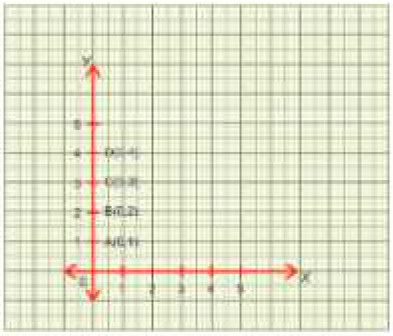
Example 13.4 **Plot the following point if they lie on a line join the points, name it.**

(i) **(0, 1), (0, 2), (0, 3), (0, 4)**
(ii) **A (1, 1), B (2, 1), C (3, 1) D (4, 1), E (5, 1)**

(iii) **G(2, 2), H(3, 2), I(4, 2); J(5, 2)**

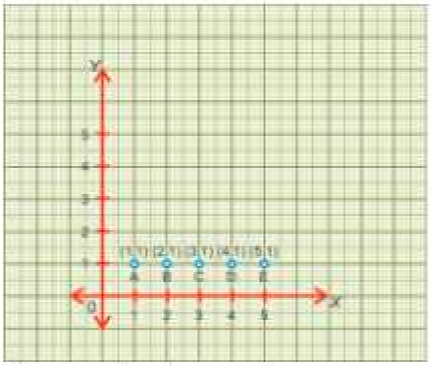
(iv) **K(2, 6) ; L(3, 5); M(5, 3); N(6, 2)**

Sol. **(i)**



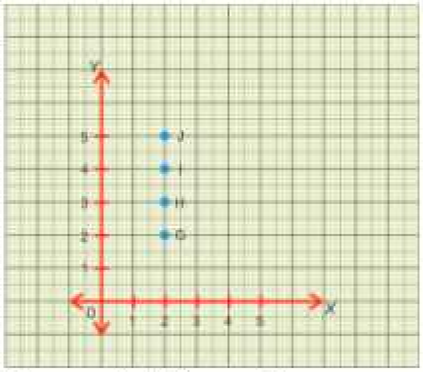
These point lie on a line and line is y-axis

(ii)



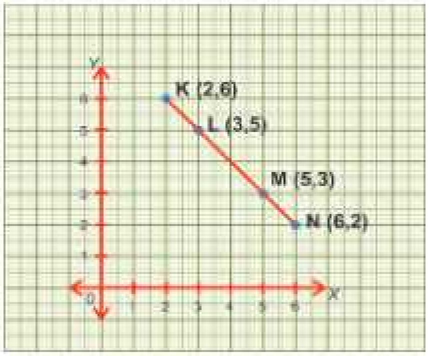
These point lie on a line we can name it AE.

(iii)



These points lie on a line, we can name it GJ.

(iv)



These points lie on a line we can name it KN.

Figure 13.6

Note *Observe that in each of the above cases, graph obtained by joining the plotted points is a line. Such graphs are known as linear graphs.*

Exercise 13.1

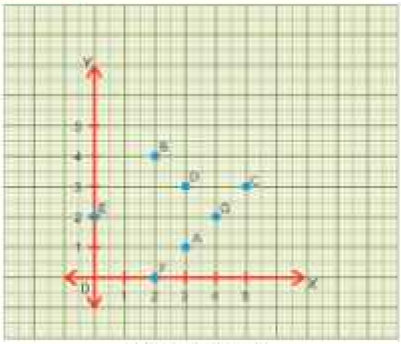


Figure 13.7

1. From the given figure choose the letter that indicates the location of the points.

(i) (0, 2) (ii) (2, 4) (iii) (3, 3)

(iv) (5, 3) (v) (4, 2) (vi) (3, 1)

(vii) (2, 0)

2. Plot the following points on the graph paper:'

(i) A(3, 5)	(ii) B(2,4)	(iii) C (5, 2)	(iv) D (0, 4)
(v) E (5, 4)	(vi) F (3, 4)	(vii) G (4, 3)	(viii) H (3, 0)

3. Plot the points (2, 3) and (3, 2) on a graph paper check both are plotted on same location.

4. Plot the following points on a graph sheet. Verify if they lie on a line

(a) (3, 7) ; (3, 4) ; (3, 2) ; (3, 0)   (b) (0, 0) ; (2, 2) ; (4, 4) ; (6, 6)

(c) (0, 4) ; (1, 4) ; (2, 4) ; (3, 4)   (d) (2, 1) ; (3, 2) ; (4, 3) ; (5, 5)

5. Write the co-ordinate of the vertices of each figure shown in graph

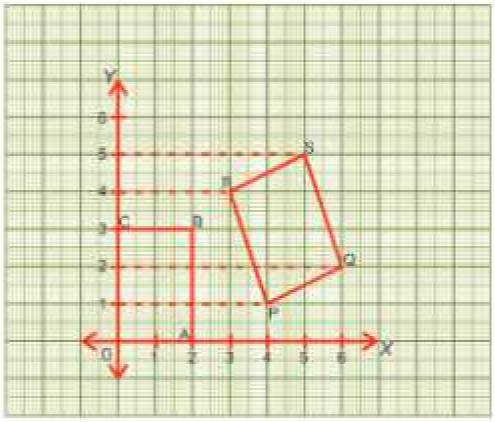


Figure 13.8

6. Draw a line joining (2, 4) and (7, 0). check whether the point (5, 1) lies on it.

7. True or False:

(i) The coordinates of origin are (0, 0).

(ii) Any point on y-axis has x-coordinate zero.

(iii) Any point on x-axis has y-coordinate zero.

(iv) The points (4, 3) and (3, 4) represents the same point.

(v) The ordinate of (5, 2) is 5.

8. Choose the correct answer:

(i) The point (1, 0) lies on

(a) x-axis   (b) y-axis   (c) origin   (d) none

(ii) Which of the following coordinate is on x-axis?

(a) (0, 3)   (b) (1, 2)   (c) (2, 3)   (d) (4, 0)

(iii) Which of the following coordinate in on y-axis?

(a) (0, 3)   (b) (1, 2)   (c) (2, 3)   (d) (4, 0)

(iv) The abscissa of (2, 7) is;

(a) 7   (b) 2   (c) 0   (d) None

(v) The ordinate of (7, 4) is:

(a) 0   (b) 7   (c) 4   (d) None

13.3. A Line Graph:

A line graph displays data that changes continuously over periods of time. for example when a person fell sick, the doctor maintains a record of his or her body temperature, taken after every fixed interval of time. Study the following table. It's Pictorial representation is known as time-temperature graph.

Time	6a.m.	8a.m.	10a.m.	12 noon	2p.m.	4p.m.	6p.m.
Body temperature in °F	102	100	101	100	99	98	98

Timings shows on x-axis and temperature on y-axis.

Plot the points (6,102), (8, 100), (10, 101), (12, 100), (2,99), (4,98), (6, 98)

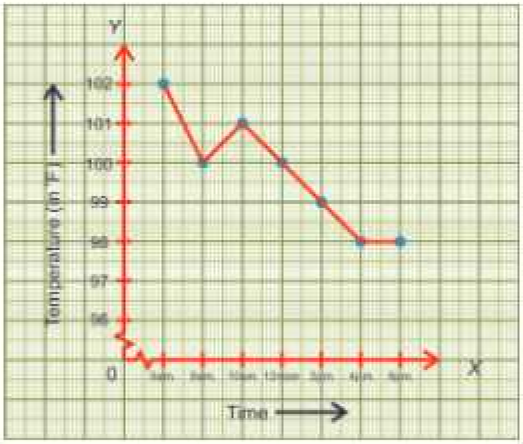


Figure 13.9

What does this graph tell you? For example you can see the pattern of highest temperature at 6a.m. then decreased till 8a.m then again increased upto 10 a.m, then decreased upto 4p.m and then remained constant upto 6p.m

Example 13.5 **Study the graph shown in figure 13.10 and answer the following questions.**

(i) What information the graph shows?

(ii) What is the time when temperature is 99° F ?

(iii) The temperature was same two times during the given period what are the times?

(iv) What is the temperature at 6p.m.?

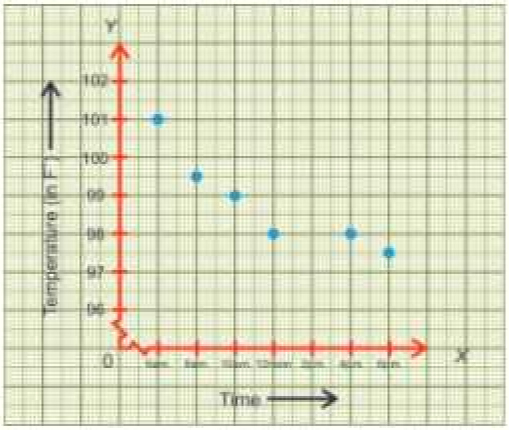


Figure 13.10

Sol. (i) The graph shows the body temperature of a person recorded after every 2 hours

(ii) The temperature is 99° F at 10 a.m.

(iii) The temperature was same at 12 noon and 4.00 p.m.

(iv) The temperature is 97.5° F at 6 p.m.

Example 13.6 The given graph (13.11) describes the distance of a car from a city A at different times when it was travelling from city A to city B which are 500km apart. Study the graph and answer the following.

(i) When the car started its journey?

(ii) How far did the car go in the first hour?

(iii) Did the car stop for some duration during its journey? For what time duration the car stopped?

(iv) When the car reached at B?

(v) What was the total distance traveled by car in first five hours?

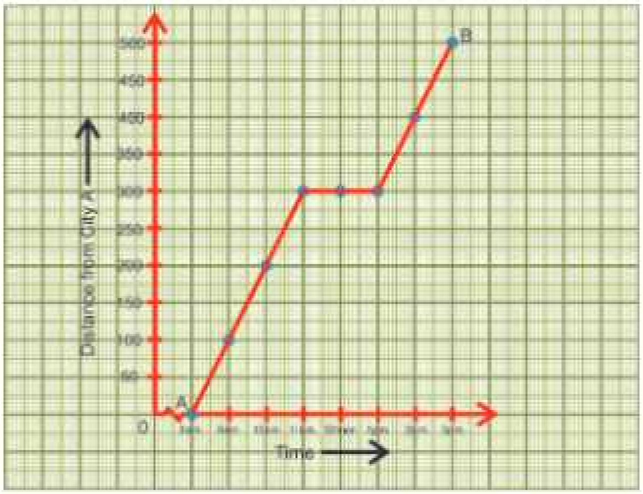


Figure 13.11

Sol. (i) The car started from point A at 8 a.m.

(ii) The car travelled 100 km in first hour

(iii) Yes, the car stopped during the journey. It stopped from 11 a.m to 1p.m. as no distance is travelled between these hours.

(iv) The car reached at point B at 3p.m.

(v) The total distance travelled in first five hours is upto 1 p.m. is 300km.

Example 13.7. The given graph (fig 13.12) represents the total runs scored by two batsman A and B during each of the five different matches in 2017. Study the graph and answer the following.

(i) What information is given on the two axes?

(ii) Which line shows the run scored by batsman B?

(iii) Whether in any match batsman A and B scored same run.

(iv) Which player is more consistent? (Give Reason)

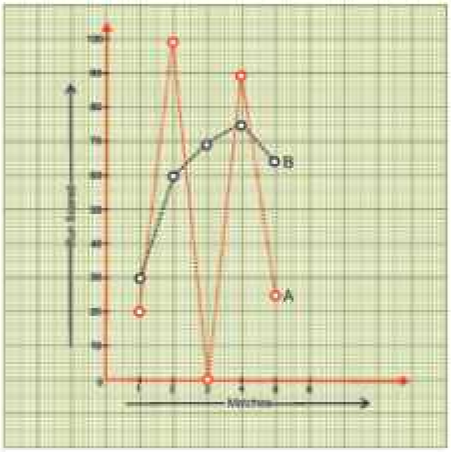


Figure 13.12

Sol. (i) The horizontal axis (or x-axis) indicates the matches played during year 2017 and the vertical axis (y-axis) shows the run scored by two batsman.

(ii) Dark line Red

(iii) No

(iv) Batsman B is more consistent. As graph of batsman A has very ups and downs, where as batsman B shows almost medium performance in all five matches.

Exercise 13.2

1. The following graph shows temperature of a patient in a hospital recorded every hour

(i) What was the patient's temperature at 2p.m. and 3p.m.

(ii) When was the patient's temperature 100°F

(iii) On which two times the patient's temperature was same?

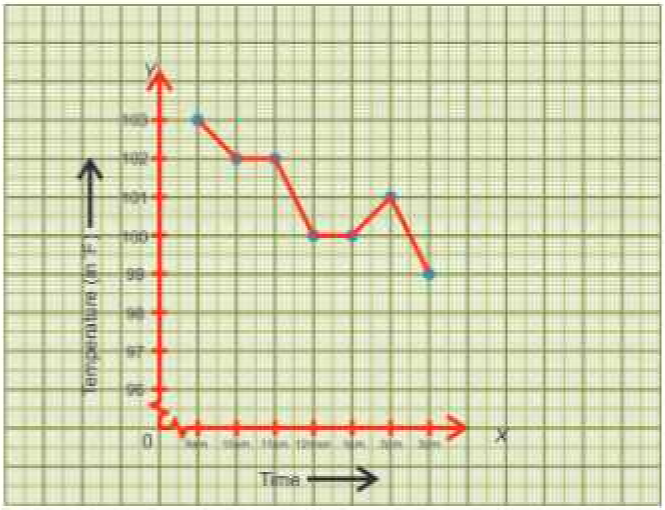


Figure 13.13

2. The following line graph shows the yearly sales figures for a manufacturing company

(i) What was the sales in 2010, 2011, 2014, 2016?

(ii) Compute the difference between the sales in 2015 and 2013?

(iii) Whether the sale was same in every year?

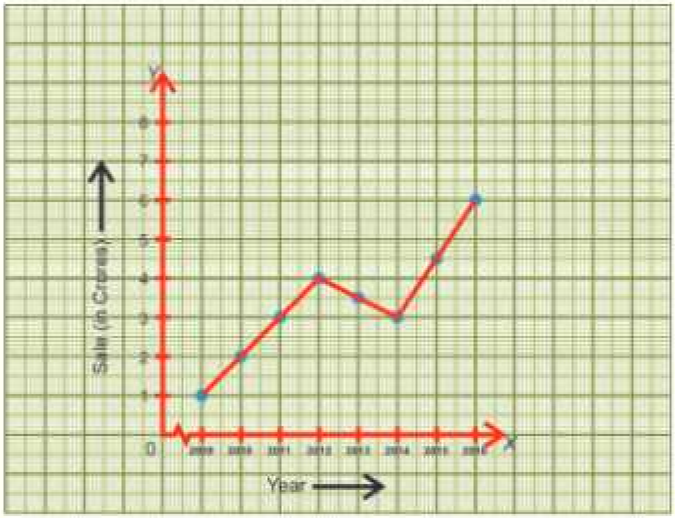


Figure 13.14

3. A person cycles from a town to a neighbouring area to deliver a packet to a merchant. His distance from the town at different times is shown by the graph.

   (i) What information is given in the graph?

   (ii) How much time did the person take for the travel?

   (iii) How far is the place of the merchant from the town?

   (iv) Did the person stop travelling on his way? Explain.

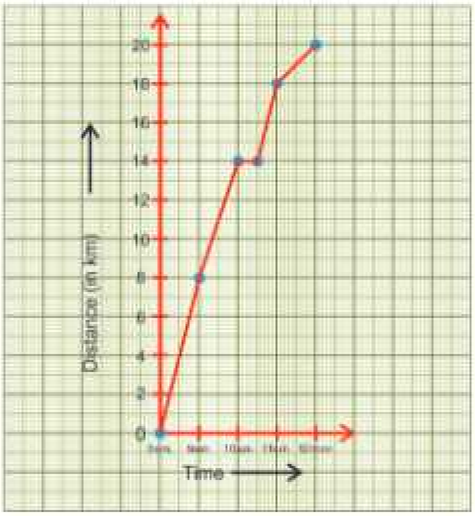


Figure 13.15

4. The graph shows the run scored by two batsmen A and B in ten matches during the year 2016. Study the graph and answer the following questions.

   (i) Whether batsmen B scored more than 100 runs in any match If yes then in which match?

   (ii) Whether in any match batsmen A and B scored same run. If yes then in which match?

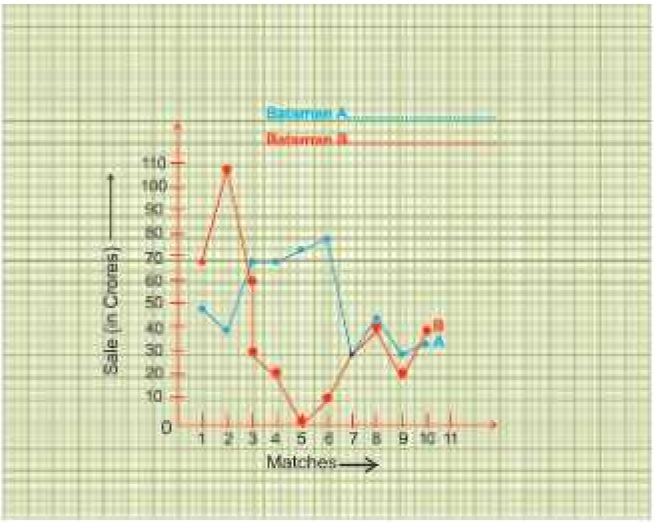


Figure 13.16

   (iii) Among the two batsmen, who is more consistent? How do you judge it?

5. The following graph shows the temperature forecast and the actual temperature for each day of a week. Study the graph and answer the following.

   (i) On which days was the forecast temperature is same as the actual temperature?

   (ii) What was the maximum actual temperature during the week.

   (iii) On which day the actual temperature differ the most from the forecast temperature?

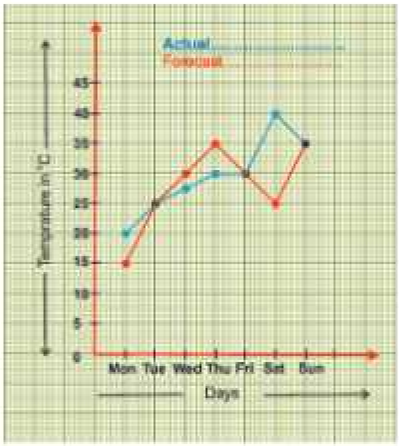


Figure 13.17

6. Observe the followig Temperature-Time graph and answer the following questions:

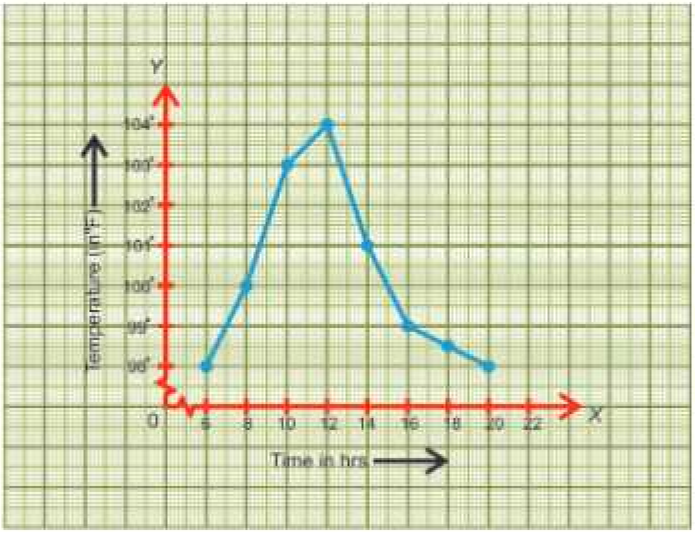


Figure 13.18

(i) On which time the temperature is maximum?

(a) 12 hours (b) 14 hours (c) 6 hours (d) 20 hours

(ii) On which time, the temperature is minimum?

(a) 8 hours (b) 12 hours (c) 14 hours (d) 6 hours and 20 hour

(iii) 103°F temperature is on the time:

(a) 10 hours (b) 12 hours (c) 14 hours (d) 20 hours

(iv) What is the difference of temperature at 6 hours and 20 hours?

(a) 0°F (b) 1°F (c) 2°F (d) 3°F

(v) What is the rise in temperature from 10 hours to 12 hours?

(a) 1°F (b) 2°F (c) 3°F (d) 4°F

13.4 Drawing a Graph

In Last section we have discussed the study of line Graph. In this section we shall learn the construction of line Graph. Here we will discuss the line Graph of variables having direct proportion.

We have learnt about the direct proportion,where increase/decrease in one quantity, changes the other quantity in same ratio for example if we use more electricity, the bill will be more. Similarly if we want to travel more miles, more fuel is required. So, Amount of electricity bill depends upon the quantity of electricity used. We say that quantity of electricity is an independent variable (or sometimes control variable) and the amount of electric bill is dependent variable. The relation between such variables can be shown through a graph.

Working Rule:

1. Take an independent variable (abscissa) along x-axis and the dependent variable (ordinate) along y-axis.
2. Plot each ordered pair (x, y) and join plotted points.

Example 13.8 **The following table shows the quantity of petrol and its cost**

No. of litres of petrol	1	5	10	15	20
Cost of petrol in ₹	70	350	700	1050	1400

Plot a graph to show the data

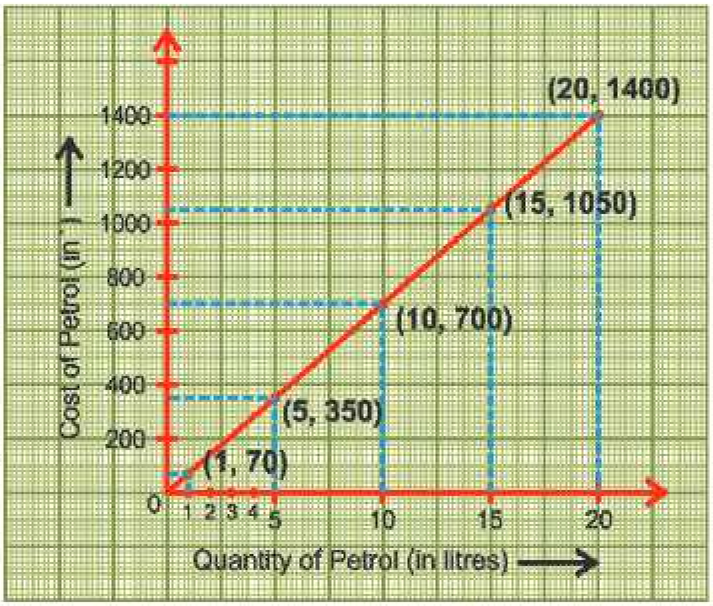


Figure 13.19

Sol. (i) Let us take a suitable scale on both axes

(ii) Mark number of litres along the horizontal (x-axis)

(iii) Mark cost of petrol along the vertical axis (y-axis)

(iv) Plot the points

(v) Join the points

We find that the graph is a line (It is a linear graph). It passes through origin because for zero litre petrol, cost will be zero.

Example 13.9 **Draw the graph for the values given in the table, with suitable scale on the axes**

Weight of aples (in kg)	1	2	5	7	10
Cost in ₹	60	120	300	420	600

From the graph find the cost of 6kg and 8kg apples.

Sol.

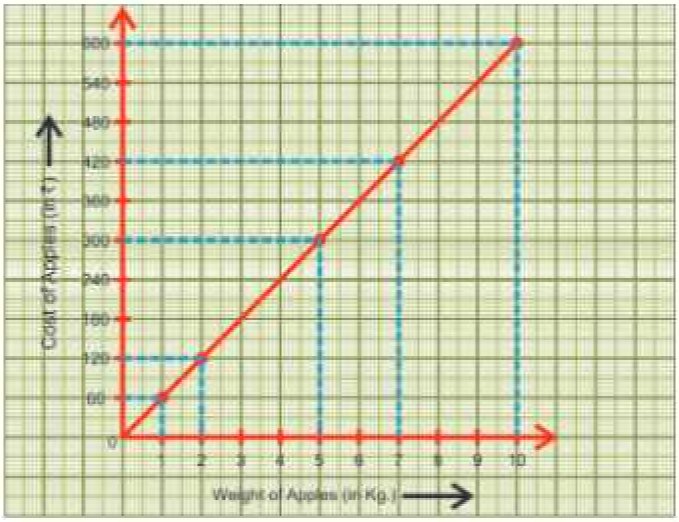


Figure 13.20

(i) Let us take suitable scale on both the axis (fig 15.20)

(ii) Mark weight of apples along x-axis

(iii) Mark cost of apples along y-axis

(iv) Plot the points (1, 60) ; (2, 120) ; (5 300) (7, 420) ; (10, 600)

(v) Join the Points we find that graph is a line.

From graph we can see that cost of 6kg of apples is ₹360 and cost of 8kg of apples is ₹480.

Example 13.10 : Gurpreet can ride a motorcycle with a uniform speed of 40 km/hr. Draw a time-distance graph for this and from graph find (a) time taken by Gurpreet to ride 100 km (b) The distance covered in 8 hours.

Sol. If Gurpreet is travelling with a uniform speed of 40 km/hr, from this we can make the following table.

Hours of Ride	Distance covered
1 hour	40km
2 hours	2 × 40 = 80km
3 hours	3 × 40 = 120km
4 hours	4 × 40 = 160km

Time (in hours)	1	2	3	4
Distance covered (in km)	40	80	120	160

Now

(i) Let us take a suitable scale on axes (fig 15.21).

(ii) Mark time along x-axis

(iii) Mark distance travelled along y-axis

(iv) Plot the point (1, 40) (2, 80) (3, 120) (4, 160)

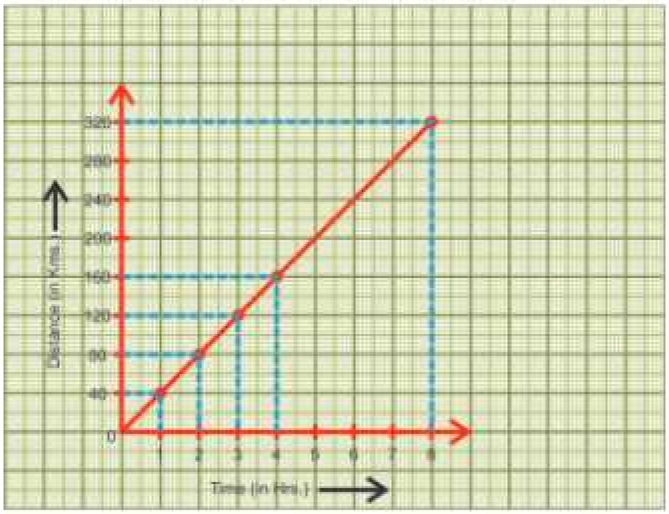


Figure 13.21

Now

(a) **Time taken by Gurpreet to ride 100km:** For 100km on y-axis, we get the corresponding time of 2.5 hours on horizontal axis. So required, time to cover a distance of 100km is 2.5 hours.

(b) Corresponding to 8 hours on x-axis there is 320km on y-axis

Exercise 13.3

1. Draw the graph of the following

(i)

Side of square in (cm)	3	4	5	6	7	8
Perimeter (in cm)	12	16	20	24	28	32

Is it a linear graph?

(ii)

Side of square (in cm)	3	4	5	6	7
Area (in cm²)	9	16	25	36	49

Is it a linear graph?

2. Draw the graphs of the following tables of values.

(i) Distance travelled by a car

Time (in hours)	6am.	7am.	8am.	9am.
Distance (in km)	50	100	150	200

Whether it is a linear graph?

(ii) Interest on a deposits for a year

Deposit (in ₹)	5000	10000	15000	20000
Simple interest (in ₹)	350	700	1050	1400

(a) Does the graph pass through the origin?

(b) Use the graph to find the interest on ₹ 30,000 for a year

(iii) Cost of Sugar

Weight (in kg)	1	2	3	4	5	6
Cost (in Rs)	17	34	51	68	85	102

(a) Use graph to find the cost of 10kg of sugar

(b) What amount of sugar can be purchased with ₹136.

3. Yash can drive a car constantly at a speed of 80km/h. Draw a time distance graph for this situation. Use it to find.

(i) The time taken by Yash to drive 200km.

(ii) The distance covered by him in $3\frac{1}{2}$ hours.

4. A bank gives 10% simple interest on deposits. Draw a graph to illustrate the sum deposited and simple interest earned. Find from graph.

(i) Annual interest obtain able for an investment of ₹250.

(ii) The investment one has to make to get an annual simple interest of ₹70.

Learning Outcomes

After completion of the chapter students will be able to:

- ***Read and interpret data from different types of graphs used in daily life.***
- ***Understand the trends and compare the information given in graphs.***
- ***Understand the concept of x-axis, Y-axis, origin etc. in cartesian co-ordinate system.***
- ***Plot points on the plane and read coordinates of a point from a graph.***

Exercise 13.1

1. (i) E (ii) B (iii) D (iv) C (v) G
(vi) A (vii) F

4. (a) lies on a line (b) lies on a line
(c) lies on a line (d) does not lie on a line

5. A (2, 0), B(2, 3), C(0, 3), P(4, 1), Q(6, 2), R(4, 4), S(5, 5)

7. (i) T (ii) T (iii) T (iv) F (v) F

8. (i) a (ii) d (iii) a (iv) b (v) c

Exercise 13.2

1. (i) 101° and 99°

(ii) At 12 noon and 1p.m.

(iii) At12 noon and 1p.m.

2. (i) (i) 2 crore (ii) 3 crore (iii) 3 crore (iv) 6 crore

(ii) 1 crore

(iii) No

3. (i) Distance covered in different times (ii) 4 hours

(iii) 20km (iv) Yes, He stopped at 10 am.

4. (i) yes in 2nd match

(ii) yes in 7th match

(iii) Batsman A

5. (i) Tuesday, Friday and Sunday

(ii) 40°C

(iii) Saturday

6. (i) a (ii) d (iii) a (iv) a (v) a

Exercise 13.3

1. (a) yes (b) No

2. (a) yes (b) (i) yes (ii) ₹2100

(c) (i) ₹170 (ii) 8kg

3. (i) $2\frac{1}{2}$ hours (ii) 280km

4. (i) ₹25 (ii) ₹700

NOTES

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